Branch:- MECHANICAL ENGINEERING

Semester:- 5th Sem

Subject:- DESIGN OF MACHINE ELEMENTS

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SYLLABUS

1.0 Introduction:

1.1 Introduction to Machine Design and Classify it.
1.2 Different mechanical engineering materials used in design with their uses and their mechanical and physical properties.
1.3 Define working stress, yield stress, ultimate stress & factor of safety and stress –strain curve for M.S & C.I.
1.4 Modes of Failure (By elastic deflection, general yielding & fracture)
1.5 State the factors governing the design of machine elements.
1.6 Describe design procedure.

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3.2 State materials for shafts.
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3.4 State standard size of shaft as per I.S.
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4.1 Design of Shaft Coupling

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1ST CHAPTER

INTRODUCTION

Machine design is an area of study in which one learns to design machine parts by utilizing its whole knowledge and experience in field of mechanical engineering.

CLASSIFICATION OF MACHINE DESIGN:

The machine design may be classified as follows :

1. Adaptive design

In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. Development design:

This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. New design:

This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.

The designs, depending upon the methods used, may be classified as follows:

(a) Rational design: This type of design depends upon mathematical formulae of principle of mechanics.

(b) Empirical design: This type of design depends upon empirical formulae based on the practice and past experience.

(c) Industrial design: This type of design depends upon the production aspects to manufacture any machine component in the industry.

(d) Optimum design: It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(e) System design: It is the design of any complex mechanical system like a motor car.

(f) Element design: It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(g) Computer aided design. This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.
2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals, and (*b*) Non-ferrous metals

The *ferrous metals* are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The *non-ferrous* metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

Physical Properties of Metals:

1. Metals are malleable and ductile.
2. Metals are good conductors of heat and electricity.
3. Metals are shiny and can be polished.
4. Metals are solids at room temperature. (except mercury, which is liquid)
5. Metals are tough and strong.

Physical properties of non-metals:

1. Mon-metals are brittle. They are neither malleable nor ductile.
2. Mon-metals are bad conductors of heat and electricity (except graphite).
3. Non-metals are dull and cannot be polished. (Expect Iodine).
4. Non-metals may be solids, liquids, gases at room temperature.
5. Non-metals are neither tough strong.

Mechanical Properties of Metals:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

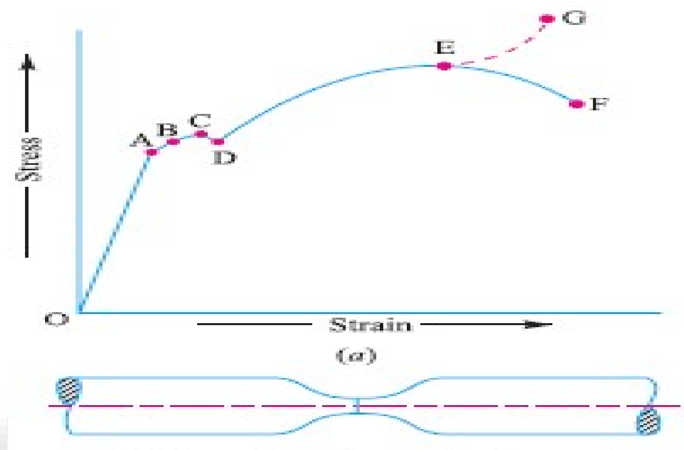
3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation. Cast iron is a brittle material.
7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.
8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.
9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting
different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.
10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for
spring materials.
11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in designing internal combustion engines, boilers and turbines.
12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.
13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:
(a) Brinell hardness test,
(b) Rockwell hardness test,
(c) Vickers hardness (also called Diamond Pyramid) test, and
(d) Shore scleroscope.

Stress-strain Diagram:



(b) Shape of specimen after elongation.

1. ***Proportional limit.*** We see from the diagram that from point *O* to *A* is a straight line, which represents that the stress is proportional to strain. Beyond point *A*, the curve slightly deviates from the straight line. It is thus obvious, that Hooke's law holds good up to point *A* and it is known as ***proportional limit***. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

2. ***Elastic limit.*** It may be noted that even if the load is increased beyond point *A* upto the point *B*, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point *B*. This point is known as ***elastic limit.*** It is defined as the stress developed in the material without any permanent set.

Note: Since the above two limits are very close to each other, therefore, for all practical purposes these are taken to be equal.

3. ***Yield point*.** If the material is stressed beyond point *B*, the plastic stage will reach *i.e*. on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point *B*, the strain increases at a faster rate with any increase in the stress until the point *C* is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to *D*, immediately after yielding commences. Hence there are two yield points *C* and *D*. The points *C* and *D* are called the ***upper*** and ***lower yield points*** respectively. The stress corresponding to yield point is known as ***yield point stress.***

4. ***Ultimate stress.*** At *D,* the specimen regains some strength and higher values of stresses are required for higher strains, than those between *A* and *D*. The stress (or load) goes on increasing till the point *E* is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At *E*, the stress, which attains its maximum value is known as ***ultimate stress*.** It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

5. ***Breaking stress.*** After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in Figure.

A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point *F.* The stress corresponding to point *F* is known as ***breaking stress.***

Working Stress:

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the ***working stress*** or ***design stress.*** It is also known as ***safe*** or ***allowable stress.***

Factor of Safety:

It is defined, in general, as the ratio of the maximum stress to the working stress.
Mathematically,

Factor of safety = Maximum stress / Working or design stress

In case of ductile materials *e.g*. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

Factor of safety = Yield point stress / Working or design stress

In case of brittle materials *e.g*. cast iron, the yield point is not well defined as for ductile materials.
Therefore, the factor of safety for brittle materials is based on ultimate stress.

Factor of safety = Ultimate stress / Working or design stress

This relation may also be used for ductile materials.
Note: The above relations for factor of safety are for static loading.

Three modes of failure:

(a) Failure by elastic deflection,
(b) Failure by general yielding,
(c) Failure of fracture.

(a) Failure of elastic deflection. In applications like transmission shaft supporting gears, the maximum acting on the shaft, without, affecting its performance, is limited by the permissible elastic deflection. Lateral or torsional rigidity is considered as the criterion of design in such cases.

(b) Failure of general yielding: A mechanical component made of ductile material loses its engineering usefulness due to large amount of plastic deformation after the yield point stress is reached. Considerable portion of the component is subjected to plastic deformation called general yielding.

(c) Failure by fracture: Components made of 1 brittle material cease to function satisfactorily because of the sudden fracture without any plastic deformation.

Factors governing the design of machine elements:

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be:
(a) Rectilinear motion which includes unidirectional and reciprocating motions.
(b) Curvilinear motion which includes rotary, oscillatory and simple harmonic.
(c) Constant velocity.
(d) Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc.

4. ***Form and size of the parts.*** The form and size are based on judgement. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. ***Frictional resistance and lubrication.*** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. ***Convenient and economical features.*** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production, or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue. Design considerations play important role in the successful production of machines expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of hand-made samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions, should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

General Procedure in Machine Design:

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

1. ***Recognition of need.*** First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. ***Synthesis*** **(*Mechanisms*).** Select the possible mechanism or group of mechanisms which will give the desired motion.

3. ***Analysis of forces.*** Find the forces acting on each member of the machine and the energy transmitted by each member.

4. ***Material selection.*** Select the material best suited for each member of the machine.

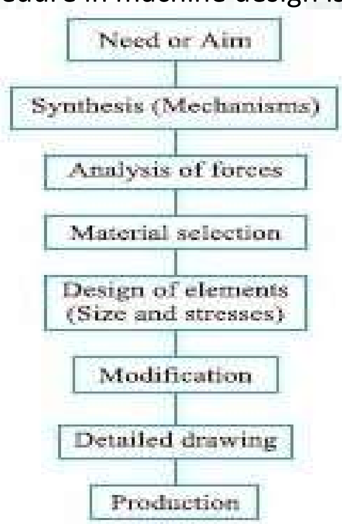
5. ***Design of elements*** **(*Size and Stresses*).** Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. ***Modification.*** Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. ***Detailed drawing.*** Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. ***Production.*** The component, as per the drawing, is manufactured in the workshop.

The flow chart for the general procedure in machine design is shown in figure.



2ND CHAPTER

Design of fastening elements

Joint: A mechanical joint is a type of joint that connects one or more mechanical surfaces. Machines such as lathes and milling machines contains hundreds or thousands of individual moving parts. In areas where these components meet, a mechanical joint is to be used. The defining characteristic of mechanical joints is that they are used exclusively to connect multiple mechanical parts.

Mechanical joints are of two types. Permanent and temporary joints.

