

## CHAPTER-1 .

### INTRODUCTION TO DESIGN AND DETAILING

#### Objectives of Design and Detailing

Every structure must be **designed** to satisfy three basic requirements;

- 1) **Stability** to prevent over turning, sliding or buckling of the structure, or parts of it, under the action of loads;
- 2) **Strengths** to resist safely the stresses induced by the loads in the various structural members.
- 3) **Serviceability** to ensure satisfactory performance under service load condition. Serviceability includes two parameters i.e deflection and cracking. The deflection should be limited to ensure the better appearance of the structure and to prevent cracking. The cracking of the reinforced concrete should not be excessive to ensure better appearance and also to prevent the access of water from cracks which may corrode there in for cement.

The re are two other considerations that a sensible designer ought to bear in mind, viz. **economy** and **aesthetics**

A good structural design often involving elaborate computations is a worthwhile exercise if only it is followed by good detailing and construction practices. In normal design practices it is often seen that analysis of structures for stress resultants and design of individual members (critical sections of beams, slabs and columns) for maximum load effects (bending moments, shear, torsion and axial forces) are done regularly with insufficient attention given to supposedly lesser important aspects e.g. termination, extending and bending of bars, anchorage and development, stirrup anchorage, splices, construction details at joints or connections (slab-beam, beam-column etc.), provision of continuity and discontinuity at connection of members, construction sequencing and reinforcement placement, deflection calculations and cont

rol, crack control, cover to reinforcement, creep and shrinkage etc.

The factors as enumerated above are very critical from the point of view of a successful structure and need to be fairly assessed with sufficient accuracy and spelt out in detail through various drawings and specifications by the designers so that the construction of the structure can be handled by the site engineer.

### **Different Methods of Design**

Over the years, various design philosophies have evolved in different parts of the world, with regard to reinforced concrete design. A design philosophy is built upon a few fundamental assumptions and is reflective of a way of thinking.

#### **Working Stress Method:**

The earliest codified design philosophy is that of **working stress method** of design (WSM). Close to a hundred years old, this traditional method of design, based on linear elastic theory, is still surviving in a number

of countries. In WSM it is assumed that structural material e.g. concrete and steel behave in linearly elastic manner and adequate safety can be ensured by restricting the stresses in the material induced by working loads (service loads) on the structure. As the specified permissible (allowable) stresses are kept well below the material strength, the assumption of linear elastic behavior considered justifiable. The ratio of the strength of the material to the permissible stress is often referred to as the factor of safety. While applying WSM the stresses under applied loads are analysed by 'simple bending theory' where strain compatibility is assumed (due to bond between concrete and steel).

### Ultimate Load Method:

With the growing realization of the shortcomings of WSM in reinforced concrete design, and with increased understanding of the behavior of reinforced concrete at *ultimate loads*, the ultimate load method of design (ULM) evolved in the 1950s and became an alternative to WSM. This method is sometimes also referred to as the *load factor method* or the *ultimate strength method*.

In this method, the stress condition at the state of impending collapse of the structure is analysed, and the nonlinear stress-strain curve of concrete and steel are made use of the concept of 'modular ratio' and its associated problems are avoided. The safety measure in the design is introduced by an appropriate choice of the load factor, defined as the ratio of the ultimate load (design load) to the working load. This method

### Advantages Of Reinforced Concrete

The following are major advantages of reinforced cement concrete (RCC)

- Reinforced Cement Concrete has good compressive stress (because of concrete).
- RCC also has high tensile stress (because of steel).
- It has good resistance to damage by fire and weathering (because of concrete).
- RCC protects steel bars from buckling and twisting at the high temperature.

- RCC prevents steel from rusting.
- Reinforced Concrete is durable.
- The monolithic character of reinforced concrete gives it more rigidity.
- Maintenance cost of RCC is practically nil.

It is possible to produce steel whose yield strength is 3 to 4 times more than that of ordinary reinforced steel and to produce concrete 4 to 5 times stronger in compression than the ordinary concrete. This high strength material offers many advantages including smaller member cross-sections, reduced dead load and longer span.

generally results in more slender sections, and often more economical design of beams and columns (compared to WSM), particularly when high strength reinforcing steel and concrete are used.

**Limit State Method:**

The philosophy of the limit state method of design (LSM) represents a definite advancement over the traditional WSM (based on service load conditions alone) and ULM (based on ultimate load conditions alone). LSM aims for a comprehensive and rational solution to the design problem, by considering safety at ultimate loads and serviceability at working loads. The LSM uses a multiple safety factor for materials which attempt to provide adequate safety at ultimate loads as well as adequate serviceability at service loads by considering all possible 'limit states'.



## CHAPTER-2

### WORKING STRESS METHOD OF DESIGN

#### **General Concept**

Working stress method is based on the behavior of a section under the load expected to be encountered by it during its service period. The strength of concrete in the tension zone of the member is neglected although the concrete does have some strength for direct tension and flexural tension (tension due to bending). The material both concrete and steel, are assumed to behave perfectly elastically, i.e., stress is proportional to strain. The distribution of strain across a section is assumed to be linear. These sections that are plane before bending remain plane after bending. Thus, the strain, hence stress at any point is proportional to the distance of the point from the neutral axis. With this a triangular stress distribution in concrete is obtained, ranging from zero at neutral axis to a maximum at the compressive face of the section. It is further assumed in this method that there is perfect bond between the steel and the surrounding concrete, the strain in both materials at any point are same and hence the ratio of stresses in steel and concrete will be the same as the ratio of elastic moduli of steel and concrete. This ratio being known as 'modular ratio', the method is also called 'Modular Ratio Method'.

In this method, external forces and moments are assumed to be resisted by the internal compressive forces developed in concrete and tensile resistive forces in steel and the internal resistive couple due to the above two forces, in concrete acting through the centroid of triangular distribution of the compressive stresses and in steel acting at the centroid of tensile reinforcement. The distance between the lines of action of

resultant resistive force is known as 'Lever arm'.

Moments and forces acting on the structure are computed from the service loads. The section of the component member is proportioned to resist these moments and forces such that the maximum stresses developed in materials are restricted to a fraction of their true strengths. The factors of safety used in getting maximum permissible stresses are as follows:

<i>Material</i>	<i>Factor of Safety</i>
For concrete	3.0
For steel	1.78

### Assumptions of WSM

The analysis and design of a RCC member are based on the following assumptions.

- (i) Concrete is assumed to be homogeneous.
- (ii) At any cross-section, plane sections before bending remain plane after bending.
- (iii) The stress-strain relationship for concrete is a straight line, under working loads.
- (iv) The stress-strain relationship for steel is a straight line, under working loads.
- (v) Concrete area on tension side is assumed to be ineffective.
- (vi) All tensile stresses are taken up by reinforcements and none by concrete except when specially permitted.
- (vii) The steel area is assumed to be concentrated at the centroid of the steel.
- (viii) The modular ratio has the value  $280/3\sigma_{cbc}$  where  $\sigma_{cbc}$  is permissible stress in concrete due to bending in concrete in  $N/mm^2$  as specified in code (IS: 456-2000)

### Moment of Resistance

- (a) *For Balanced section:* When the maximum stresses in steel and concrete simultaneously reach their allowable values, the section is said to be a 'Balanced Section'. The moment of resistance shall be provided by the couple developed by compressive force acting at the centroid of stress diagram on the area of concrete in compression and tensile force acting at the centroid of reinforcement multiplied by the distance between these forces. This distance is known as 'lever arm'.



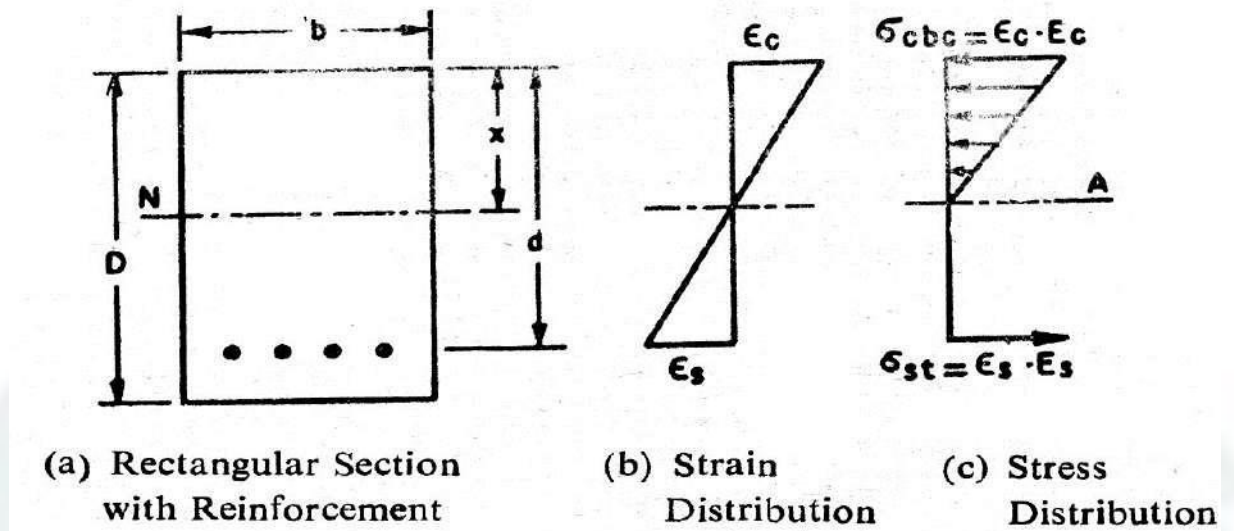


Fig.2.1(a-c)

$D$  = overall depth of section

$d$  = effective depth of section (distance from extreme compression fiber to the centroid of steel area,

$A_s$  = area of tensile steel

$\epsilon_c$  = Maximum strain in concrete,

$\epsilon_s$  = maximum strain at the centroid of the steel,

$\sigma_{cbc}$  = maximum compressive stress in concrete in bending

$\sigma_{st}$  = Stress in steel

$E_s/E_c$  = ratio of Young's modulus of elasticity of steel to concrete

=modularratio ' $m$ '

$$\frac{c}{d} \text{ or } \frac{d}{c} = \frac{c}{d} \text{ or } \frac{d}{c}$$

$$\text{Or } \frac{x \cdot \frac{1}{m} \cdot \frac{1}{bc} \cdot \frac{1}{st}}{1} \text{ or } x \cdot \frac{1}{m} \cdot \frac{1}{bc} \cdot \frac{1}{st} \cdot d = k \cdot d$$

Where  $k = \text{neutral axis constant} = \frac{1}{12}$

$$\frac{\text{Total compressive force}}{\text{Total tensile forces}} = \frac{b \cdot s \cdot o_c b c}{o_c t d^x d^{k.d} = d_{[?]1 - k_{[?]} = j.d}$$

3

Where  $j$  is called the lever arm constant.

$$\text{Moment of resistance} = MR = \frac{f_c b d^2}{4} \left( 1 - \frac{f_c}{f_y} \right) \left( 1 - \frac{f_c}{2f_y} \right) = \frac{f_c b d^2}{4} Q$$

Where  $Q$  is called moment of resistance constant and is equal to  $\frac{f_c}{4} \left( 1 - \frac{f_c}{f_y} \right) \left( 1 - \frac{f_c}{2f_y} \right)$ .

### (b) Under reinforced section

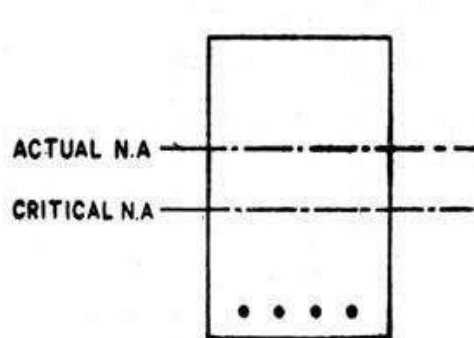
When the percentage of steel in a section is less than that required for a balanced section, the section is called 'Under-reinforced section.' In this case (Fig.2.2) concrete stress does not reach its maximum allowable value while the stress in steel reaches its maximum permissible value. The position of the neutral axis will shift upwards, i.e., the neutral axis depth will be smaller than that in the balanced section as shown in Figure 2.2. The moment of resistance of such a section will be governed by allowable tensile stress in steel.