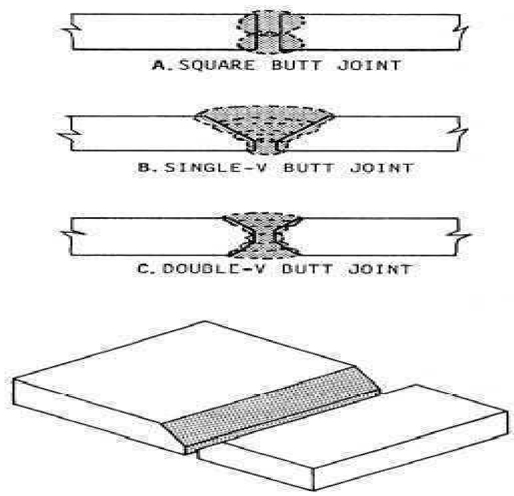
Temporary Joining	Permanent Joining
A temporary joint can be dismantled without breaking the assembled parts.	A permanent joint cannot be dismantled without breaking parts.
Temporary joining is beneficial where frequent assembly and disassembly are required.	Permanent joining is beneficial where joint is intended to stay fixed for longer period.
Strength of temporary joint is comparatively lower.	Permanent joint offers stronger joining.
Temporary joints are not leak-proof.	Most permanent joining processes provide leak-proof joints.
Temporary joining processes offer easy and cost efficient inspection, repair and maintenance as parts can be dismantled without breaking.	Inspection, repair and maintenance are difficult when structures are joined permanently as disassembly is not possible without breaking.
Examples of various temporary joining techniques: • Fasteners • Press fit • Cotter joint • Knuckle joint.	Examples of various permanent joining techniques: • Welding • Brazing and soldering • Riveting • Coupling

Welded joint-

The joints that are made using welding process is known as welded joint. Welded joints are classified into two categories.

1) **Butt joint** –A butt joint is a joint where two pieces of metal are placed together in the same plane, and the side of each metal is joined by welding. A butt weld is the most common type of joint that is used in the fabrication of structures and piping systems. It's fairly simple to prepare, and there are many different variations that can be applied to achieve the desired result.
   - ✓ When thickness of plate is less than 5 mm, there is no need to bevel the edges of the plate and it is called square butt joint.
   - ✓ When the thickness of the plates is between 5 to 25 mm, The the edges of the plates are bevelled before welding. The edge of the plate takes the shape of letter V. It is called V joint or single welded V joint.

When the thickness of the plates is more than 20 mm, the edges of the plates are machines to take the shape of U. It is done only from one side. It is called U joint.



- ✓ When the thickness of the plates is more than 30 mm, double welded V joint is used.

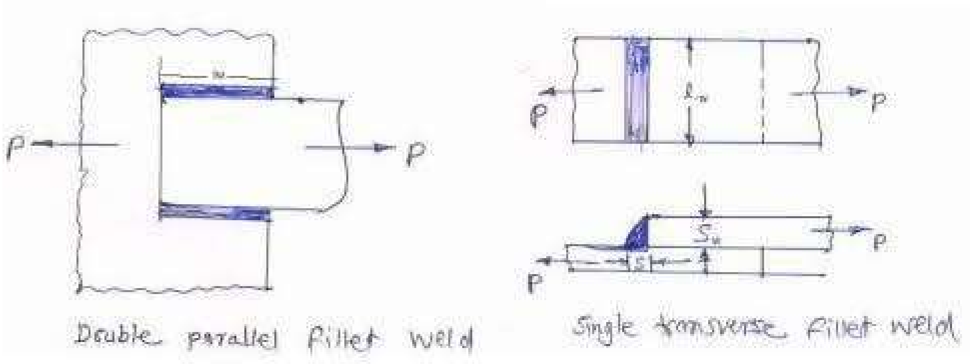
Lap joint - A lap welded joint is a joint in which two or more materials that are overlapped on top of one another. The edge of one material is melted and fused with the surface of another material. A lap weld is categorized in the fillet weld category.

Lap welds are commonly used in welding processes that involve automation. The lap joint, which is used when making a lap weld, is very forgiving to varying part dimensions. That is because satisfactory lap weld fit-up is easily achieved and the member whose flat surface is being welded to the edge of the other material acts as a backing material that reduces the risk of blowing through the weld joint.

Types of lap weld/fillet weld:

- ✓ **Longitudinal (parallel) and transverse fillet weld.**

  When the applied force is parallel to the length of the weld is called longitudinal fillet weld. When the applied force is perpendicular to the length of the weld is called transverse fillet weld



Advantages and Disadvantages of Welded Joints over Riveted Joints:

Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.
6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (i.e. circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

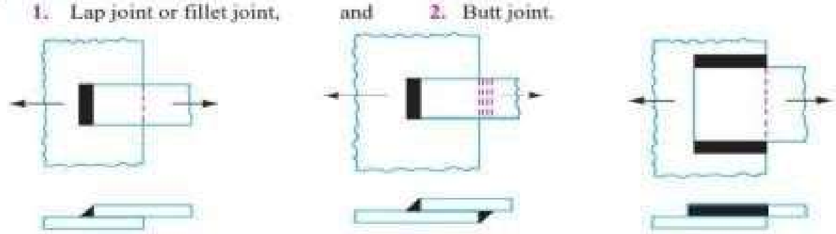
Disadvantages

1. As there is an uneven heating and cooling during fabrication, therefore the members gets distorted or additional stresses can be developed.
2. It requires a highly skilled labour and supervision.
3. As there is no provision kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

Types of Weld joint.

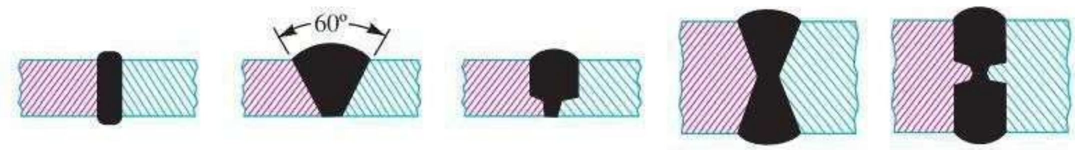
Type of Lap joint (plate over plate joints)

1. Lap joint or fillet joint, and 2. Butt joint.



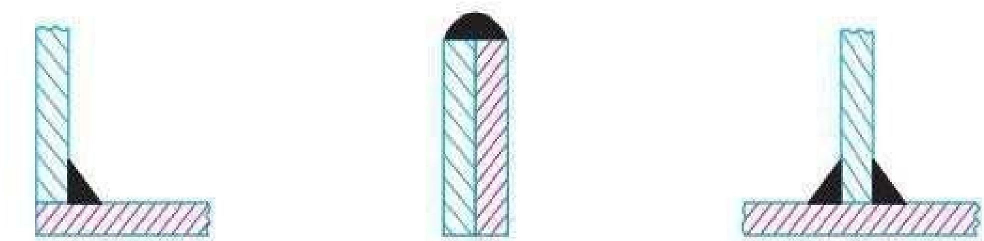
(a) Single transverse. (b) Double transverse. (c) Parallel fillet.

Butt joint (End on end plate joints)



(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

Special type of weld joint:



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

Strength of Transverse Fillet Welded Joints

We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. 10.6 (*a*) and (*b*) respectively.

Transverse fillet welds.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*. The enlarged view of the fillet is shown in figure. The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e. BD*) is known as ***throat thickness.*** The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),

s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

From Fig. 10.7, we find that the throat thickness,

$t = s \times \sin 45° = 0.707\ s$

∴ Minimum area of the weld or throat area,

A = Throat thickness × Length of weld = $t \times l = 0.707\ s \times l$

If σt is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

P = Throat area × Allowable tensile stress = 0.707 $s \times l \times \sigma t$
and tensile strength of the joint for double fillet weld,
$P = 2 \times 0.707\ s \times l \times \sigma t = 1.414\ s \times l \times \sigma t$
Note: Since the weld is weaker than the plate due to slag and blow holes, therefore the weld is given a reinforcement which may be taken as 10% of the plate thickness.

Strength of Parallel Fillet Welded Joints:

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig. 10.8 (*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

$A = 0.707\ s \times l$

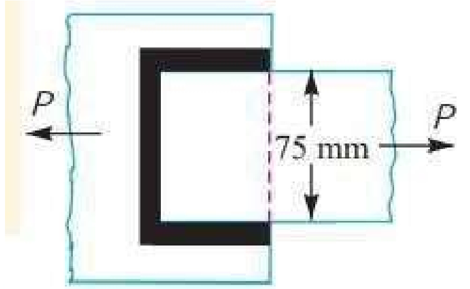
If l is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,
P = Throat area × Allowable shear stress = 0.707 $s \times l \times \tau$
and shear strength of the joint for double parallel fillet weld,
$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\ s \times l \times \tau$

1. A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

   Solution. Given: *Width = 100 mm ; Thickness = 10 mm ; P = 80 kN = 80 × 103 N; τ = 55 MPa = 55 N/mm2
   Let l =Length of weld, and s = Size of weld = Plate thickness = 10 mm ... (Given)
   We know that maximum load which the plates can carry for double parallel fillet weld (P), 80 × 103 = 1.414 × s × l × τ
   = 1.414 × 10 × l × 55 = 778 l
∴ l = 80 × 103 / 778 = 103 mm
   Adding 12.5 mm for starting and stopping of weld run, we have l = 103 + 12.5 = 115.5 mm (Ans.)

2. A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa.
   Solution. Given: *Width = 100 mm ; Thickness = 12.5 mm ; P = 50 kN = 50 × 103N ; τ = 56 MPa = 56 N/mm2.
   Let l = Length of weld,
   s = Size of weld = Plate thickness = 12.5 mm ... (Given)
   We know that the maximum load which the plates can carry for double parallel fillet welds P = 50 × 103 =1.414 s × l × τ
= 1.414 × 12.5 × l × 56
= 990 l
∴ l = 50 × 103 / 990
= 50.5 mm
   Adding 12.5 mm for starting and stopping of weld run, we have l = 50.5 + 12.5 = 63 mm (Ans.)

3. A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. 10.15. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld.



Solution. Given : Width = 75 mm ; Thickness = 12.5 mm ; σ_t = 70 MPa = 70 N/mm2; τ = 56 MPa = 56 N/mm2.

The effective length of weld (L 1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

∴ L 1 = 75 – 12.5 = 62.5 mm

Length of each parallel fillet for static loading

Let L2 = Length of each parallel fillet.

We know that the maximum load which the plate can carry is P = Area × Stress

=75×12.5×70=65625N

Load carried by single transverse weld,

P1 = 0.707 s × L 1 × σ_t

= 0.707 × 12.5 × 62.5 × 70

= 38 664 N

and the load carried by double parallel fillet weld,

P2 = 1.414 s × L2 × τ

= 1.414×12.5×L2×56=990L2N

Load carried by the joint (P),

65625=P1+P2

= 38664+990L2

L2 = 27.2 mm

Adding 12.5 mm for starting and stopping of weld run, we have L2 = 27.2 + 12.5 = 39.7 say 40 mm (Ans.)

Eccentrically Loaded Welded Joints

An external load whose line of action is parallel but does not coincide with the centroidal axis of the component/structure on which load is applied is known as an eccentric load. Whereas, the perpendicular distance between the centroidal axis and line of action of the load is known as eccentricity.

1. **Eccentrically loaded transverse fillet joint:**

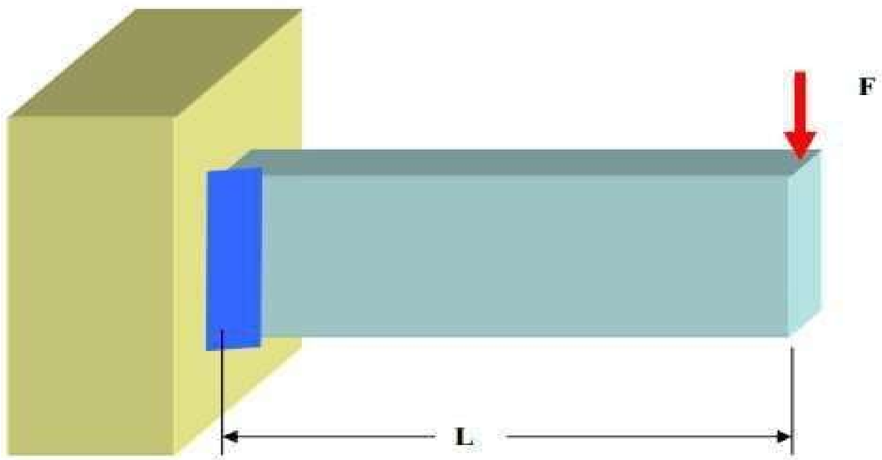


Figure 11.2.1: Eccentrically loaded welded joint

The joint will be subjected to the following two types of stresses:
1. Direct shear stress due to the shear force P or F acting at the welds, and
2. Bending stress due to the bending moment P (or F) × e.

(a) direct shear stress of magnitude $\frac{F}{2bt}$,

where b = length of the weld
t = thickness at the throat

(b) Indirect shear stress due to bending of the beam

the magnitude of the shear stress is

$$\tau = \frac{FLy}{I_y}$$

$I_y = tb^3/12$

$$\tau_{max} = \sqrt{\left(\frac{F}{2bt}\right)^2 + \left(\frac{3FL}{tb^2}\right)^2}.$$

For Shown eccentrically welded joint.

In order to design a safe welded joint

$$\tau_{max} \le S_S,$$

Where Ss = shear stress of the weld.

2. **Eccentrically loaded parallel fillet joint:**

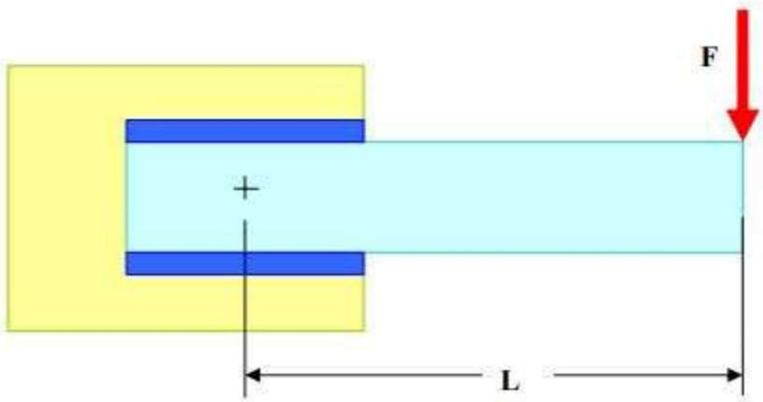


Figure 11.2.3: Eccentrically loaded parallel fillet joint

(a) Direct shear of magnitude $\frac{F}{2lt}$

where l = length of the weld

t = thickness of the throat.

(b) Indirect shear stress owing to eccentricity of the loading.

The shear stress at a point at a distance r from the centroid is given by

τ = cr

where the proportionality constant = c is to calculated using the moment equilibrium equation.

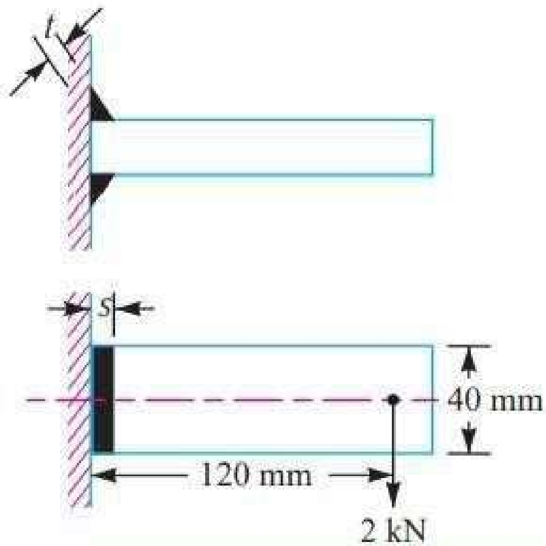
C = FL/J

J = polar moment of inertia of throat section about the centroid.

The net shear stress at a point is calculated by vector addition of the two kinds of shear stresses

The net stress is calculated using trigonometry or resolution of vectors. And then it is equated with the given stress value to get the required thickness.

1. A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.



Solution. Given: P = 2kN = 2000 N ; e = 120 mm ; l = 40 mm ; τmax = 25 MPa = 25 N/mm2

Let s = Size of weld in mm,

and t = Throat thickness.

The joint, as shown will be subjected to direct shear stress due to the shear force, P=2000N

and bending stress due to the bending moment of P × e.

We know that area at the throat,

A = 2t × l

= 2 × 0.707 s × l

= 1.414 s × l

= 1.414 s × 40

= 56.56 × s mm^2

∴ Shear stress, τ = 2000 /56.56 × s

=(35.4 /s)N/mm^2

Bending moment, M = P × e

= 2000 × 120

= 240 × 103 N-mm

Section modulus of the weld metal through the throat,

$$Z = \frac{t \times l^2}{6} \times 2 \qquad \text{...(For both sides weld)}$$

$$= \frac{0.707\, s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

$$Z = \frac{s \times l^2}{4.242} = \frac{s\ (40)^2}{4.242} = 377 \times s\ \text{mm}^3$$

$$\therefore \text{ Bending stress, } \sigma_b = \frac{M}{Z} = \frac{240 \times 10^3}{377 \times s} = \frac{636.6}{s}\ \text{N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore \qquad s = 320.3 / 25 = 12.8$ mm **Ans.**

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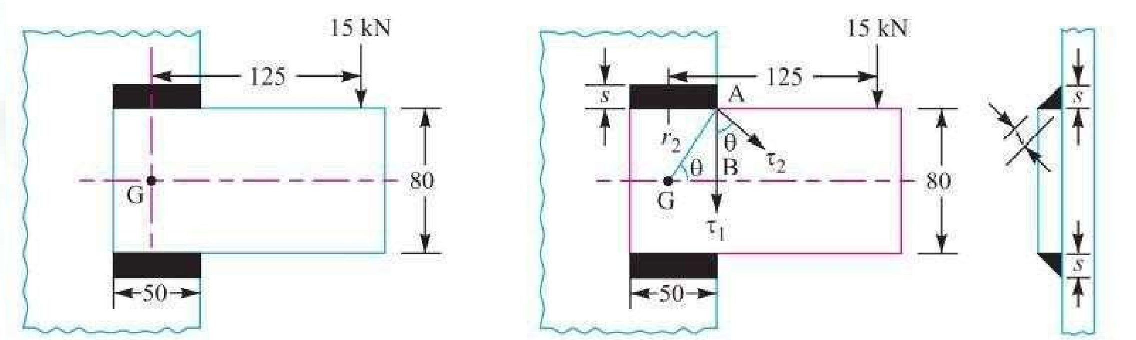
2. A bracket carrying a load of 15 kN is to be welded as shown in Fig. 10.28. Find the size of weld required if the allowable shear stress is not to exceed 80 MPa.



All dimensions in mm.