Moment of resistance =  $\sigma_{st} A_s \cdot j \cdot d$  where  $j = 1 - \frac{3}{2} \frac{p}{100}$

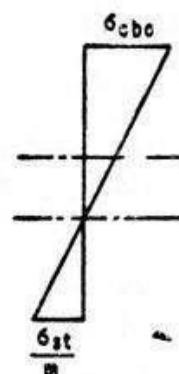
Since  $p = \frac{A_s \cdot 100}{b \cdot d}$

Moment of resistance =  $\sigma_{st} A_s \cdot j \cdot d$

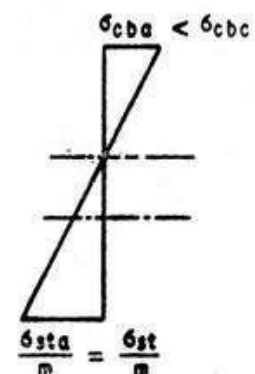
$= \sigma_{st} \cdot \frac{p}{100} \cdot b \cdot d \cdot j \cdot d$  where  $Q = \frac{j \cdot d}{b \cdot d}$



(a) Rectangular Section with Reinforcement



(b) Strain Distribution



(c) Stress Distribution

Fig.2.2(a-c)

(c) **Over reinforced section:**

When the percentage of steel in a section is more than that required for a balanced section, the section is called 'Over-reinforced section'. In this case (Fig.2.3) the stress in concrete reaches its maximum allowable value earlier than that in steel. As the percentage steel is more, the position of the neutral axis will shift towards steel from the critical or balanced neutral axis position. Thus the neutral axis depth will be greater than that in case of balanced section.



Moment of resistance of such a section will be governed by compressive stress in concrete,

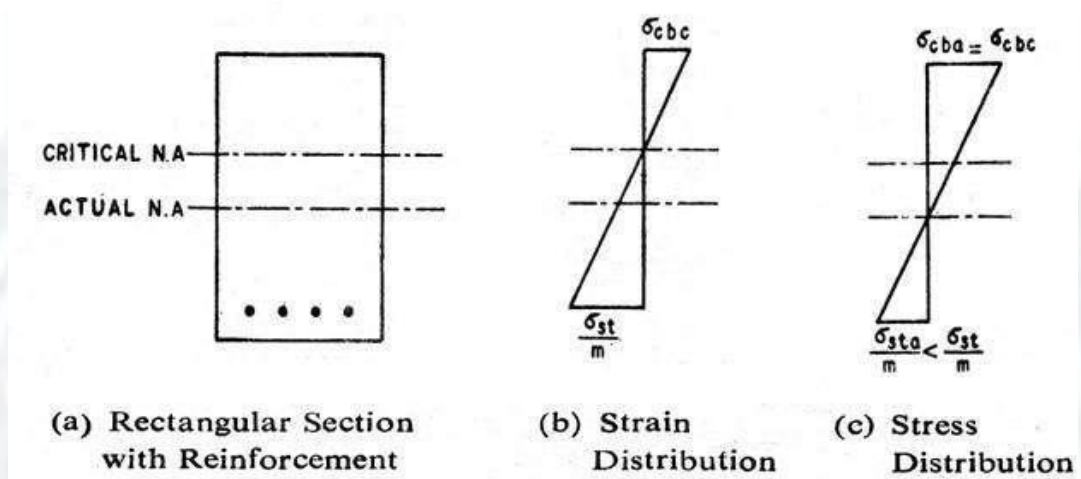


Fig.2.3(a-c)

$$\text{Moment of resistance} = \frac{1}{2} \cdot \frac{d}{3} \cdot \frac{c}{2} \cdot b \cdot x \cdot d \cdot \frac{1}{3} \cdot k'$$

$$\begin{array}{ccccccc} & & & & c & & \\ & & & & b & & \\ & & & & c & & \\ cbc & .b.x.d.j' & 1 & & & & \\ & \frac{?}{2} & \frac{?}{2} & .k.j.b.d & \frac{?}{2} & Q.b.d & 2 \\ & & & & & cbc & \end{array} \quad \text{where } Q = \frac{?}{2} \frac{?}{2} cbc \frac{?}{2} .k.j \frac{?}{2} \text{Constant}$$

## Basic concept of design of singly reinforced members

The following types of problems can be encountered in the design of reinforced concrete members.

### (A) Determination of Area of Tensile Reinforcement

This section, bending moment to be resisted and the maximum stresses in steel and concrete are given.

*Stepstobefollowed:*

- (i) Determine  $k, j.Q$  (or  $Q'$ ) for the given stress.
- (ii) Find the critical moment of resistance,  $M = Q.b.d^2$  from the dimensions of the beam.
- (iii) Compare the bending moment to be resisted with  $M$ , the critical moment of resistance.

- (a) If  $B.M.$  is less than  $M$ , design the section as underreinforced.

$$M_{u\max} = \frac{f_y A_s d}{3}$$

$$st. A_s = \frac{M_{u\max}}{f_y d}$$

To find  $A_s$  in terms of  $x$ , take moments of areas about N.A.

$$b \cdot x \cdot \frac{x}{2} = m \cdot A_s \cdot d \left( \frac{x}{3} \right)$$

$$b \cdot x^2$$

$$= \frac{f_y \cdot b \cdot x^2}{6} \cdot \frac{x}{3}$$

$$A_s =$$

$$\frac{2 M_{u\max}}{f_y m d}$$

$$M_{u\max} = \frac{f_y A_s d^2}{2}$$

$$= \frac{f_y B.M.}{3}$$

Solve for 'x', and then A<sub>s</sub> can be calculated.

- (b) If B.M. is more than M<sub>cbc</sub>, design the section as over-reinforced.

M<sub>cbc</sub> =  $\frac{1}{2} b x_c^2$ . B.M. to be resisted. Determine 'x'. Then A<sub>s</sub> can be obtained by taking

moment of areas (compressive and tensile) about using the following expression.

$$A_s = \frac{b x^2}{2 m \cdot j \cdot d}$$

### (B) Design of Section for a Given loading

Design the section as a balanced section for the given loading.

Steps to be followed:

- Find the maximum bending moment (B.M.) due to given loading.
- Compute the constants  $k, j, Q$  for the balanced section for known stresses.
- Fix the depth to breadth ratio of the beam section as 2 to 4.
- From  $M = Q \cdot b \cdot d^2$ , find 'd' and then 'b' from depth to breadth ratio.
- Obtain overall depth 'D' by adding concrete cover to 'd' the effective depth.
- Calculate A<sub>s</sub> from the relation

$$A_s = \frac{B.M.}{\sigma_{st} \cdot j \cdot d}$$

### (C) To Determine the Load carrying Capacity of a given Beam

The dimension of the beam section, the material stresses and area of reinforcing steel are given.

Steps to be followed:

- Find the position of the neutral axis from section and reinforcement given.
- Find the position of the critical N.A. from known permissible stresses of concrete and steel.

$$x = \frac{1}{2} d$$

$\sigma_{st} m. cbc$

(iii) Check if (i) > (ii) - the section is over-reinforced

(i) < (ii) - the section is under-reinforced

(iv) Calculate  $M_f$  from relation

$$M_f = \frac{1}{2} \cdot \left( \frac{\sigma_{st}}{m} \right) \cdot \left( \frac{b \cdot d^2}{3} \right) \cdot cbc. \quad \text{for over-reinforced section}$$

and  $M_{st} = \frac{wL^2}{8} \left( \frac{x}{L} - \frac{x^2}{L^2} \right)$  for under-reinforced section.

- (v) If the effective span and the support condition of the beam are known, the load carrying capacity can be computed.

#### (D) To Check The Stresses Developed In Concrete And Steel

This section, reinforcement and bending moment are given.

Steps to be followed:

- (i) Find the position of N.A. using the following relation.

$$\frac{m A_s (d - x)}{b x} = \frac{x^2}{2}$$

- (ii) Determine lever arm,  $z = d - \frac{x}{2}$

- (iii)  $B.M. = \sigma_{st} A_s z$  is used to find out the actual stress in steel  $\sigma_{sa}$ .

- (iv) To compute the actual stress in concrete  $\sigma_{cba}$ , use the following relation.

$$BM = \frac{\sigma_{cba} b x}{2} \cdot z$$

#### Doubly Reinforced Beam Sections by Working Stress Method

Very frequently it becomes essential for a section to carry bending moment more than it can resist as a balanced section. Such a situation is encountered when the dimensions of the cross section are limited because of structural, head room or architectural reasons. Although a balanced section is the most economical section but because of limitation of size, section has to be sometimes over-reinforced by providing extra reinforcement on tension face than that required for a balanced section and also some reinforcement on compression face. Such sections reinforced both in tension and compression are also known as "Doubly Reinforced Sections". In some loading cases reversal of stresses in the section take place (this happens



when wind blows in opposite

directions at different timings), there is reinforcement required on both faces.



## MOMENT OF RESISTANCE OF DOUBLY REINFORCED SECTIONS

Consider a rectangular section reinforced on tension as well as compression faces as shown in Fig.2.4 (a-c) Let  $b$  = width of section,

$d$  = effective depth of section,

$D$  = overall depth of section,

$d'$  = cover to centre of compressive steel,

$M$  = Bending moment or total moment of resistance,

$M_{bal}$  = Moment of resistance of a balanced section with tension reinforcement,

$A_{st}$  = Total area of tensile steel,

$A_{st1}$  = Area of tensile

steel required to develop  $M_{bal}$   $A_{st2}$  = Area

of tensile steel required to develop  $M_2$

$A_{sc}$  = Area of compression steel,

$\sigma_{st}$  = Stress in steel, and

$\sigma_{sc}$  = Stress in compressive steel

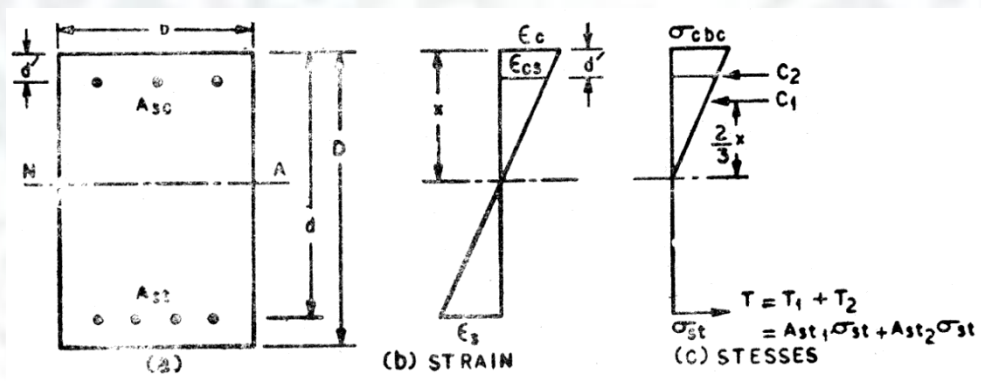


Fig.2.4

(a-c) Since strains are proportional to the distance

from N.A.,

$$\frac{\text{Strain in top fibre of concrete}}{\text{Strain in Compression Steel}} = \frac{x}{d - d'}$$

cbc

sc

sc  $E_c$

sc cbc

$$\begin{array}{r} \overline{EC} \times \\ E_s \overline{x} d' \\ \hline \overline{cbc} E_s \overline{x} \\ \overline{x} d' \\ \hline \overline{x} \end{array} .m$$

Since  $\sigma_{cb} \frac{x}{x-d'}$  is the stress in concrete at the level of compression steel, it can be denoted as

$$\sigma'_{cb} = \sigma_{sc} \frac{x}{x-d'}$$

As per the provisions of IS:456-2000 Code, the permissible compressive stress in bars, in a beam or slab when compressive resistance of the concrete is taken into account, can be taken as 1.5 times the compressive stress in surrounding concrete ( $1.5 \sigma'_{cbc}$ ) or permissible stress in steel in compression ( $\sigma_{sc}$ ) whichever is less.

$$\sigma_{sc} \leq 1.5 \sigma'_{cbc}$$

Total equivalent concrete area resisting compression

$$= (b - A_{sc})x + 1.5 A_{sc}x$$

$$= .b + (1.5m - 1)A_{sc}$$

Taking moment about centre of

tensile steel Moment of resistance

$$M = C_1 \cdot (d - x/3) + C_2 (d - d')$$

total compressive force in concrete,

$C_2$  = total compressive force in compression steel,

$$M = b \cdot x \cdot \sigma_{cb} \cdot \left( \frac{x}{x-d'} \right) \cdot \left( \frac{1.5m-1}{3} \right) \cdot A_{sc} \cdot \left( \frac{x-d'}{x} \right)$$

$$= \frac{b \cdot x \cdot \sigma_{cb} \cdot (1.5m-1) \cdot A_{sc} \cdot (x-d')}{3}$$

$$M_1 = M_2$$

Where  $M_1$  = Moment of resistance of the

balanced section  $M_{bal}$  Moment of  
resistance of the  
compression steel

$$A_{st1} = \frac{M_1}{j \cdot d}$$

$$A_{st2} = \frac{M_2}{\sigma_{st} (d - d')}$$

Thus the total tensile steel  $A_{st}$  shall be  
:

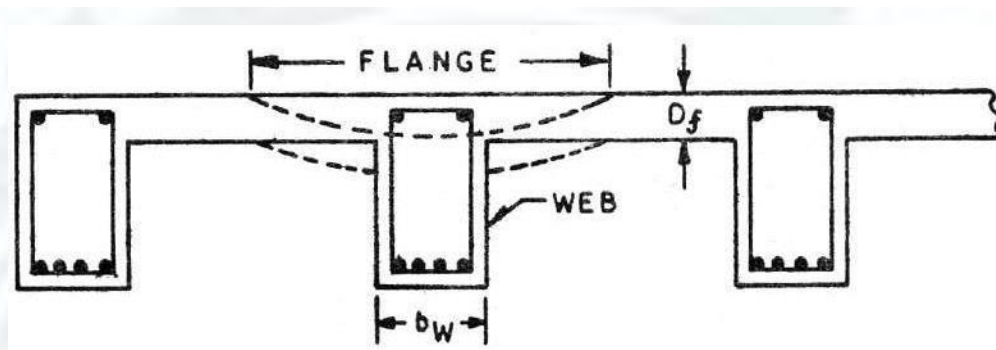
$$A_{st} = A_{st1} + A_{st2}$$

The area of compression steel can be  
obtained as

$$(1.5m - 1) A_{sc} (d - d') = m A_{st2} (d - x)$$



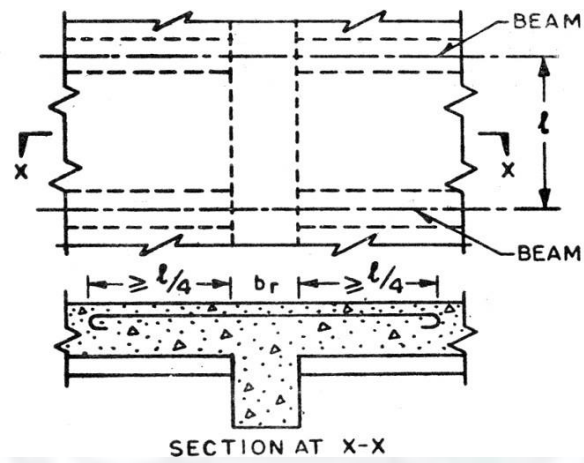
## Design Concept of T-Beam



**Fig.2.5**

Flanged beam sections comprise T-beams and L-beams where the slabs and beams are cast monolithically having no distinction between beams and slabs. Consequently the beams and slabs are so closely tied that when the beam deflects under applied loads it drags along with it a portion of the slab also as shown in Fig.2.5. This portion of the slab assists in resisting the effect of the loads and is called the 'flange' of the T-beams. For design of such beams, the profile is similar to a T-section for intermediate beams. The portion of the beam below the slab is called 'web' or 'Rib'. A slab which is assumed to act as a flange of a T-beam shall satisfy the following conditions:

- (a) The slab shall be cast integrally with the web or the web and the slab shall be effectively bonded together in any other manner; and
- (b) If the main reinforcement of the slab is parallel to the beam, transverse reinforcement shall be provided as shown in Fig.2.6, such reinforcement shall not be less than 60% of the main reinforcement at mid-span of the slab.



**Fig.2.6**

## **CHAPTER-**

### **3 LIMIT STATE METHOD**

#### **D**

#### **SAFETY AND SERVICEABILITY REQUIREMENTS**

In the method of design based on limit state concept, the structure shall be designed to withstand safely all loads liable to act on it throughout its life; it shall also satisfy the serviceability requirements, such as limitations on deflection and cracking. The acceptable limit for the safety and serviceability requirements before failure occurs is called a 'limit state'. The aim of design is to achieve acceptable probabilities that the structure will not become unfit for the use for which it is intended that it will not reach a limit state.

All relevant limit states shall be considered in design to ensure an adequate degree of safety and serviceability. In general, the structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

For ensuring the above objective, the design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on experience. The 'design values' are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads. In the absence of special considerations these factors should have the values given in 36 according to the material, the type of loading and the limit state being considered.

### **LimitStateofCollapse**

The limit state of collapse of the structure or part of the structure could be assessed from rupture of one or more critical sections and from buckling due to elastic or plastic in stability (including the effects of sway where appropriate) or overturning. The resistance to bending, shear, torsion and axial load at every section shall not be less than the appropriate value at that section produced by the probable most unfavourable combination of loads on the structure using the appropriate partial safety factors.

### **LimitStateDesign**

For ensuring the design objectives, the design should be based on characteristic values of material strengths and applied loads (actions), which take into account the probability of variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data, if available. Where such data is not available, they should be based on experience. The design values are derived from the characteristic values through the use of partial safety factors, both for material strength and for loads. In the absence of special considerations, the safety factors should have the values given in this section according to the material, the type of load and the limit state being considered. The reliability of design is ensured by requiring that Design Action  $\leq$  Design Strength

Limit states are the states beyond which the structure no longer satisfies the performance requirements specified. The limit states are classified as

- a) Limit state of strength
- b) Limit state of serviceability

a) The limit state of strength are those associated with failures (or imminent failure), under the action of probable and most unfavorable combination of loads on the structure using the appropriate partial safety factors, which may endanger the safety of life and property. The limit state of strength includes:

- a) Loss of equilibrium of the structure as a whole or any of its parts or components.
- b) Loss of stability of the structure (including the effect of sway where appropriate and overturning) or any of its parts including supports and foundations.
- c) Failure by excessive deformation, rupture of the structure or any of its parts or components.
- d) Fracture due to fatigue.
- e) Brittle fracture.

b) The limit state of serviceability include

- a) Deformation and deflections, which may adversely affect the appearance or, effective, use of the structure or may cause improper functioning of equipment or services or may cause damages to finishes and non-structural members.
- b) Vibrations in the structure or any of its components causing discomfort to people, damages to the structure, its contents or which may limit its functional effectiveness. Special considerations shall be given to floor vibration systems susceptible to vibration, such as large open floor areas free of partitions to ensure that such vibration is acceptable for the intended use and occupancy.

c) Repairable damage due to fatigue.



d) Corrosion and durability.

### **Limit States of Serviceability**

To satisfy the limit state of serviceability the deflection and cracking in the structure shall not be excessive. This limit state corresponds to deflection and cracking.

#### **Deflection**

The deflection of a structure or part shall not adversely affect the appearance or efficiency of the structure or finishes or partitions.

#### **Cracking**

Cracking of concrete should not adversely affect the appearance or durability of the structure; the acceptable limit of cracking would vary with the type of structure and environment. The actual width of cracks will vary between the wide limits and predictions of absolute maximum width are not possible. The surface width of cracks should not exceed 0.3mm.

In members where cracking in the tensile zone is harmful either because they are exposed to the effects of the weather or continuously exposed to moisture or in contact soil or ground water, an upper limit of 0.2 mm is suggested for the maximum width of cracks. For particularly aggressive environment, such as the 'severe' category, the assessed surface width of cracks should not in general, exceed 0.1 mm.

## CHARACTERISTIC AND DESIGN VALUES AND PARTIAL SAFETY FACTORS

### 1. Characteristic Strength of Materials

Characteristic strength means that value of the strength of the material below which not more than 5 percent of the test results are expected to fall and is denoted by  $f$ . The characteristic strength of concrete ( $f_{ck}$ ) is as per the mix of concrete. The characteristic strength of steel ( $f_y$ ) is the minimum stressor 0.2 percent of proof stress.

### 2. Characteristic Loads

Characteristic load means that value of load which has a 95 percent probability of not being exceeded during the life of the structure. Since data are not available to express loads in statistical terms, for the purpose of this standard, dead loads given in IS 875 (Part I), imposed loads given in IS 875 (Part 2), wind loads given in IS 875 (Part 3), snow loads given in IS 875 (Part 4) and seismic forces given in IS 1893-2002 (part I) shall be assumed as the characteristic loads.

### Design

### Values

### Materials

The design strength of the materials  $f_d$  is given by

$$f_d = \frac{f}{\gamma_m}$$

**d**

**L**

**o**

**a**

The design load,  $F_d$ , is given by

$F$

$$F_d = \frac{F}{\gamma_f}$$

Where,  $F$  = characteristic load

and  $\gamma_f$  = partial safety factor appropriate to the nature of loading and the limit state being considered.

### Consequences of Attaining Limit State

Where the consequences of a structure attaining a limit state are of a serious nature such as huge loss of life

and disruption of the economy, high values of  $\gamma_f$  than those given under 36.4.1 and 36.4.2 may be

applied.

## Partial Safety Factors:

### 1. Partial Safety Factor $\gamma_f$ for Loads

Sr. No.	Load Combination	Ultimate Limit State	Serviceability Limit State
1	DL + LL	$1.5(DL + LL)$	DL + LL
2	DL + WL i) DL contribute to stability ii) DL assist overturning	$0.9 DL + 1.5 WL$ $1.5 (DL + WL)$	DL + WL DL + WL
3	DL + LL + WL	$1.2(DL + LL + WL)$	DL + 0.8 LL + 0.8 WL

### 2. Partial Safety Factor $\gamma_m$ for Material Strength

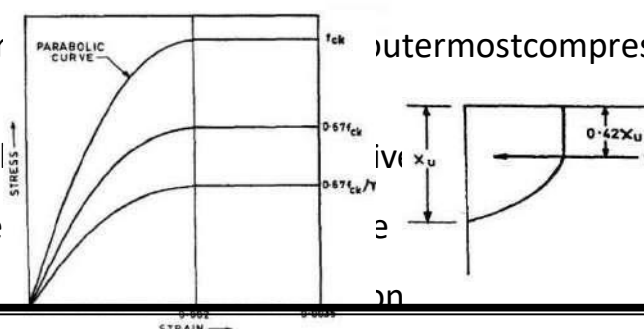
Sr. No.	Material	Ultimate Limit State	Serviceability Limit State
1	Concrete	1.50	$E_c = 5000 f_{ck} \text{ MPa}$
2	Steel	1.15	$E_s = 2 \times 10^5 \text{ MPa}$

When assessing the strength of a structure or structural member for the limit state of collapse, the values of partial safety factor, should be taken as 1.5 for concrete and 1.15 for steel.

## LIMIT STATE OF COLLAPSE: FLEXURE

### Assumptions for Limit State of Collapse (Flexure):

- Plane section normal to the axis remains plane even after bending. i.e. strain at any point on the cross section is directly proportional to the distance from the N.A.
- Maximum strain in bending.
- The relationship between stress and strain in concrete is taken as parabolic or any other shape.



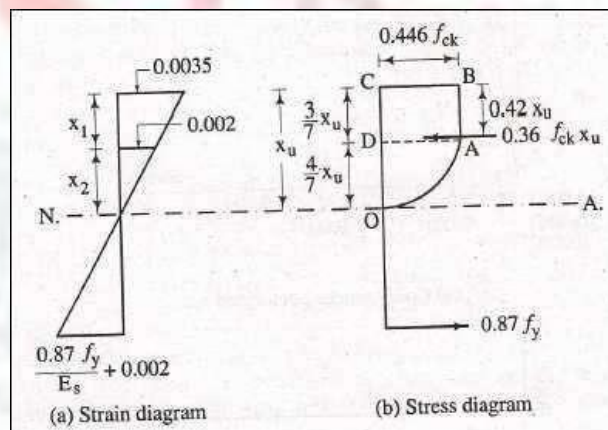
with the results of test. An acceptable stress strain curve is as shown below.



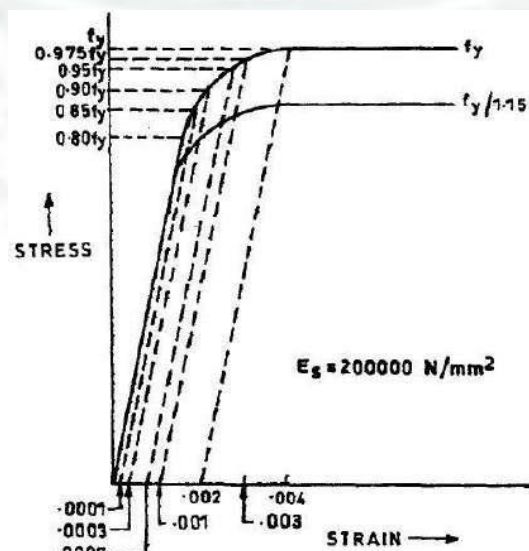


For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor  $\gamma_m = 1.5$  shall be applied in addition to this.