Solution. Given : P = 15 kN = 15 × 103 N ; τ = 80 MPa = 80 N/mm2 ; b = 80 mm ; l = 50 mm; e = 125 mm

Let s = Size of weld in mm,

and t = Throat thickness.

We know that the throat area,

$A = 2 \times t \times l = 2 \times 0.707\ s \times l = 1.414\ s \times l = 1.414 \times s \times 50 = 70.7\ s\ \text{mm}^2$

∴ Direct or primary shear stress,

$$\tau_1 = \frac{P}{A} = \frac{15 \times 10^3}{70.7\ s} = \frac{212}{s}\ \text{N/mm}^2$$

From design Data book we can find the polar moment of inertia of this type of section as

$$J = \frac{t.l\,(3b^2 + l^2)}{6} = \frac{0.707\ s \times 50\,[3\,(80)^2 + (50)^2]}{6}\ \text{mm}^4$$

$$= 127\,850\ s\ \text{mm}^4 \qquad \ldots (\because\ t = 0.707\ s)$$

From the picture,we find that AB = 40 mm and BG = r1 = 25 mm

∴ Maximum radius of the weld,

$$r_2 = \sqrt{(AB)^2 + (BG)^2} = \sqrt{(40)^2 + (25)^2} = 47\ \text{mm}$$

Shear stress due to the turning moment *i.e.* secondary shear stress,

$$\tau_2 = \frac{P \times e \times r_2}{J} = \frac{15 \times 10^3 \times 125 \times 47}{127\,850\ s} = \frac{689.3}{s}\ \text{N/mm}^2$$

and $$\cos\theta = \frac{r_1}{r_2} = \frac{25}{47} = 0.532$$

We know that resultant shear stress,

$$\tau = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\,\tau_1 \times \tau_2\ \cos\theta}$$

$$80 = \sqrt{\left(\frac{212}{s}\right)^2 + \left(\frac{689.3}{s}\right)^2 + 2 \times \frac{212}{s} \times \frac{689.3}{s} \times 0.532} = \frac{822}{s}$$

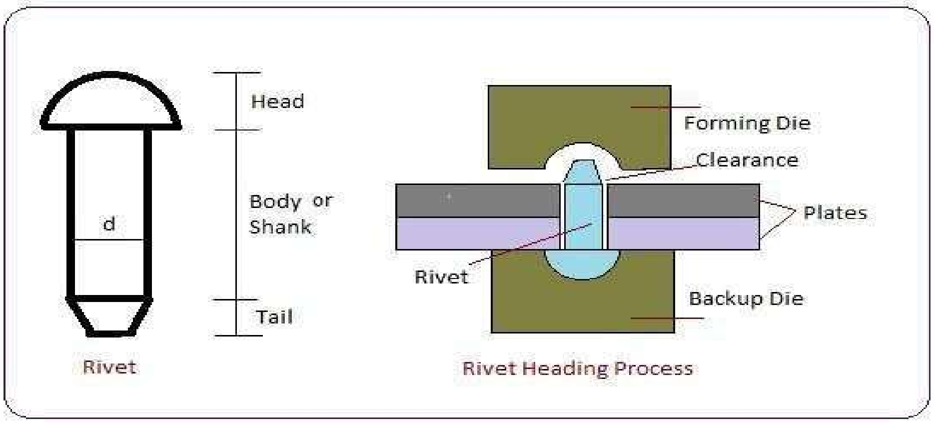
∴ $$s = 822 / 80 = 10.3\ \text{mm}$$ **Ans.**

Riveted joints

A riveted joint is a permanent joint with mainly two components (parts to be joined) which are held together by a rivet with the head at top and tail at the bottom.

Geometry of rivet

In simple language rivet is short cylindrical bar with integral head at top.



Rivet Heading Process(Riveting):

- Rivet heading process is done with the help of forming die and backup die which keep a rivet in between them and by application of force, the rivet is set in the parts to be joined.
- Equal and opposite force makes rivet to deform and tail part of the rivet is converted to head at the bottom so, the complete rivet is seated in the plates.
- In this riveting process, the tail of rivet is converted to 'head' which is sometimes called 'shop head'.
- For riveting parts to be joined are first drilled with the help of drilling machine. Clearance is taken into consideration while riveting because by pressing application diameter of the rivet is somewhat increased.
- Usually clearance is considered as per following:

  If diameter of rivet, d = 12 to 24 mm, Clearance, C = 1.5 mm If
  diameter of rivet, d = 24 to 48 mm, Clearance, C = 2 mm

Rivet material:

- Usually rivet is made up of wrought iron or soft steel due to lower hardness which is necessary to have easy deformation during riveting.

- Sometimes copper, aluminum are used in corrosive environment. Only material requirements are ductility, toughness and hardness.

Manufacturing process- Rivets are made from rolled bars by process of cold heading or cold upsetting.

Types of riveted joints:

There are mainly two types of riveted joints, based on the rivet arrangement.

1. Lap joints

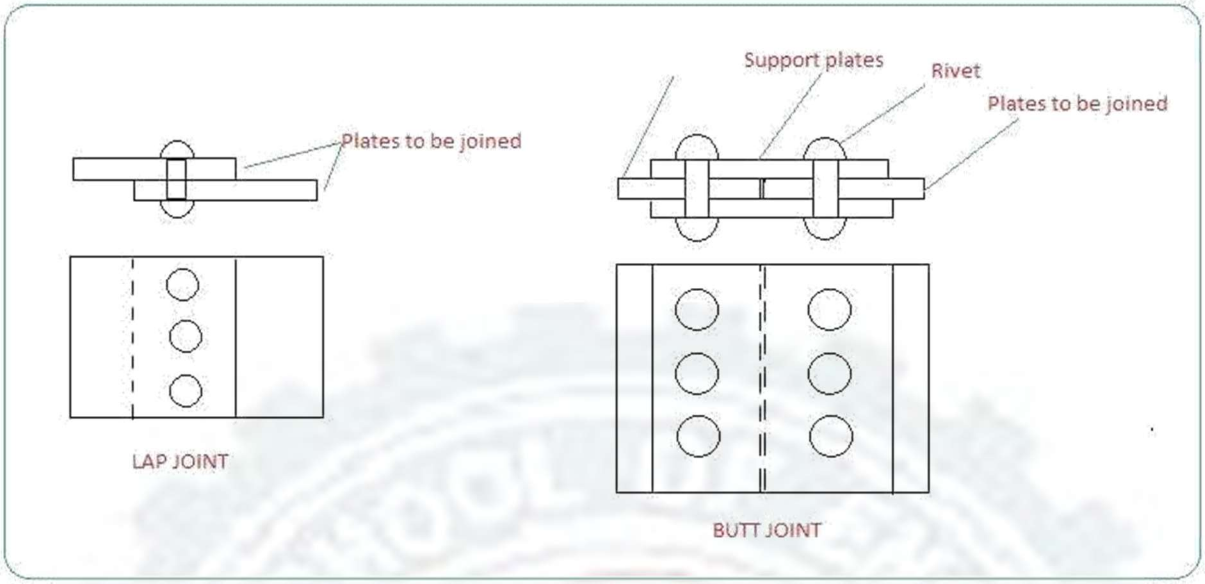
2. Butt joints

- A Lap Joint Is a joint in which one Plate is kept over the other And the Two Plates Are Riveted together.

When The Joint Is Made With Only One Row Of Rivets, It Is Called A Single-riveted Lap Joint.

. A butt joint is a joint in which the two plates to be connected are kept in alignment butting (touching) each other and a cover plate is placed on one side or on both the sides of the two main plates to be joined . The cover plate(s) are then riveted to the main plates. . At least two rows of rivets, one in each connected plate, are necessary to make the joint. The butt joints may be single strap butt joint or double strap butt joints

Both joints are also sub-classified into single riveted and double riveted. Sometimes based on joints strengths, triple-riveted are also possible. Single riveted means one row of the rivet in joint.



Advantages of riveted joints:

- Cheaper fabrication cost
- Low maintenance cost
- Dissimilar metals can also be joined, even non-metallic joints are possible with riveted joints.
- Ease of riveting process.