**NOTE** - For the above stress-strain curve the design stress block parameters are as follows: Area of stress block  $= 0.36 \cdot f_{ck} \cdot x_u$   
 Depth of centre of compressive force  $= 0.42 x_u$  from the extreme fibre in compression  
 Where  $f_{ck}$  = characteristic compressive strength of concrete, and  $x_u$  = depth of neutral axis.



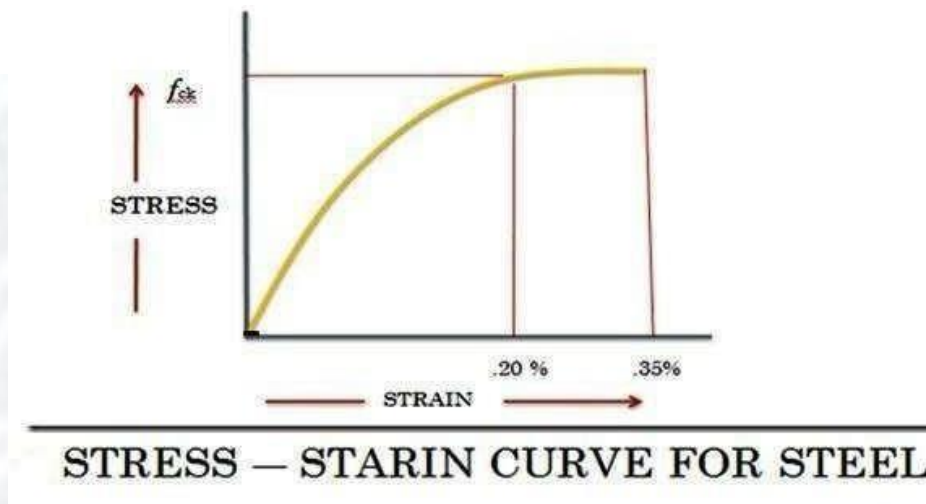
- 4) the tensile strength of the concrete is ignored.
- 5) the stresses in the reinforcement are derived from representative stress –





strain curve for the type of steel used.





- 6) the maximum strain in tension reinforcement in the section at failure shall not be less than
- $$\frac{f_y}{E_s} \geq 0.002 = \frac{0.87 f_y}{E_s} \geq 0.002$$

## CHAPTER 4

### LIMIT STATES OF COLLAPSE OF SINGLE REINFORCED MEMBERS IN BENDING

#### Limit state method of design

- The object of the design based on the limit state concept is to achieve an acceptable probability, that a structure will not become unsuitable in its lifetime for the use for which it is intended, i.e. It will not reach a limit state
- A structure with appropriate degree of reliability should be able to withstand safely.
- All loads, that are reliable to act on it throughout its life and it should also satisfy the serviceability requirements, such as limitation on deflection and cracking.
- It should also be able to maintain the required structural integrity, during and after accident, such as fires, explosion & local failure. i.e. limit state must be considered in design to ensure an adequate degree of safety and serviceability
- The most important of these limit states, which must be examined in design areas follows

#### Limit state of collapse

- Flexure
- Compression
- Shear
- Torsion

On this state corresponds to the maximum load carrying capacity.

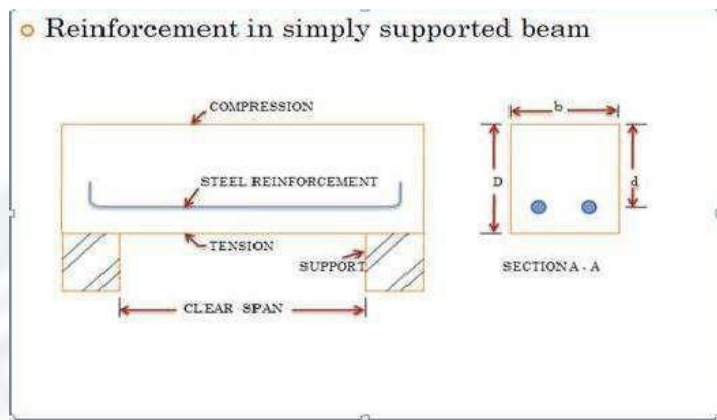
#### Types of reinforced concrete beams

- a) Singly reinforced beam
- b) Doubly reinforced beam
- c) Singly or Doubly reinforced flanged beams

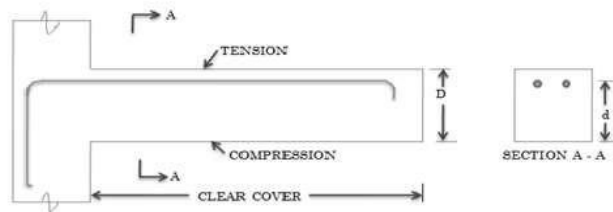
#### Singly reinforced beam

In singly reinforced simply supported beams or slabs reinforcing steel bars are

placed near the bottom of the beam or slabs where they are most effective in resisting the tensile stresses.



### Reinforcement in a cantilever beam



### TYPES OF BEAM SECTIONS

Section in which, tension steel also reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called, **'Balanced Section'**.

Section in which, tension steel also reaches yield strain at loads lower than the load at which concrete reaches the failure strain in bending are called, **'Under Reinforced Section'**.

Section in which, tension steel also reaches yield strain at loads higher than the load at which concrete reaches the failure strain in bending are called, **'Over Reinforced Section'**.

Sr. No.	Types of Problems	Data Given	Data Determine
			<p>If <math>\frac{x_u}{d} = \frac{x_{u\max}}{d}</math> [?] Balanced</p> <p>If <math>\frac{x_u}{d} &lt; \frac{x_{u\max}}{d}</math> [?] Under Reinforced</p> <p>If <math>\frac{x_u}{d} &gt; \frac{x_{u\max}}{d}</math> [?] Over Reinforced</p>
	Identify the type of beam	Grade of Concrete	$x_u = 0.87 f_y A_{st}$ $u = 0.56 b u_{ck}$

1	section, balance, under reinforced or over reinforced	&Steel, Size of beam & Reinforcement provided	$d$	$f_y$	$X_{u\max}$
				250	0.53
				415	0.48
				500	0.46



2	Calculate Moment of Resistance	Grade of Concrete & Steel, Size of beam & Reinforcement provided	$1) \text{ If } \frac{x_u}{d} \geq \frac{x_{u,max}}{d}, \text{ balanced}$ $M.R = M_u = 0.36 \cdot \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) b \cdot d^2 \cdot f_{ck}$
			$2) \text{ If } \frac{x_u}{d} < \frac{x_{u,max}}{d} \text{ Under Reinforced}$ $M.R = M_u = 0.87 f_{yk} \cdot A_{st} \cdot d (1 - \frac{A_{st} f_{yk}}{b \cdot d \cdot f_{ck}}) \text{ or } M.R = 0.87 f_{yk} \cdot A_{st} \cdot d (1 - 0.42 \frac{x_u}{d})$
3	Design the beam. Find out the depth of Beam & Reinforcement required $A_{st}$ .	Grade of Concrete & Steel, width of beam & Bending Moment or loading on the beam with the span of the beam Reinforcement provided	$3) \text{ If } \frac{x_u}{d} > \frac{x_{u,max}}{d} \text{ overreinforced, Revisethe depth}$
			<p>We have to design the beam as a 'Balanced Design'. For finding 'd' effective depth use the equation;</p> $M.R = M_u = 0.36 \cdot \frac{x_{u,max}}{d} (1 - 0.42 \frac{x_{u,max}}{d}) b \cdot d^2 \cdot f_{ck}$ <p>For finding <math>A_{st}</math> use the equation</p> $0.87 f_{yk} \cdot A_{st} \cdot d (1 - \frac{A_{st} f_{yk}}{b \cdot d \cdot f_{ck}}) \text{ or } M.R = 0.87 f_{yk} \cdot A_{st} \cdot d (1 - 0.42 \frac{x_u}{d})$

Where

$d$

= effective depth of beam in

mm.  $b$

= width of beam in mm

$x_u$

= depth of actual neutral axis in mm from extreme compression

fibre.  $x_{u,max}$

= depth of critical neutral axis in mm from extreme compression

fibre.

$A_{st}$  = area of tensile reinforcement

$f_{ck}$  = characteristic strength of concrete in MPa.

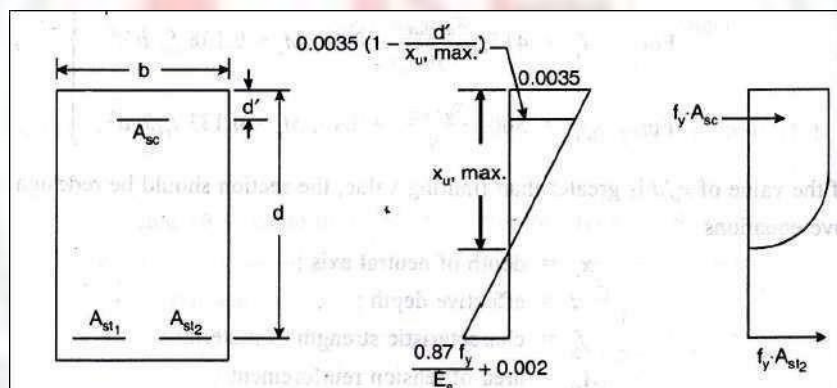
$f_y$  = characteristic strength of steel in MPa.

$M_{u,lim}$  = Limiting Moment of Resistance of a section without compression reinforcement

## Doubly Reinforced Section or sections with Compression Reinforcement

Doubly Reinforced Section sections are adopted when the dimension of the beam has been predetermined from other considerations and the design moment exceeds the moment of resistance of a singly reinforced section. The additional moment of resistance is carried by providing compression reinforcement and additional reinforcement in tension zone. The moment of resistance of a doubly reinforced section is the sum of the limiting moment of resistance  $M_{u,lim}$  of a singly reinforced section and the additional moment of resistance  $M_{u2}$ .

$$M_{u2} = M_u - M_{u,lim}$$



The lever arm for the additional moment of resistance is equal to the distance between the centroids of tension and compression reinforcement,  $(d - d')$ .

$$M_{u2} = 0.87 f_y A_{st2} (d - d') = A_{sc} (f_{sc} - f_{cc}) (d - d')$$

Where:

$A_{st2}$  = Area of additional tensile

reinforcement  $A_{sc}$  = Area of compression

reinforcement

$f_{sc}$  = Stress in compression reinforcement

$f_{cc}$  = Compressive stress in concrete at the level of compression

reinforcement Since the additional reinforcement is balanced by the additional compressive force.

$$A_{sc} (f_{sc} - f_{cc}) = 0.87 f_y A_{st2}$$

The strain at level of compression reinforcement is

$0.0035 \left(1 - \frac{d'}{x_{u, \max.}}\right)$  Total area of reinforcement shall be obtained by

$$A_{st} = A_{st1} + A_{st2}$$

$A_{st1}$  = Area of reinforcement for singly reinforced section for  $M_{u, \lim}$

$$A_{st2} = \frac{A_{sc} (f_{sc} - f_{cc})}{0.87 f_y}$$

### EXAMPLE 4.1

Calculate the area of steel of grade Fe 415 required for section of 250mm wide and overall depth 500mm with effective cover 40mm in M20, if the limit state of moment to be carried by the section is

a) 100 kN m    b) 146 kN m    c) 200 kN m

**SOLUTION:**

$$\text{For } f_y = 415 \text{ N/mm}^2, \quad \frac{x_u}{d} = 0.48$$

$$M_{u, \text{lim}} = 0.36 \cdot \frac{x_{u, \text{max}}}{d} \left( 1 - 0.42 \frac{x_{u, \text{max}}}{d} \right) b \cdot d^2 \cdot f_{ck}$$

$$= 0.36 \times 0.48 \left( 1 - 0.42 \times 0.48 \right) \times 250 \times 460^2 \times 20$$

$$= 146 \times 10^6 \text{ N.mm}$$

a) For  $M_u = 100 \text{ kN.m} < 146 \text{ kN.m}$

Area of steel required is obtained from,  $M_u$

$$M_u = 0.87 \cdot f_{st} \cdot A_{st} \cdot d \left( 1 - \frac{A_{st} f_y}{b \cdot d \cdot f_{ck}} \right)$$

$$100 \times 10^6 = 0.87 \times 415 \times A_{st} \times 460 \left( 1 - \frac{A_{st} \times 415}{250 \times 460 \times 20} \right)$$

$A_{st} = 686 \text{ or } 4850 \text{ mm}^2$ , taking minimum steel  $686 \text{ mm}^2$

b)  $M_u = 146 \text{ kN.m} = M_{u, \text{lim}} = 146$

$$M_u = M_{u, \text{lim}} = 146 \text{ kN.m}$$

Area of tension reinforcement required

$$X_{u,max} = \frac{0.87 f_y A_{st}}{0.36 b d f_{ck}}$$

$$A_{st} = \frac{0.48 \times 0.36 \times 20 \times 250 \times 460}{0.87 \times 415} = 1100 \text{ mm}^2$$

c)  $M_u = 200 \text{ kN.m} > M_{u,lim} = 146 \text{ kN.m}$

Reinforcement is to be provided in the compression zone also along with the reinforcement in tension zone.  $M_u = M_{u,lim} = f_{sc} A_{sc} (d - d')$



$$f_{sc} = \frac{0.0035(x_{u,max} d')}{0.48x_{u,lim}} = \frac{0.0035(0.48 \times 460 - 40)}{0.48 \times 460} = 0.002866$$

$$f_{sc} = 360.8 \text{ N/mm}^2$$

$$(200 - 146) \times 10^6 = 360.8 \times A_{sc} (460 - 40)$$

$$A_{sc} = 356 \text{ mm}^2$$

Ast1 = Area of tension reinforcement corresponding to  $M_{u,lim}$

$$146 \times 10^6 = 0.87 \times 460 \times \left(1 - \frac{A_{st1} \times 415}{250 \times 460 \times 20}\right) \times 415 \times 20 \times A_{st1}$$

$$A_{st1} = 1094 \text{ mm}^2$$

$$A_{st2} = A_{sc} \times f_{sc} / 0.87 \times 415 = 356 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1094 + 356 = 1450 \text{ mm}^2$$

#### EXAMPLE:4.2

Design a rectangular beam which carries a maximum limiting bending moment of 65 kN.m. Use M20 and Fe415 as reinforcement.

At balanced failure condition

$$M_{u,lim} =$$

$$M_{u,lim} =$$

$$M_{u,lim} = 0.36 \times \frac{x_{u,max}}{d} \left(1 - 0.42 \times \frac{x_{u,max}}{d}\right) b \times d^2 \times f_{ck}$$

$$M_{u,lim} = 0.36 \times 0.48 \times 20 \left(1 - 0.42 \times 0.48\right) b d^2$$

$$=2.759bd^2$$

Assuming width of beam as 250 mm

$$d = 307 \text{ mm}$$

Area of reinforcement

$$\frac{X_u}{d} \leq \frac{0.87 f_y A_{st}}{6b.d f_{ck}}$$

$$0.48 = \frac{0.87 \times 415 \times A_{st}}{0.36 \times 250 \times 307}$$

$$A_{st} = 734.66 \text{ mm}^2$$

### EXAMPLE:4.3

Find out the factored moment of resistance of a beam section 300mm wide X 450mm effective depth reinforced with 2 X 20mm diameter bars as compression reinforcement at an effective cover of 50mm and 4 X 25mm diameter bars as tension reinforcement. The materials are M20 grade concrete and Fe 415 HYSD bars.

#### Solution:

Given;

Width =  $b = 300\text{mm}$

Effective depth =  $d = 450\text{mm}$

Cover to compression reinforcement =  $d' = 50\text{mm}$

$$\frac{d'}{d} = \frac{50}{450} \approx 0.11, \text{ next higher value } 0.15 \text{ may be adopted.}$$

$A_{sc}$  = area compression reinforcement =  $2 \pi$

$$16^2 =$$

$628\text{mm}^2$   $A_{st}$  = area of reinforcement in tension = 4

$\times 25^2 = 1964\text{mm}^2$   $f_{sc}$  = stress in compression steel

$$= 342\text{N/mm}^2$$

Equating total force

$$0.36 f_{ck} \cdot b \cdot x_u + f_{sc} \cdot A_{sc} = 0.87 f_y \cdot A_{st}$$

$$0.36 \times 20 \times 300 \times x_u + 628 \times 342 = 0.87 \times 415 \times 1964$$

$$x_u = 228.85\text{mm}$$

But  $x_{u,max} = 0.48d$  for Fe415

$$x_{u,max} = 0.48 \times 450 =$$

216mm So  $x_u > x_{u,max}$ ,

∴ overreinforced

The moment of resistance can be found out by taking moments of compressive forces about centroid of tensile reinforcement.

$$M_u = 2160x_u(450 - 0.42x_u) + 214776(450 - 50) \times 10^{-6}$$

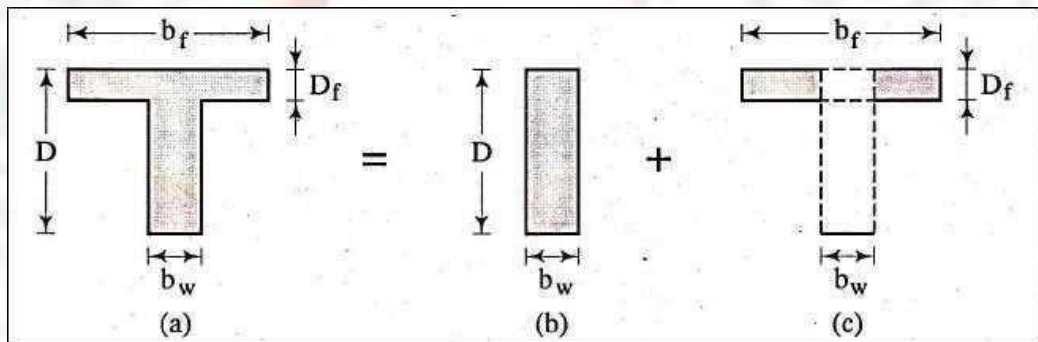
Putting  $x_u =$

$$216 \text{mm } M_u = 25$$

$$3.54 \text{ kN.m}$$

## BEHAVIOR OF 'T' AND 'L' BEAMS (FLANGED BEAM)

A 'T' beam or 'L' beam can be considered as a rectangular beam with dimensions  $b_w$ ,  $D$  plus a flange of size  $(b_f - b_w) \times D_f$ . It is shown in the figure beam (a) is equivalent to beam (b) + beam (c).



The flanged beam analysis and design are analogous to doubly reinforced rectangular beam. In doubly reinforced beams additional compressive is provided by adding reinforcement in compression zone, whereas in flanged beams, this is provided by the slab concrete, where the spanning of the slab is perpendicular to that of beam and slab is in compression zone.

If the spanning of the slab is parallel to that of the beam, some portion of slab can be made to span in the direction perpendicular to that of the beam by adding some reinforcement in the slab.

A flanged beam can be also doubly reinforced.

The moment of resistance of a T beam is sum of the moment of resistance of beam (a) is the sum of moment of resistance of beam (b) and moment of resistance of beam (c).

## CHAPTER-5

### LIMIT STATE OF COLLAPSE IN SHEAR (Design of Shear by LSM)

#### **5.1. SHEAR STRESS IN REINFORCED CONCRETE BEAMS:-**

When a beam is loaded with transverse loads, the Bending Moment (BM) varies from section to section. Shearing stresses in beams are caused by this variation of BM in the beam span. Due to the variation of BM at two sections distance  $dx$  apart, there are unequal bending stresses at the same fibre. This inequality of bending stresses produces a tendency in each horizontal fibre to slide over adjacent horizontal fibre causing horizontal shear stress, which is accompanied by complementary shear stress in vertical direction.

#### **SHEAR CRACKS IN BEAMS:-**

Under the transverse loading, at any section of the beam, there exists both Bending Moment (BM) and Shear Force (V). Depending upon the ratio of Bending Moment (BM) to Shear Force (V) at different sections, there may be three regions of shear cracks in the beam as follows.

- (a) Region I : Region of flexure Cracks.
- (b) Region II : Region of flexure shear Cracks.
- (c) Region III : Region of web shear Cracks or diagonal tension cracks.

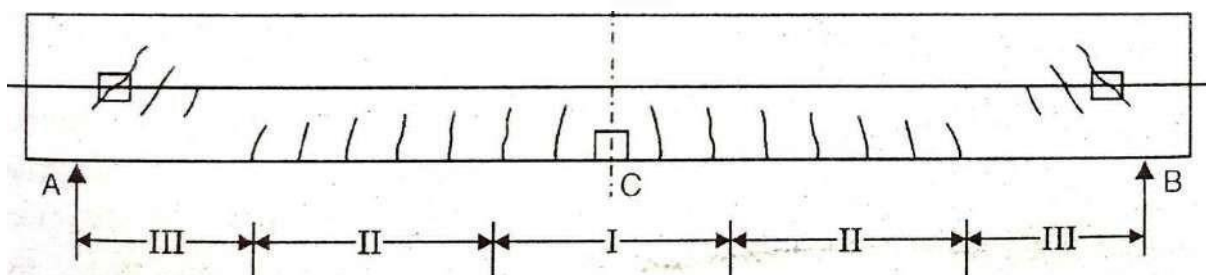




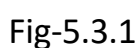
Fig-5.2.1 DIFFERENT REGION OF CRACKS IN BEAMS

**(a) Region I : Region of flexure cracks.**

This region normally occurs adjacent to mid-span where BM is large and shear force is either zero or very small. The principal planes are perpendicular to beam axis. When the principal tensile stress reaches the tensile strength of the concrete (which is quite low) tensile cracks develop vertically. The cracks are known as flexural cracks resulting primarily due to flexure.

(c) RegionII:RegionofwebshearCracks ordiagonaltensioncracks.

## MECHANISM OF SHEAR TRANSFER IN REINFORCE CONCRETE BEAMWITHOUTSHEAR.



- Shear resistance  $V_{cz}$  of the uncracked portion of concrete.
- Vertical Component  $V_{ay}$  of the interface shear or aggregate interlock force  $V_a$  and
- Dowel force  $V_d$  in the tension reinforcement, due to dowel action.

Thus  $V = V_{cz} + V_{ay} + V_d$ .



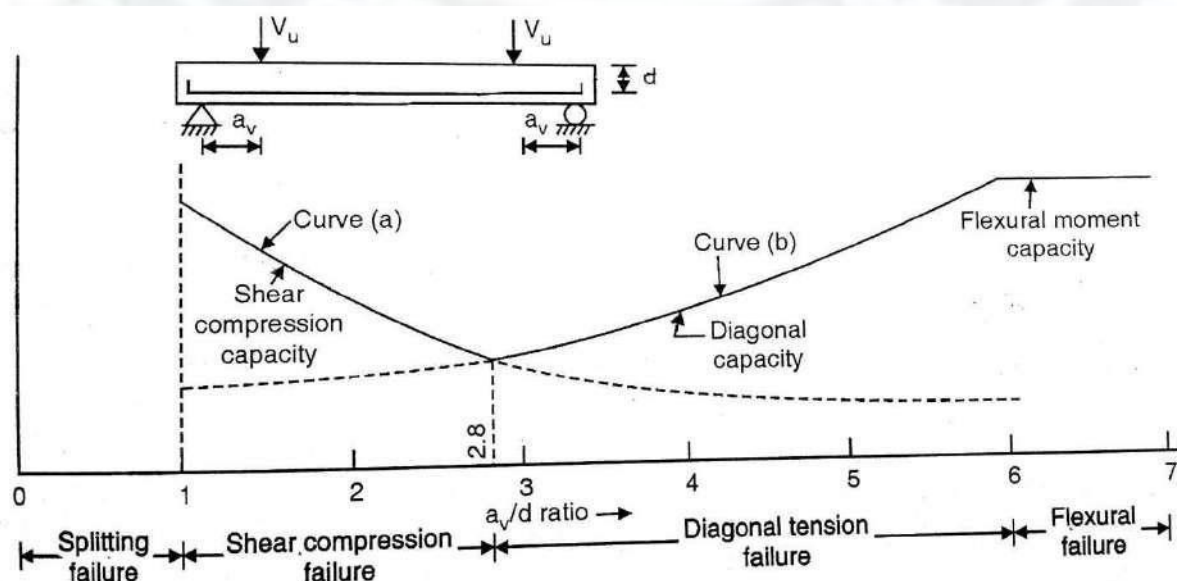
The relative contribution of each of the above three mechanism depend upon the stage of loading and extent of cracking. In the initial stage before the flexural cracking starts, the entire shear is resisted by the shear resistance of the concrete (i.e.  $V = V_c$ ).