Disadvantages of riveted joints:

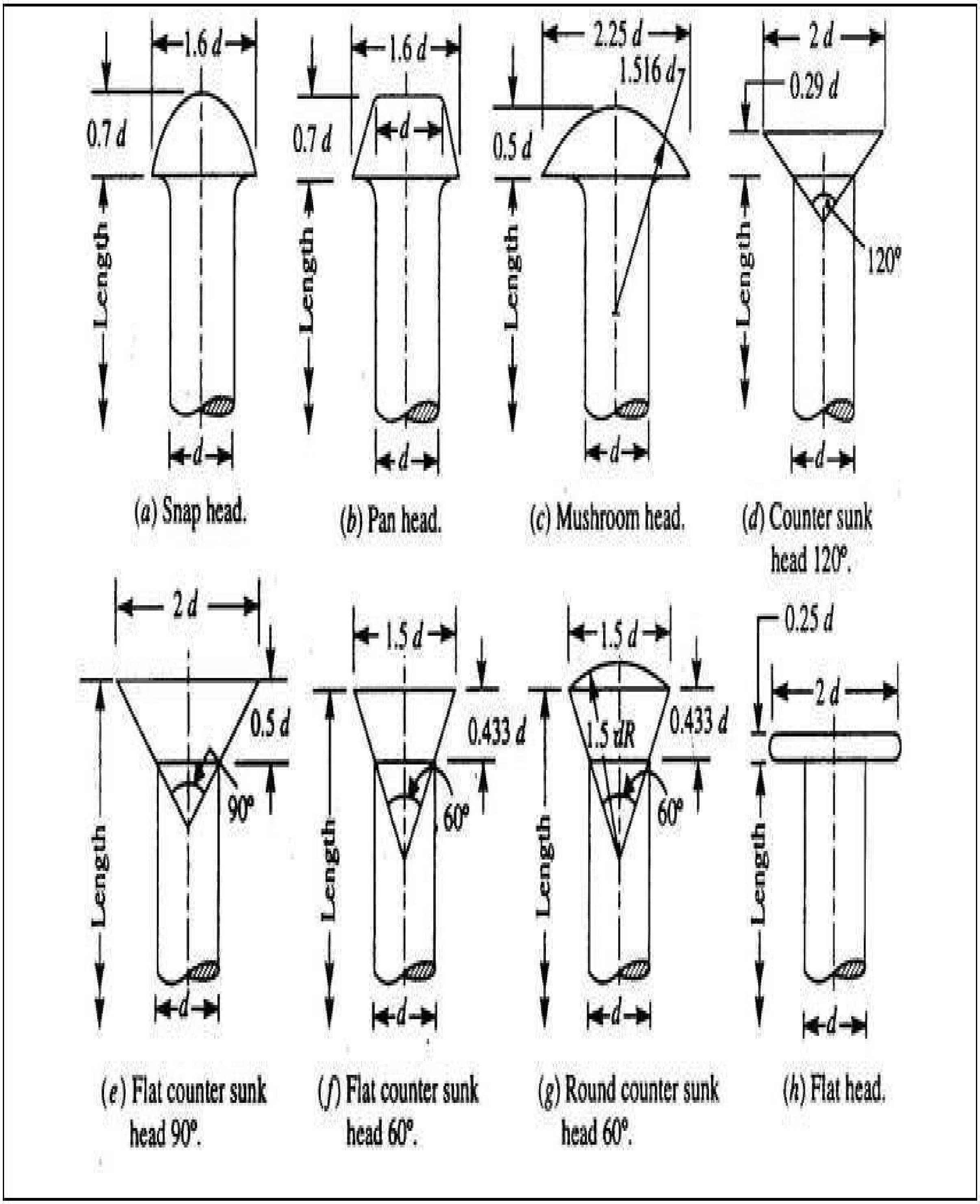
- Skilled workers required
- Leakage may be a problem for this type of joints, but this is overcome by special techniques.

Applications of riveted joints:

- Boiler shells
- Structures members and bridges parts

- Railway wagons and coaches
- Buses and trucks

Types of rivets(according to rivet heads)



Process of producing leak proof joints

The riveted joints are used to produce leak proof joints in case of boilers and ship building works. These joints are made by the processes namely caulking and fullering. The edges of the plates are hammered and driven-in by a caulking tool or a fullering tool. The caulking tool is in the shape of a blunt chisel. Leakage through the hole is prevented by the caulking operation on the edge of the rivet-head The thickness of the fullering tool is about the same as that of the plates. To facilitate these operations the edges of the plates are usually machined to an angle of about 80° before joining them together. This angle is increased to about 85° after the fullering process.

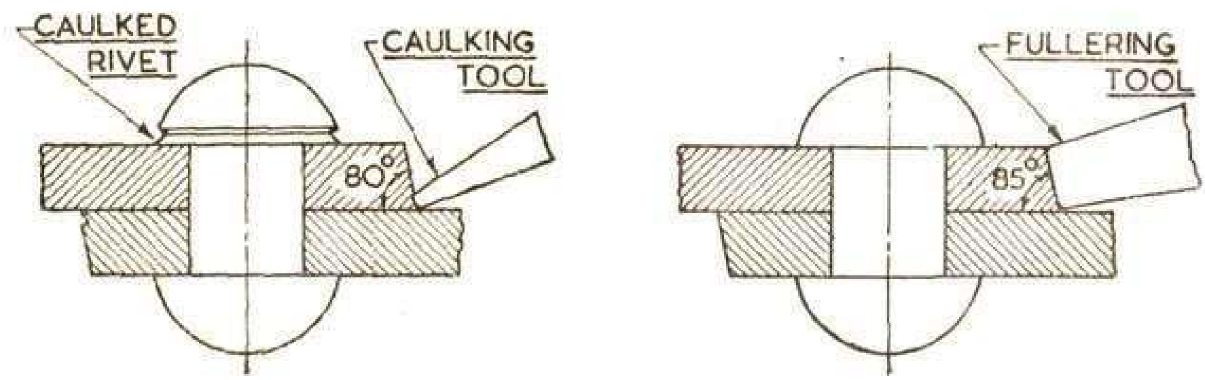


Fig.6.6.Caulking and Fullering

Important Formulae of Riveted joints

<u>Tearing strength or Ultimate tearing resistance of the plate per pitch</u>

$P_t = (p - d)t \times \sigma_t$

<u>Shearing Strength or Ultimate shearing resistance of the rivets per pitch</u>

$P_s = n \times \pi / 4 \times d^2 \times \tau$

……..(single shear)

$= n \times 2 \times \pi / 4 \times d^2 \times \tau$……..(Theoretically, in double shear)

$= n \times 1.875 \times \pi / 4 \times d^2$

$\times \tau$……. (In double shear, according to IBR)

<u>Crushing strength or Ultimate crushing resistance of the rivets per pitch</u>

$P_c = n \times d \times t \times \sigma_c$

Strength of the riveted jont = Least of Pt, Ps and Pc

<u>Strength of the un-riveted or solid plate per pitch length</u>

$P = p \times t \times \sigma_t$

<u>Efficiency of the riveted joint</u>

η = Least of (pt, ps & pc) / $(p \times t \times \sigma_t)$

<u>Design of rivet joints</u>:

The design parameters in riveted joints are d, p, and m .

Diameter of the hole (d):

When thickness of the plate (t) is more than 8 mm, Unwin's formula is

used,

$d = 6\sqrt{t}$ mm.

Otherwise is obtained by equating crushing strength to the shear strength of the joint. In a double riveted zigzag joint, this implies

$P_c = n \times d \times t \times \sigma_c = P_s = n \times \pi / 4 \times d^2 \times \tau$

(valid for $t < 8$ mm)

Pitch (p):

Pitch is designed by equating the tearing strength of the plate to the shear strength of the rivets. In a double riveted lap joint, this takes the following form.

$\sigma_t \times (p-d)\, t = 2 \times \pi / 4\, d^2 \times \sigma_s$

But $p \geq 2d$ in order to accommodate heads of the rivets.

Margin (m):

m =1.5 d

In order to design boiler joints, a designer must also comply with

Indian Boiler Regulations (I.B.R.).

(pb usually 0.33 +0.67 d mm)

1. A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume

   σ_t = 80 MPa ; τ = 60 MPa ; and σ_c = 120 MPa. State how the joint will fail and find the efficiency of the joint.

Solution. Nature of Joint: double riveted lap joint with zig-zag riveting

Given data : t = 13 mm ; σ_t = 80 MPa = 80 N/mm2; τ = 60 MPa = 60 N/mm2; σ_c = 120 MPa = 120 N/mm2
1**. Diameter of rivet**

Since the thickness of plate is greater than 8 mm,

therefore diameter of rivet hole, $d = 6\sqrt{t} = 6\sqrt{13} = 21.6$ mm
From design data book, the standard size of the

rivet hole (d) is 23 mm and the corresponding diameter of the rivet is 22 mm.

Pitch of rivets

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting , therefore there are two rivets per pitch length, i.e. n = 2. Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,
$P_t = (p - d)\ t \times \sigma_t = (p - 23)\ 13 \times 80 = (p - 23)\ 1040$ N..........................(i)

and shearing resistance of the rivets, $P_s = n \times \pi/4\ d^2 \times \tau$
$= 2 \times \pi/4\ (23)^2$

$60 = 49\ 864$N ……………..(ii) …

(There are two rivets in single shear)

From equations (i) and (ii),

$p - 23 = 49864/1040 = 48$ or

$p = 48 + 23 = 71$ mm

The maximum pitch is given by, $p_{max} = C \times t + 41.28$ mm

From design data book, we find that for 2 rivets per pitch length, the value of C is 2.62.

$\therefore\ p_{max} = 2.62 \times 13 + 41.28 = 75.28$ mm

Since p max is more than p, therefore we shall adopt p = 71 mm Ans.

Distance between the rows of rivets

We know that the distance between the rows of rivets (for zig-zag riveting), p_b
$= 0.33\ p + 0.67\ d = 0.33 \times 71 + 0.67 \times 23$ mm $= 38.8$ say 40 mm Ans.

Margin

We know that the margin,

$m = 1.5\ d = 1.5 \times 23 = 34.5$ say 35 mm Ans.

Tearing resistance of the plate

$P_t = (p - d)\ t \times \sigma_t = (71 - 23)13 \times 80 = 49\ 920$ N

Shearing resistance of the rivets

$P_s = n \times \pi/4 \times d^2 \times \tau = 2 \times \pi/4\ (23)^2 \times 60 = 49\ 864$ N

Crushing resistance of the rivets

$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\ 760$ N

The least of Pt, Ps and Pc is Ps = 49 864 N.

Hence the joint will fail due to shearing of the rivets.

Strength of the un riveted plate per pitch length

$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73840$ N

Efficiency of joint =Least of (P_t,P_s and P_c)/p= 49 864 / 738

Efficiency of the joint = 0.675 or 67.5%

3RD CHAPTER
Design of shafts and Keys

Design of shaft

Shaft is a common machine element which is used to transmit rotary motion or torque. It generally has circular cross-section and can be solid or hollow. Shafts are supported on the bearings and transmit torque with the help of gears, belts and pulleys etc. Shafts are generally subjected to bending moment, torsion and axial force or a combination of these three. So the shafts are designed depending upon the combination of loads it is subjected to. Spindle stub and axle are some important types of shaft. Small shaft is called spindle. Shaft integral part of the prime mover is called stub shaft. An axle is a non-rotating member that carries no torque and is used to support rotating wheels, pulleys etc. And therefore it is subjected to bending moment only.

Shaft Materials

Hot-rolled plain carbon steel is the least expensive material used for shafts. These essentially require machining to remove the scales of hot rolling process. Cold rolled plain carbon steel provides better yield strength and endurance strength but the cold working induces residual stresses. Surface is smooth in this case and amount of machining therefore is minimal. It is used for general purpose transmission shafts. When a shaft is to work under severe loading and corrosive conditions and require more strength, alloy steels are used, generally having Ni, Cr, Mo and V as alloying elements. Alloy steels are expensive. Sometimes shafts are heat treated to improve hardness and shock resistance and surface hardening techniques are also used if high wear resistance is the requirement. As the shafts transmitting power are subjected to fatigue loading, therefore higher factor of safety of 3 to 4 is used on the basis of yield strength for static load analysis.

Design of shafts

Shafts are designed on the basis of strength or rigidity or both. Design based on strength is to ensure that stress at any location of the shaft does not exceed the material yield stress. Design based on rigidity is to ensure that maximum deflection (because of bending) and maximum twist (due to torsion) of the shaft is within the allowable limits. Rigidity consideration is also very important in some cases for example position of a gear mounted on the shaft will change if the shaft gets deflected and if this value is more than some allowable limit, it may lead to high dynamic loads and noise in the gears.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to torque

(b) Shafts subjected to bending moment

(c) Shafts subjected to combination of torque and bending moment

(d) Shafts subjected to axial loads in addition to combination of torque and bending moment

Shafts Subjected to Torque

Maximum shear stress developed in a shaft subjected to torque is given by,

$$\tau = \frac{T r}{J} \leq [\tau]$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation

$= \frac{\pi d^4}{32}$ for solid shafts with diameter d

$= \frac{\pi(d_o^4 - d_i^4)}{32}$ for hollow shafts with do and di as outer and inner

diameter.

r = Distance from neutral axis to the outer most fibre = d/2 (or do/2)

So dimensions of the shaft subjected to torque can be determined from above relation for a known value of allowable shear stress, [τ].

1. Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Ans-Given : P = 20 kW = 20 × 103 W ; N = 200 r.p.m.

τu = 360 MPa = 360 N/mm2 ; Factor of safety. = 8

$k = di / do = 0.5$

We know that the allowable shear stress,

$\tau = 360 /8$

$\tau = 45$ N/mm2

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

$\therefore$ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\,d_o = 0.5 \times 50 = 25$ mm **Ans.**

Lecture notes

Subject-Design of machine elements

Sem-5th Mechanical diploma

<u>Design of shaft</u>

Shafts Subjected to Bending Moment

Maximum bending stress developed in a shaft is given by,

$$\sigma_b = \frac{M\ y}{I} \leq [\sigma_t]$$

where M = Bending Moment acting upon the shaft,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation

$$\frac{\pi d^4}{64}$$

= for solid shafts with diameter d

= for hollow shafts with do and di as outer and inner

diameter.

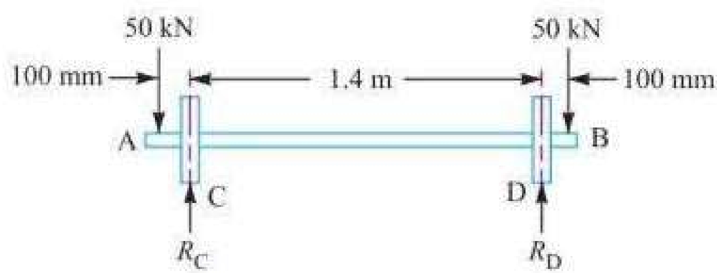
y = Distance from neutral axis to the outer most fibre = d / 2 (or do/2)

So dimensions of the shaft subjected to bending moment can be determined from above relation for a known value of allowable tensile stress.

1. A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Given : $W = 50$ kN $= 50 \times 10^3$ N ; $L = 100$ mm ; $x = 1.4$ m ; $\sigma_b = 100$ MPa $= 100$ N/mm^2

Ans-



Maximum bending moment- load × distance

$M = W \times l$

$= 50 \times 10^3 \times 100$

$= 5 \times 10^6$ N-mm

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

Shafts Subjected to Combination of Torque and Bending Moment

When the shaft is subjected to combination of torque and bending moment, principal stresses are calculated and then different theories of failure are used. Bending stress and torsional shear stress can be calculated using the above relations.

$$\tau = \frac{T\,r}{J} = \frac{T\,\frac{d}{2}}{\frac{\pi}{32} d^4} = \frac{16\,T}{\pi d^3}$$

$$\sigma_b = \frac{M\,y}{I} = \frac{M\,\frac{d}{2}}{\frac{\pi}{64} d^4} = \frac{32\,M}{\pi d^3}$$

Maximum Shear Stress Theory

Maximum shear stress is given by,

$$\tau_{max.} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \sqrt{\left(\frac{16\,M}{\pi d^3}\right)^2 + \left(\frac{16\,T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2} \leq [\tau]$$

$\sqrt{M^2 + T^2}$ is called equivalent torque, T_e, such that

$$\tau_{max.} = \frac{T_e\, r}{J} \leq [\tau]$$

Maximum Principal Stress Theory

Maximum principal stress is given by,

$$\sigma = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau)^2} = \frac{16\, M}{\pi d^3} + \sqrt{\left(\frac{16\, M}{\pi d^3}\right)^2 + \left(\frac{16\, T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[M + \sqrt{M^2 + T^2}\right] \leq [\sigma_t]$$

$\left[M + \sqrt{M^2 + T^2}\right]$ is called equivalent bending moment, M_e, such that

$$\sigma = \frac{M_e\, y}{I} \leq [\sigma_t]$$

1. A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa.

   Assuming a factor of safety as 6, determine the diameter of the shaft.

Ans-

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \text{ or } d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \text{ or } d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Sub-Design of machine elements

Sem-5th sem mechanical Diploma

Design of coupling-

Coupling-

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Use of Coupling

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and gen erator and to provide for disconnec tion for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

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Requirements of a Good Shaft Coupling

1. It should be easy to connect or disconnect.

2. It should transmit the full power from one shaft to the other shaft without losses.

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Types of Shafts Couplings

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. e.g- sleeve coupling,flange coupling,compression coupling

2. Flexible coupling-It is used to connect two shafts having both lateral and angular misalignment. E.g-Bushed ,Oldham couplings

Sleeve or Muff-coupling-

It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key.

The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. So all the elements must be strong enough to transmit the torque.

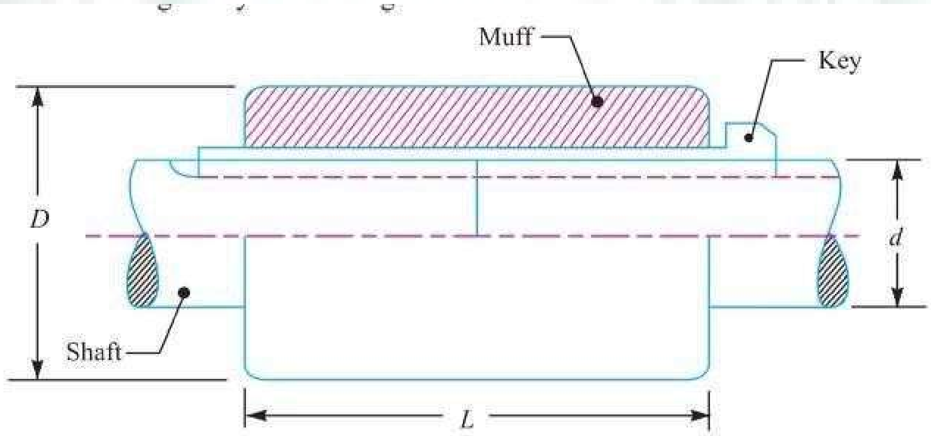
The usual proportions of a cast iron sleeve coupling are as follows :

Outer diameter of the sleeve, D = 2d + 13 mm and

length of the sleeve, L = 3.5 d

where d is the diameter of the shaft.

Design for sleeve



Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \ (1 - k^4) \qquad \ldots (\because \ k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

Design for key

The coupling key is usually made into two parts so that the length of the key in each shaft,

$l = L/2$

$= 3.5d/2$

The torque transmitted is

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Q) Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Ans

Solution. Given : $P = 40$ kW $= 40 \times 10^3$ W; $N = 350$ r.p.m.; $\tau_s = 40$ MPa $= 40$ N/mm^2; $\sigma_{cs} = 80$ MPa $= 80$ N/mm^2; $\tau_c = 15$ MPa $= 15$ N/mm^2

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

Using Design data book the diameter of sleeve is taken .