As the flexural cracking starts, the interface shear comes into action resulting in the redistribution of stresses. Further extension of flexural cracks results in sharing the shear by the dowel force  $V_d$  of the tension reinforcement. Thus at the final stage of collapse, the shear is transferred by the shear is borne by all the three mechanism expressed by the equation above.

### MODES OF SHEAR FAILURE

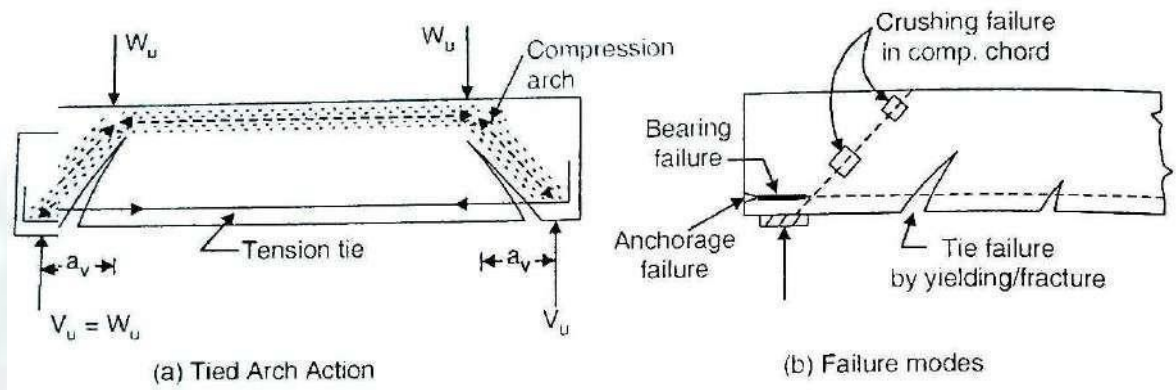
The shear Failure of a R C beam, without shear reinforcement is governed by  $a_v / d$  ratio. A beam may experience following types of shear failure.

1. Case I:  $a_v / d < 1$  : Splitting or compression failure.
2. Case II:  $1 < a_v / d < 2.8$  : Shear compression or shear tension failure.
3. Case III:  $2.8 < a_v / d < 6$  : Diagonal tension failure.
4. Case IV:  $a_v / d > 6$  : Flexure failure



Fig– 5.4.1. EFFECT OF  $a_v / d$  ON SHEAR STRENGTH OF R C BEAM

**CASE I:  $a_v/d < 1$  (Deep Beams): Splitting or compression failure:**



Fig– 5.4.2. CASE I:  $a_v/d < 1$  (DEEP BEAMS)

This case corresponds to a deep beam without shear reinforcements where the inclined cracking transforms the beam into a tied arch (Fig-a). The load is carried by (i) direct compression in the concrete between the load and reaction point by crossing of concrete and by (ii) tension in the longitudinal steel by yielding or fracture or anchorage failure or bearing failure.

**CASE II:  $1 < a_v/d < 2.8$  : Shear compression or shear tension failure.**

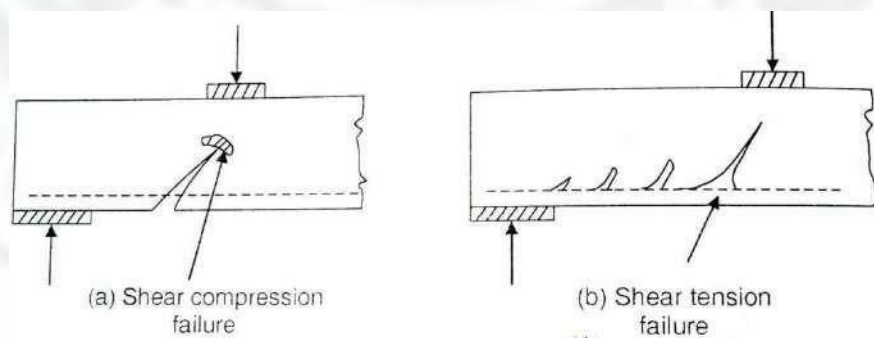


Fig-5.4.3 CASE II:  $1 < a_v/d < 2.8$

This case is common in short beams with  $a_v/d$  ratio between 1 to 2.8, where failure is initiated by an inclined crack— more commonly a flexural shear crack. Fig-a shows the shear compression



failure due to vertical compressive stresses developed in the vicinity of the load. Similarly the vertical compressive stress over the reaction limits the bond splitting and diagonal cracking along the steel. The crack extends towards the tension reinforcement and then propagates along the reinforcements (Fig-b) resulting in the failure of the beam by anchorage failure.





**CASE III:  $2.8 < a_v / d < 6$  : Diagonal tension failure.**

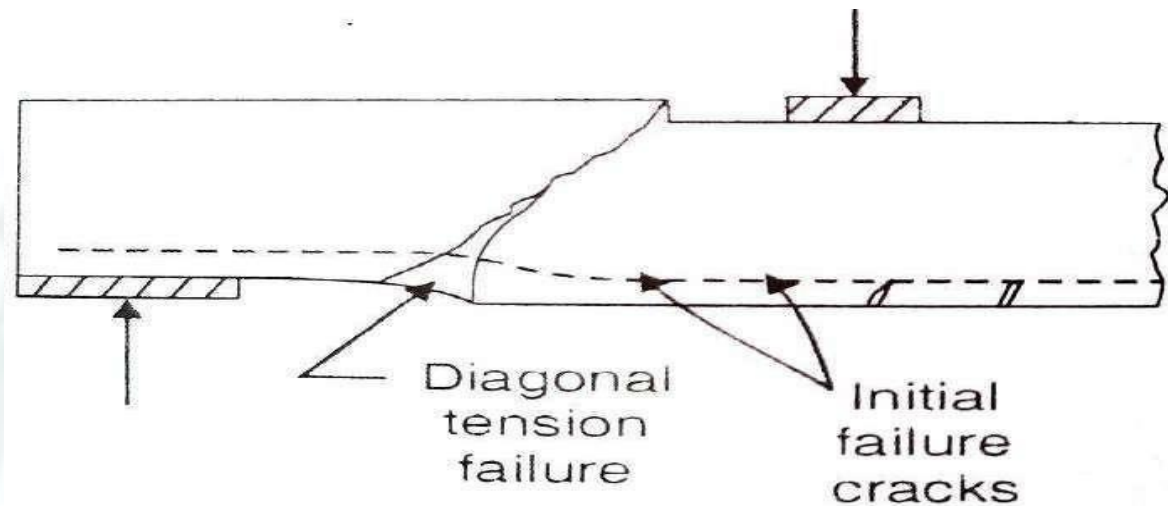


Fig-5.4.4 CASE III:  $2.8 < a_v / d < 6$

Diagonal tension failure occurs when the shear span to the effective depth ratio is in the range of 2.8 to 6. Normal beams have  $a_v / d$  ratio in excess of 2.8. Such beams may fail either in shear or in flexure.

**CASE- IV:  $a_v / d > 6$  : Flexure failure**

Flexural failure is encountered when  $a_v / d$  ratio  $> 6$ . Two cases may be encountered; (i)

under reinforced beam and (ii) over reinforced beam. In the case of under reinforced beam, tension reinforcement is less than the limiting one, due to which failure is initiated by yielding of tension reinforcement, leading to the ultimate failure due to crushing of concrete in compression zone. Such a ductile failure is known as flexural tension failure, which is quite slow giving enough warning. In the over reinforced sections failure occurs due to crushing of concrete in compression zone before yielding of tension reinforcement. Such a failure, known as flexural compression failure is quite sudden.

#### **FACTORS AFFECTING THE SHEAR RESISTANCE OF A RC MEMBER.**

The shear resistance of rectangular beams, without shear reinforcements depends

on the following factors.

1. **Grade of concrete:** Higher grade of concrete has higher characteristic strength which in turn results in (i) higher tensile strength (ii) greater dowel shear resistance (iii) greater aggregate interlock capacity, and (iv) greater concrete strength in compression zone. Hence shear resistance increases with the increase in the grade of concrete.

**2. Percentage and grade of longitudinal tensile reinforcement :** The increase in percentage ( $p_t$ ) of longitudinal tensile reinforcement results in the increase in dowel shear ( $V_d$ ). Due to this reason, the design Codes make the shear strength ( $\tau_c$ ) of concrete a function of  $p_t$  and grade of concrete (see Table 5.1). However, higher grade of steel results in lesser shear resistance of R.C. beam because the percentage of steel ( $p_t$ ) corresponding to a higher grade steel is less than that required for a low grade steel, say mild steel.

**3. Ratio of shear span to effective depth (i.e.  $a_v/d$  ratio) :** As discussed in the previous article, for  $a_v/d$  ratio between 6 and 2.8, the shear capacity, being governed by inclined crack resistance, decrease with decrease in  $a_v/d$  ratio (curve *b* of Fig.5.4.1). However, for a value of  $a_v/d$  less than 2.8, the shear capacity, being dependent on *shear-compression* or *shear-bond capacity*, increases rapidly. The minimum shear capacity is at  $a_v/d$  ratio around 2.8.

**4. Compressive force :** Presence of axial compressive force result in increase of shear capacity. The effect of axial compression on the design shear strength has been taken into account by I.S. Code by increasing the design shear strength by a modification factor  $\delta$ .

**5. Compressive reinforcement :** The shear resistance is found to increase with the increase in the percentage of compressive steel ( $p_c$ ).

**6. Axial tensile force:** Axial tensile force reduces marginally the shear resistance of concrete as per

$$\text{the equation } \delta = 1 - \frac{P_w}{3.45 A E_g}$$

**7. Shear reinforcement:** The shear resistance of a R C Beam increases with the increase in shear reinforcement ratio. This is due to two reasons (i) concrete gets conformed between stirrup spacing and (ii) the shear/web reinforcement itself provides shear resistance.

## 5.6 . DESIGN SHEAR STRENGTH OF CONCRETE WITHOUT SHEAR REINFORCEMENT (IS 456: 2000)

The magnitude of design shear strength ( $v_c$ ) depends basically on the grade of concrete ( $f_{ck}$ ) and the percentage of tension steel ( $P_t$ ). As per IS 456 : 2000 the design shear strength of concrete in beams without shear reinforcement shall be given in table 5.1.

TABLE 5.1 DESIGN SHEAR STRENGTH ( $\tau_c$ ) OF CONCRETE, ( N/mm<sup>2</sup> )

$100 \frac{A_{st}}{bd}$	Grade of concrete					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\leq 0.15$	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.0 and above	0.71	0.82	0.92	0.96	0.99	1.01

### Analytical expression for design shear strength:

The Values of  $\tau_c$  given in the above table by the code are based on the following semi empirical expression (SP24, 1983).

$$\tau_c = \frac{0.85 \sqrt{0.8 f_{ck}} (\sqrt{1 + 5 \beta} - 1)}{6 \beta} \quad \dots\dots 5.6.1$$

where  $\beta = \frac{0.8 f_{ck}}{6.89 p_t}$ , but not less than 1  
 $p_t = \frac{100 A_{st}}{bd}$  (percentage steel in rib width only)  
 $0.8 f_{ck}$  = cylinder strength in terms of cube strength



0.85 = reduction factor similar to  $1/\gamma_m$

The formula in BS 8110 for design shear strength of concrete is slightly different, and is given by the expression

$$\tau_c = 0.79 (p_t)^{1/3} \left( \frac{400}{d} \right)^{1/4} \left( \frac{1}{\gamma_m} \right) \left( \frac{f_{ck}}{25} \right)^{1/3} \quad \dots 5.6.2$$

where  $\left( \frac{400}{d} \right)$  = the correction factor for depth and should not be less than 1  
 $\left( \frac{f_{ck}}{25} \right)$  = the correction factor for the strength of concrete and should not be greater than 40

$$\gamma_m = 1.25$$

$p_t$  = percentage steel, the value of which should not exceed 3

#### Design shear strength for solid slabs

For solid slabs, the design shear strength for concrete shall be  $\tau_c \cdot k$ , where  $k$  has the values given in Table 5.2

TABLE 5.2 VALUES OF  $k$  (IS 456 : 2000)

Overall depth of slab (mm)	300 or more	275	250	225	200	175	150 or less
$k$	1.00	1.05	1.10	1.15	1.20	1.25	1.30

Note : The above provision shall not apply to flat slabs.