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

So the torque transmitted is

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

It is less than the given value i.e 15 Mpa. So design is safe.

Design of key-

From design data book we can get directly value of keys

w =t =18 mm

l= L/2

= 97.5 mm

Now ,calculating the induced crushing stress and shear stress

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

Considering Crushing strength we have

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \, \sigma_{cs}$$

$$\sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

It is less than the given value i.e 80 Mpa.

Considering shear strength we have,

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8\ \text{N/mm}^2$$

It is less than the given value i.e 40 Mpa .

So design of coupling is safe.

Keys and key ways

A key is a piece of mild steel inserted between the

shaft and hub or boss of the pulley to connect these together

in order to prevent relative motion between them. It is

always inserted parallel to the axis of the shaft. Keys are

used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or

recess in a shaft and hub of the pulley to accommodate a

key.

Types of keys

1. Sunk keys

2. Saddle keys

3. Tangent keys

4. Round keys

5. Splines

Sunk key

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. Rectangular sunk key. A rectangular sunk key is shown in the picture. The usual proportions of this key are

   Width = w = d/4
   Thickness= t= 2w/3 = d/6

   Diameter of shaft or Hole in the hub.

2. Square sunk key- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal.

W = t = d/4

3. Parallel sunk key. The parallel sunk keys may be of rectangular or square section

uniform in width and thickness throughout. It can be taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. . Gib-head key. It is a rectangular sunk key with a head at one end known as gib head.

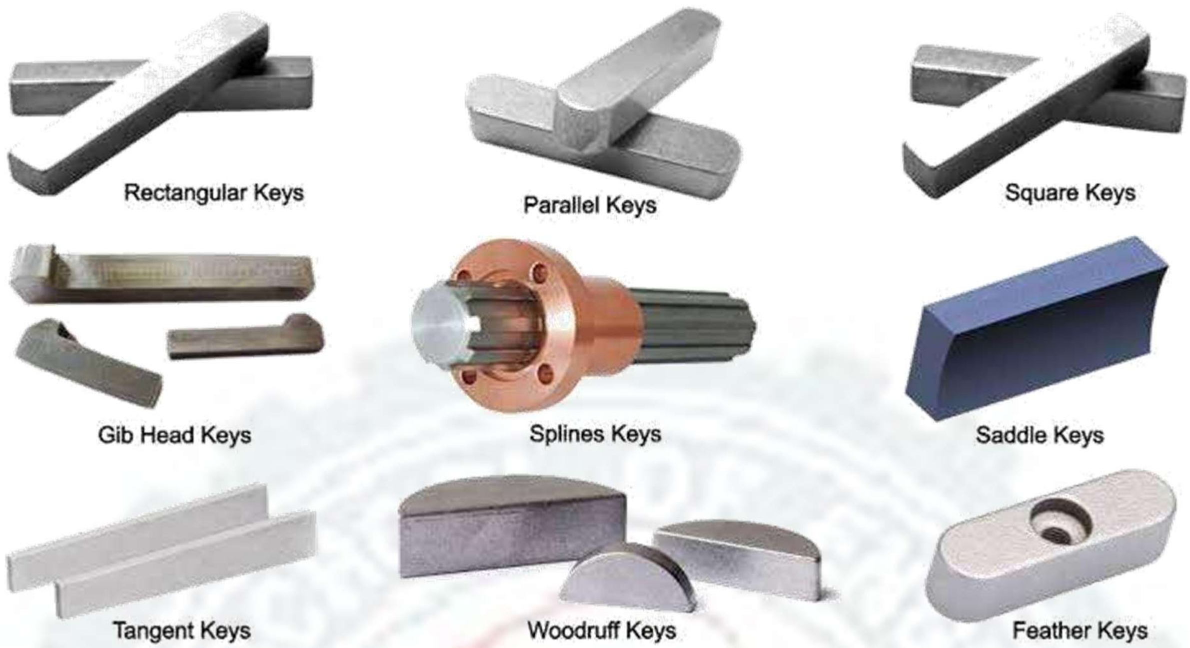
It is usually provided to facilitate the removal of key.

W = d/4

t = 2w/3 = d/6

5. Feather key. A key attached to one member of a pair and which permits relative axial

movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.


Rectangular Keys
Parallel Keys
Square Keys
Gib Head Keys
Splines Keys
Saddle Keys
Tangent Keys
Woodruff Keys
Feather Keys

Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and 2. Hollow saddle key.

A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft.

A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key

is shaped to fit the curved surface of the shaft.

Tangent Keys

The tangent keys are fitted in pair at right angle. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

Round keys

These are circular in section and fit into holes drilled partly

in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and

reamed after the mating parts have been assembled. Round keys are usually considered to be most

appropriate for low power drives.

<u>Splines</u>

When keys are made integral with the shaft which fits in the

keyways broached in the hub. Such shafts are known as splined shafts. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

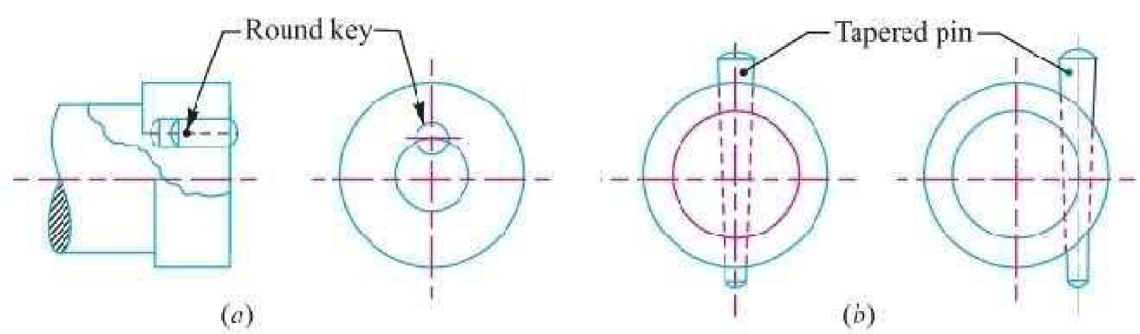
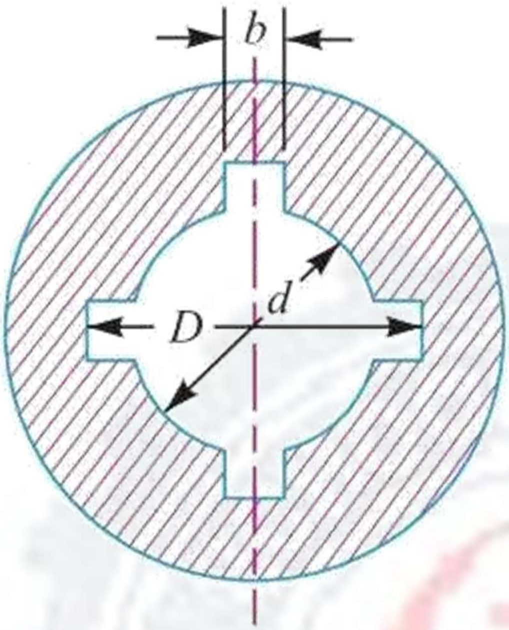


Fig. 13.7. Round keys.



$D = 1.25\ d$ and $b = 0.25\ D$

Fig. 13.8. Splines.

Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor

or hub, the following two types of forces act on the key.

1. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered

key driven in place. These forces produce compressive stresses in the key which are difficult

to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and

compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces areconcentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting

of the shaft within the hub.

Strength of a Sunk Key

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

If we consider the shearing of the key, the tangential shearing force acting at the circumference of the

shaft,

F = Area resisting shearing × Shear stress = $l \times w \times \tau$

Torque transmitted by the shaft,

$T = F \times d/2$

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...[\text{Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad ...(iii)$$

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad ...(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad ...(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad ...\ (\text{Taking } w = d/4) \quad ...(vi)$$

1.Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.

Ans-

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa = 42 N/mm^2 ; $\sigma_c = 70$ MPa = 70 N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \qquad ...(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad ...(ii)$$

From equations *(i)* and *(ii)*, we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \qquad ...(iii)$$

From equations *(ii)* and *(iii)*, we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$$l = 117.7 \text{ say } 120 \text{ mm}$$ **Ans.**

A design data book should be used to have the proper dimension of key.

2. A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.

Ans-

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$\therefore \quad l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

Effect of Keyways- The keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

3. A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.

Ans-

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ; τ = 56 MPa = 56 N/mm^2 ; σ_c = 112 MPa = 112 N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let $\qquad w$ = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$$\therefore \quad w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least d / 4.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm } \textbf{Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \text{... } (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

$\therefore$ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\ 0.8125 = 571\ 844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\ 000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\ 000}{571\ 844} = 1.47 \textbf{ Ans.}$$

4TH CHAPTER
Design of Coupling

Design of coupling-

Coupling-

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Use of Coupling

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2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

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1. It should be easy to connect or disconnect.