#### Shear strength of members under axial compression (IS 456 : 2000)

For members subjected to axial compression  $P_{uc}$ , the design shear strength of concrete, given in Table 7.1, shall be multiplied by the following factor :

$$\delta = 1 + \frac{3 P_{uc}}{A_g \cdot f_{ck}}, \text{ but not exceeding } 1.5 \quad \dots 5.6.3$$

where  $P_{uc}$  = factored axial compressive force in Newtons

$A_g$  = gross area of concrete section in  $\text{mm}^2$ , and

$f_{ck}$  = characteristic compressive strength of concrete, in  $\text{N/mm}^2$

#### Shear strength of members under axial tension (ACI Code, 1989) :

Though it is evident that there is some reduction in design shear strength of a member under axial tension, IS Code (IS 456 : 2000) does not explicitly mention this case. However, the following simplified expression for  $\delta$ , based on ACI Code (1989) may be used :

$$\delta = 1 - \frac{P_{ut}}{3.45 A_g} \quad \dots 5.6.4$$

where  $P_{ut}$  = factored axial tensile force in Newtons.

#### Maximum shear stress in concrete with shear reinforcement (IS 456 : 2000)

##### (a) Maximum shear stress in beams :

Under no circumstances, even with shear reinforcement, shall the nominal shear stress ( $\tau_v$ ) in beams exceed  $\tau_{c, \max}$  given in Table 5.3

**TABLE 5.3. MAXIMUM SHEAR STRESS  $v_{c,max}$  (N/mm<sup>2</sup>)**

Grade of concrete	M15	M20	M25	M30	M35	M40 & above
$v_{c,max}$ (N/mm <sup>2</sup> )	2.5	2.8	3.1	3.5	3.7	4.0

**(b) Maximum shear stress in solid slabs**

For solid slab the nominal shear stress shall not exceed half the appropriate values given in table 5.3.

**WEB REINFORCEMENT FOR DIAGONAL TENSION:**

As stated earlier, proper reinforcement must be provided to resist the diagonal tension. The shear resisted by shear reinforcement can be worked out by considering the equilibrium of forces across a potential diagonal crack, which is assumed to be inclined at an angle of 45° with axis of the beam. Fig. 7.11 shows a diagonal crack AB. Let the web reinforcement be inclined at angle  $\alpha$  with the axis of the beam, and be spaced at distance  $s_v$  apart. Let the diagonal crack AB intersect  $n$  number of web reinforcing bars.

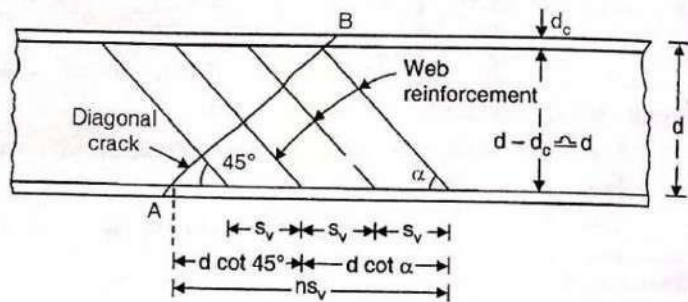


FIG. 5.7.1 SHEAR RESISTED BY WEB STEEL

Let  $V_{us}$  = Ultimate shear carried by shear (or web) reinforcement  
 $f_{yd}$  = design yield stress in web steel =  $0.87 f_y$   
 $n$  = number of bars/links crossing the crack  
 $\alpha$  = inclination of web steel  
 $A_{sv}$  = total cross-section area of each set of bar or link.

The web reinforcement is anchored to the main tensile steel at the bottom, and to the holding bars (at a cover  $d_c$ ) at the top. Hence the vertical component of length of inclined bar =  $(d - d_c)$ . Since  $d_c$  is normally quite small in comparison to  $d$ , we can take  $(d - d_c) \approx d$ , as marked in Fig. 5.7.1. Now, for equilibrium

Shear carried by shear reinforcement = Sum of vertical components of tensile forces developed in shear reinforcement

$$\therefore V_{us} = n A_{sv} f_{yd} \sin \alpha \quad \dots 5.7.1 (a)$$

In order to get the value of  $n$ , we have from geometry,

$$n s_v = d \cot 45^\circ + d \cot \alpha \quad \text{or} \quad n = \frac{d \cot 45^\circ + d \cot \alpha}{s_v} = \frac{d (1 + \cot \alpha)}{s_v} \quad \dots 5.7.1 (b)$$

Substituting the value of  $n$  and  $f_{yd} (= 0.87 f_y)$  in Eq. 5.7.1 (a) we get

$$V_{us} = \frac{d (1 + \cot \alpha)}{s_v} \cdot A_{sv} (0.87 f_y) \sin \alpha = \frac{0.87 f_y A_{sv} d}{s_v} (\sin \alpha + \cos \alpha) \quad \dots 5.7.2$$

Rearranging the above, we get

$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} (\sin \alpha + \cos \alpha) \quad \dots 5.7.2 (a)$$



The above equation gives the spacing of the bars inclined at  $\alpha$  with horizontal.



Here  $A_{sv}$  = Area of C/S of bars  $\times$  No of legs  $= A\phi \times$  No of legs.

### Special Cases:

#### (i) Bars inclined at $45^\circ$ . (i.e. $\alpha = 45^\circ$ )

If  $\alpha = 45^\circ$ , 
$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} (\sqrt{2}) \quad \dots 5.7.3$$

OR 
$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} \sqrt{2} \quad \dots 5.7.3 (a)$$

#### (ii) Bars inclined at $90^\circ$ (i.e. vertical stirrups)

$$V_{us} = \frac{0.87 f_y A_{sv} d}{s_v} \quad \dots 5.7.4$$

OR 
$$s_v = \frac{0.87 f_y A_{sv} d}{V_{us}} \quad \dots 5.7.4 (a)$$

#### (iii) Single bar or single group of bars

For a single bar, or single group of bars, all bent up at the same cross-section, we get from Eq. 5.7.1 (a) taking  $n = 1$

$$V_{us} = 0.87 f_y \cdot A_{sv} \sin \alpha \quad \dots 5.7.5$$

### TYPES OF SHEAR REINFORCEMENT.

Shear reinforcement is necessary if the nominal shear stress ( $v_v$ ) exceeds the design shear stress  $v_c$ . In general shear reinforcement is provided in any one of the following three forms.

- Vertical stirrups
- Bent up bars along with the stirrups.
- Inclined stirrups.

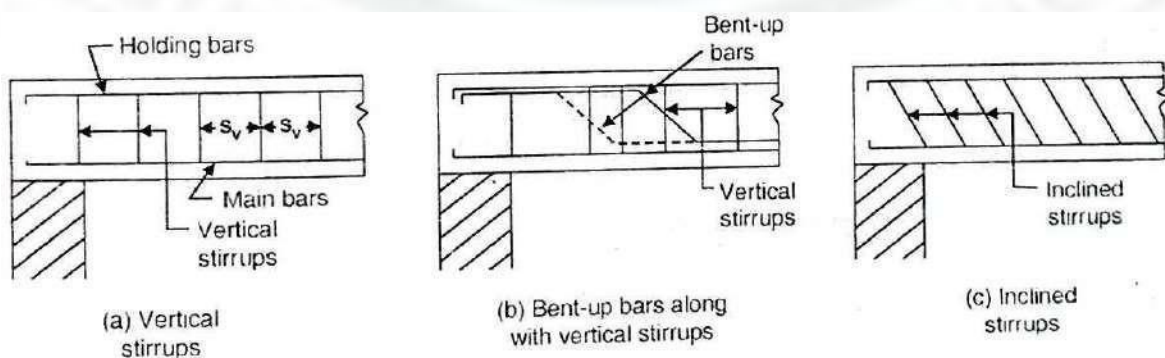


FIG. 5.8 TYPES OF SHEAR REINFORCEMENT

Where bent-

up bars are provided, their contribution towards shear resistance shall not be more than half that of total shear reinforcement.



The total external shear  $V_u$  is jointly resisted by concrete as well as shear reinforcement and is represented by the expression

$$V_u = V_{uc} + V_{us}$$

Where  $V_{uc}$  = Shear strength of concrete  
and  $V_{us}$  = Shear reinforcement.

### VERTICAL STIRRUPS:

Shear reinforcement in the form of vertical stirrups consists of 5 mm to 15 mm dia steel bars bent round the tensile reinforcement where it is anchored to 6 to 12 mm dia. Anchor bars or holding bars. Depending upon the magnitude of the shear stress to be resisted, a stirrup may be one legged, two legged, four legged or multi legged, as shown in Figure.

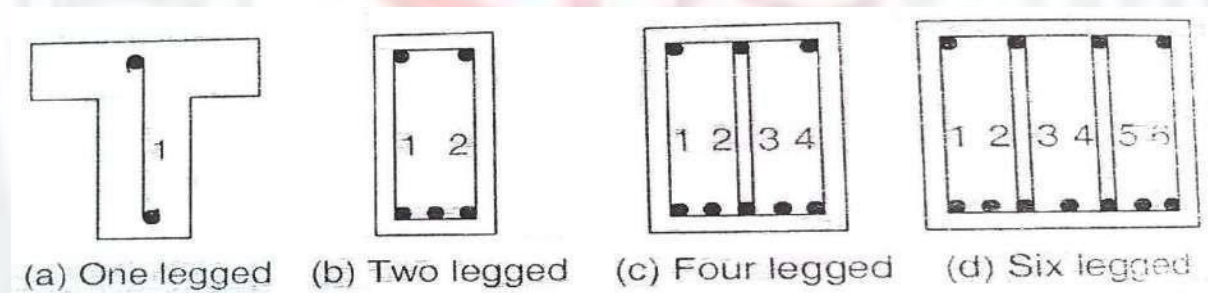


FIG-5.9. FORMS OF VERTICAL STIRRUPS

The strength of shear reinforcement in the form of vertical stirrups is given by

$$V_{uc} = \frac{0.87 f_y A_s}{c d s_v} \quad \dots\dots\dots 5.9.1$$

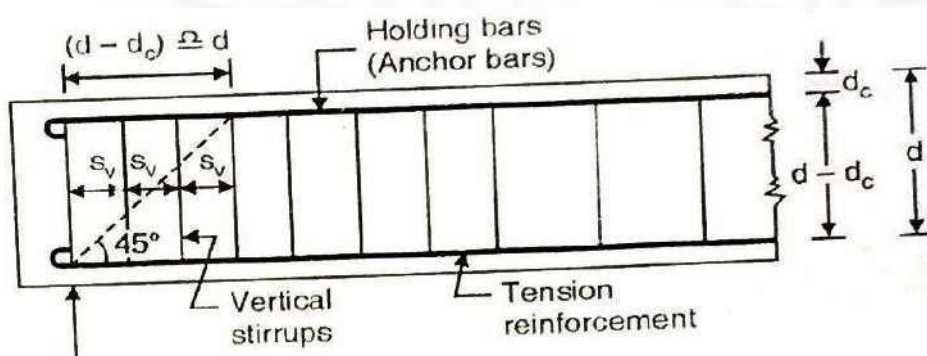


FIG. 5.9.1 SPACING OF VERTICAL STIRRUPS

Here  $A_{sv} = m A_{\phi}$ , where  $m$  = no of legs in the stirrups and  $A_{\phi}$  = Area of c/s of the bar stirrups.



Let us assume that in absence of shear reinforcement, the beam fails in diagonal tension, the inclination of the tension crack being at  $45^\circ$  to the axis of the beam and extended up to a horizontal distance equal to  $(d - d_c) = d$

Hence No of stirrups resisting shear force  $= d/S_v$ , Or,

---

### **MINIMUM SHEAR REINFORCEMENT (IS 456 : 2000)**

The shear reinforcement in the form of stirrups remain unstressed till the diagonal crack occurs at the critical location. However, the instant a diagonal crack occurs. The web reinforcement receives sudden increase in stress. If web reinforcement is not provided. Shear failure may occur without giving any warning. The code therefore, specifies that all the beams should be provided with at least some minimum reinforcement called nominal shear reinforcement even if nominal shear stress is less than the design shear stress of concrete.



## Reasons for providing minimum shear reinforcement:

1. It prevents *sudden* shear failure with the formation of diagonal tension crack, and imparts *ductility* to provide sufficient warning of impending failure. Thus *brittle shear failure* is prevented.
2. It guards against any sudden failure of a beam if concrete cover bursts and bond to tension steel is lost.
3. It holds the main reinforcements in place while pouring the concrete. Thus minimum requirement of cover and clear distance between longitudinal bars are maintained.
4. It acts as necessary ties for the compression steel (if any) and makes it effective.
5. It prevents pressing down of the longitudinal reinforcement, thereby maintaining the *dowel capacity*.
6. It confines the concrete, thereby increasing its strength and rotation capacity.
7. It prevents failure that can be caused by tension due to shrinkage and thermal stresses and internal cracking in the beam.