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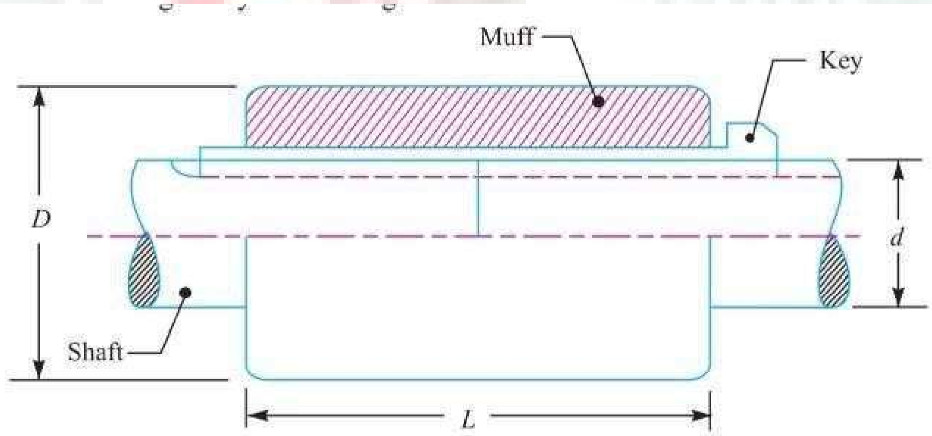


Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \ (1 - k^4) \qquad \ldots (\because \ k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

Design for key

The coupling key is usually made into two parts so that the length of the key in each shaft,

$l = L/2$

$= 3.5d/2$

The torque transmitted is

$$T = l \times w \times \tau \times \frac{d}{2}$$ ... (Considering shearing of the key)

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$ ... (Considering crushing of the key)

Q) Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Ans

Solution. Given : P = 40 kW = 40 × 10^3 W; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm^2; σ_{cs} = 80 MPa = 80 N/mm^2; τ_c = 15 MPa = 15 N/mm^2

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ d^3 = 1100 × 10^3/7.86 = 140 × 10^3 or d = 52 say 55 mm **Ans.**

Using Design data book the diameter of sleeve is taken .

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm} \quad \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm} \quad \textbf{Ans.}$$

So the torque transmitted is

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

It is less than the given value i.e 15 Mpa. So design is safe.

Design of key-

From design data book we can get directly value of keys

w =t =18 mm

l= L/2

= 97.5 mm

Now ,calculating the induced crushing stress and shear stress

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Considering Crushing strength we have

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

$$\sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6\ \text{N/mm}^2$$

It is less than the given value i.e 80 Mpa.

Considering shear strength we have,

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

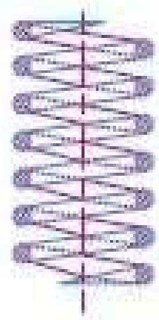
$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8\ \text{N/mm}^2$$

It is less than the given value i.e 40 Mpa .

So design of coupling is safe.

5^{TH} CHAPTER
DESIGN A CLOSED COIL HELICAL SPRING

1. *Helical springs*. The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. and ***tension helical spring*** as shown in Fig.



(*a*) Compression helical spring.

(*b*) Tension helical spring.

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring. In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(a) These are easy to manufacture.
(b) These are available in wide range.
(c) These are reliable.
(d) These have constant spring rate.
(e) Their performance can be predicted more accurately.
(f) Their characteristics can be varied by changing dimensions.

Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used *i.e*. severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 percent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance. Table shows the values of allowable shear stress, modulus of rigidity and modulus of elasticity for various materials used for springs.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Standard Size of Spring Wire:

SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)	SWG	Diameter (mm)
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. ***Solid length*****.** When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid*****.** The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$L_S = n'.d$

where n' = Total number of coils, and

d = Diameter of the wire.

2. ***Free length*****.** The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).

Mathematically,

Free length of the spring,

L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\ \delta_{max}$

The following relation may also be used to find the free length of the spring, *i.e.*

$L_F = n'.d + \delta_{max} + (n' - 1) \times 1$ mm

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. ***Spring index*****.** The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. ***Spring rate*****.** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. ***Pitch*****.** The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil,

$$p = \frac{L_F - L_S}{n'} + d$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil,

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted:

(a) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

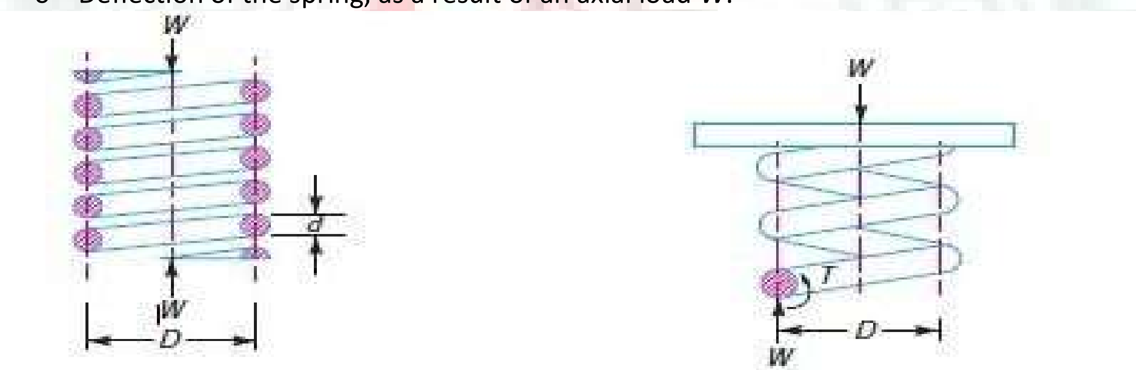
(b) The spring should not close up before the maximum service load is reached.
Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,
$L_F = n.d + (n - 1)$
and pitch of the coil,

$$p = \frac{L_F}{n-1}$$

Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load *W*, as shown in Fig.

Let *D* = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ = Maximum shear stress induced in the wire,
C = Spring index = *D*/*d*,
p = Pitch of the coils, and
δ = Deflection of the spring, as a result of an axial load *W*.



(a) Axially loaded helical spring.
(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig.(*b*). The load *W* tends to rotate the wire due to the twisting moment (*T*) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig.(*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8 W.D}{\pi d^3} \qquad ...(i)$$

The torsional shear stress diagram is shown in Fig.(*a*). In addition to the torsional shear stress ($\tau 1$) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load *W*, and

2. Stress due to curvature of wire.

$$= \frac{8\,W.D}{\pi\,d^3}\left(1+\frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad \text{...(iii)}$$

.... (Substituting $D/d = C$)

where $\quad K_S$ = Shear stress factor $= 1 + \dfrac{1}{2C}$

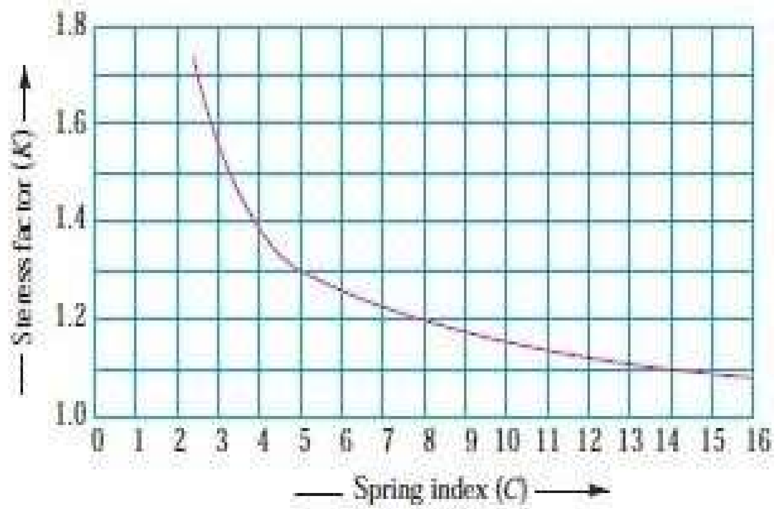
From the above equation, it can be observed that the effect of direct shear $\left(\dfrac{8\,WD}{\pi\,d^3} \times \dfrac{1}{2C}\right)$ is appreciable for springs of small spring index *C*. Also we have neglected the effect of wire curvature in equation **(*iii*)**. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses. In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (*K*) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad \text{...(iv)}$$

where $\quad K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$

The values of *K* for a given spring index (*C*) may be obtained from the graph as shown in Fig.



Wahl's stress factor for helical springs

We see from Figure that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (*K*) may be considered as composed of two sub-factors, K_S and K_C, such that

$K = K_S \times K_C$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \ldots(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T\,l}{J.G} \qquad \ldots\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T\,l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\, D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16 W.D^2.n}{G.d^4} \qquad \ldots(ii)$$

Substituting this value of θ in equation (i), we have

$$\delta = \frac{16 W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \ldots(\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. In the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called surge.

The natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

where
d = Diameter of the wire,
D = Mean diameter of the spring,
n = Number of active turns,
G = Modulus of rigidity,
g = Acceleration due to gravity, and
ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : $D = 50$ mm ; $d = 5$ mm ; $^*n = 20$; $W = 500$ N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$= 534.7$ MPa **Ans.**

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $\quad C = \dfrac{D}{d} = \dfrac{69}{6} = 11.5$

Let $\quad W$ = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$\therefore \quad W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$\therefore \quad W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$