As per IS 456 : 2000, minimum shear reinforcement in the form of stirrups shall be provided such that

$$\frac{A_{sv}}{b s_v} \geq \frac{0.4}{0.87 f_y} \quad \dots 5.10.1$$

where

$A_{sv}$  = total cross-sectional area of stirrup legs effective in shear.

$s_v$  = stirrup spacing along the length of the member

$b$  = breadth of beam or breadth of the web of flanged beam.

$f_y$  = characteristic strength of stirrup reinforcement in  $\text{N/mm}^2$ , which shall not be taken greater than  $415 \text{ N/mm}^2$ .

Hence spacing based on minimum shear reinforcement is given by

$$s_v \leq \frac{0.87 f_y \cdot A_{sv}}{0.4 b} \leq \frac{2.175 f_y A_{sv}}{b} \quad \dots 5.10.2$$

However, where the maximum shear stress calculated is less than half the permissible value, and in members of minor structural importance such as lintels, this provision need not be complied with.

### Shear resistance of minimum shear reinforcement

The shear resistance of minimum reinforcement envisaged in Eq. 5.10.1 is found by substituting the value of  $\frac{0.87 f_y \cdot A_{sv}}{s_v} = 0.4 b$

$$\text{Thus, } V_{us, min} = \left( 0.87 \frac{f_y A_{sv}}{s_v} \right) d = (0.4 b) d = 0.4 b d \quad \dots 5.10.3$$

Thus, shear carried by concrete and that carried by minimum stirrups is given by

$$V_{u, min} = \tau_c \cdot b d + 0.4 b d \quad \dots 5.10.4$$

### MAXIMUM SPACING OF SHEAR REINFORCEMENT:-

The maximum spacing of shear reinforcement measured along the axis of the member shall not exceed  $0.75d$  for vertical stirrups and  $d$  for inclined stirrups at  $45^\circ$ , where  $d$  is the effective depth of the section under consideration. In no case shall the spacing exceed 300 mm.

#### Example-5.1.

A reinforced concrete beam 250 mm wide and 400 mm effective depth is subjected to a factored shear force of 150 kN at the critical section near supports. The tensile reinforcement at this section near supports is 0.5 percent. Design the shear stirrups near the supports also design the minimum shear reinforcement at the mid-span. Assume M20 concrete and Fe250 mild steel.

**Solution :** Given :  $b = 250$  mm ;  $d = 400$  mm;  $A_{st}/bd = 0.5\% = 0.005$

$$\tau_v = \frac{V_u}{bd} = \frac{150 \times 10^3}{250 \times 400} = 1.5 \text{ N/mm}^2$$

From Table 5.1

$$\tau_c = 0.48 \text{ N/mm}^2 \text{ for M 20 concrete and } 100 A_{st}/bd = 0.5$$

Also, from Table 5.3  $\tau_{c, max} = 2.8 \text{ N/mm}^2$  for M 20 concrete.

Thus,  $\tau_v$  is less than  $\tau_{c, max}$ , but greater than  $\tau_c$ . Hence shear reinforcement is necessary.

$$V_{uc} = \tau_c bd = 0.48 \times 250 \times 400 = 48000 \text{ N}$$

Hence

$$V_{us} = V_u - V_{uc} = 150000 - 48000 = 102000 \text{ N}$$

The shear resistance of nominal stirrups is given by

$$V_{us, min} = 0.4 bd = 0.4 \times 250 \times 400 = 40000 \text{ N} < V_{us}$$

Hence nominal stirrups are *not* sufficient at the section near supports.

We know that 
$$s_v = \frac{0.87 f_y A_{sv}}{V_{us}} \cdot d$$

Using two legged stirrups of 10 mm dia. bars,  $A_{sv} = 2 \frac{\pi}{4} (10)^2 = 157.08 \text{ mm}^2$

$$\therefore s_v = \frac{0.87 \times 250 \times 157.08}{102000} \times 400 \approx 134 \text{ mm}$$

Again we know 
$$s_v = \frac{0.87 f_y A_{sv}}{(\tau_v - \tau_c) b} = \frac{0.87 \times 250 \times 157.08}{(1.5 - 0.48) 250} \approx 134 \text{ mm}$$

Maximum spacing =  $0.75 d$  or 300 mm, which ever is less.

Hence provide 10 mm dia. two legged stirrups @ 130 mm c/c at the section near supports.

At mid-span, the spacing of minimum shear reinforcement for 10 mm  $\phi$  - 2 lgd stirrups is given by Eqn 5.10.2

$$s_v = 0.87 \frac{f_y A_{sv}}{0.4 b} = \frac{0.87 \times 250 \times 157.08}{0.4 \times 250} = 341.6 \text{ mm}$$

However, maximum spacing is limited to  $0.75 d$  or 300 mm which ever is less.

Hence  $s_v = 300$  mm.

Hence provide 10 mm dia. two legged stirrups @ 300 mm c/c at the mid-span.



### **Example- 5.2 -**

A simply supported beam, 300 mm wide and 500 mm effective depth carries a uniformly distributed load of 50 kN/m, including its own weight over an effective span of 6 m. Design the shear reinforcement in the form of vertical stirrups. Assume that the beam contains 0.75 % of reinforcement throughout the length. The concrete is of M 20 grade and steel for stirrups is of Fe 250 grade. Take width of support as 400 mm.

Solution:-  $W_u = 1.5 \times 50 = 75$

kN/m.  $V_{u\max} = W_u L / 2 = (75$

$\times 6) / 2 = 225$  kN

The critical section lies at a distance of  $d = 500$  mm from the face of support or at a distance of  $500 + 400 / 2 = 700$  mm from the centre of the support.

$V_{uD} = 225 - 75 \times 0.7 = 172.5$  kN.

And  $v_v = (172.5 \times 10^3) / (300 \times 500) = 1.15$  N / mm<sup>2</sup>.

From Table-5.1 for  $100 A_s / b d = 0.75\%$ , we get  $v_c = 0.56$  N / mm<sup>2</sup> for

M20 Concrete.  $V_{uc} = 0.56 \times 300 \times 500 = 84000$  N = 84 kN.



Also,  $\tau_v, max = 2.8 \text{ N/mm}^2$  for M 20 concrete. Since  $\tau_v < \tau_v, max$  it is OK.

However,  $\tau_v > \tau_c$ ; hence shear reinforcement is necessary.

$$V_{us} = V_{uD} - V_{uc} = 172500 - 84000 = 88500 \text{ N}$$

Using 10 mm  $\phi$  2-lgd vertical stirrups,  $A_{sv} = 2 \frac{\pi}{4} (10)^2 = 157.1 \text{ mm}^2$

$$\therefore \text{Spacing } s_v = \frac{0.87 f_y \cdot A_{sv} \cdot d}{V_{us}} = \frac{0.87 \times 250 \times 157.1 \times 500}{88500} = 193 \text{ mm} \approx 190 \text{ mm (say)}$$

Spacing corresponding to minimum shear reinforcements is

$$s_v = \frac{0.87 f_y A_{sv}}{0.4 b} = \frac{0.87 \times 250 \times 157.1}{0.4 \times 300} = 284.7 \text{ mm} \approx 280 \text{ mm (say)}$$

However in no case should the spacing exceed  $0.75 d = 0.75 \times 500 = 375 \text{ mm}$ , or 300 mm whichever is less. Hence the spacing is to vary from 190 mm at the end section @ 280 mm at a section distant  $x$  m (say) from the mid-span. Let us locate this section where the S.F. is  $V_{ux}$ .

$$\therefore V_{ux} = \frac{V_{u, max}}{3} x = \frac{225000}{3} x = 75000 x$$

$$\therefore V_{us} = V_{ux} - V_{uc} = 75000 x - 84000$$

$$s_v = 280 = \frac{0.87 \times 250 \times 157.1 \times 500}{75000 x - 84000}$$

from which, we get  $x = 1.93 \text{ m}$  from mid-span or  $1.07 \text{ m}$  from supports. Hence provide 8 mm  $\phi$  2 lgd stirrups at a spacing of 190 mm c/c from supports to a section distant 1.07 m from the centre of either supports. For the remaining length, provide the stirrups @ 280 mm c/c.

## **CHAPTER-6**

### **BOND, ANCHORAGE, DEVELOPMENT LENGTHS, AND SPLICING**

#### **5. BOND**

:

One of the most important assumption in the behavior of reinforced concrete structure is that there is proper 'bond' between concrete and reinforcing bars. The force which prevents the slippage between the two constituent materials is known as bond. In fact, bond is responsible for providing 'strain compatibility' and composite action of concrete and steel. It is through the action of bond resistance that the axial stress (tensile or compressive) in a reinforcing bar can undergo variation from point to point along its length. This is required to accommodate the variation in bending moment along the length of the flexural member.

When steel bars are embedded in concrete, the concrete, after setting, adheres to the surface of the bar and thus resists any force that tends to pull or push this rod. The intensity of this adhesive force is bond stress. The bond stresses are the longitudinal shearing stress acting on the surface between the steel and concrete, along its length. Hence bond stress is also known as interfacial shear. Hence bond stress is the shear stress acting parallel to the reinforcing bar on the interface between the bar and the concrete.

#### **TYPES OF BOND:-**

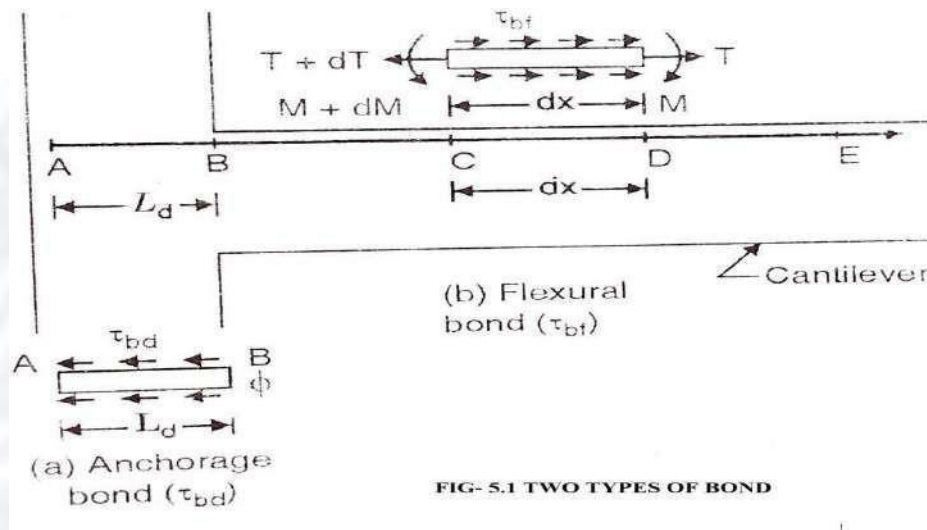
Bond stress along the length of a reinforcing bar may be induced under two loading situations, and accordingly bond stresses are two types:

1. Flexural bond or Local bond



## 2. Anchorage bond or development bond

**Flexural bond ( $\tau_{bf}$ )** is one which arises from the change in tensile force carried by the bar, along its length, due to change in bending moment along the length of the member. Evidently, flexural bond is critical at points where the shear ( $V = dM/dx$ ) is significant. Since this occurs at a particular section, flexural bond stress is known as local bond stress [Fig-5.1(b)].



**Anchorage bond ( $\tau_{bd}$ )** is that which arises over the length of anchorage provided for a bar. It also arises near the end or cutoff point of reinforcing bar. The anchorage bond resists the 'pulling out' of the bar if it is in tension or 'pushing in' of the bar if it is in compression. Fig. [8.1 (a)] shows the situation of anchorage bond over a length  $AB (=L_d)$ . Since bond stresses are developed over specified length  $L_d$ , anchorage bond stress is also known as developed over a specified length  $L_d$ , anchorage bond stress is also known as development bond stress.

Anchoring of reinforcing bars is necessary when the development length of the reinforcement is larger than the structure. Anchorage is used so that the steel's intended tension load can be reached and pop-outs will not occur. Anchorage shapes can take the form of 180 or 90 degree hooks.

## 5.2. ANCHORAGE BOND STRESS:

Fig- 5.2 shows a steel bar embedded in concrete and subjected to a tensile force  $T$ . Due to this force there will be a tendency of bar to slip out and this tendency is resisted by the bond stress developed over the perimeter of the bar, along its length of embedment.

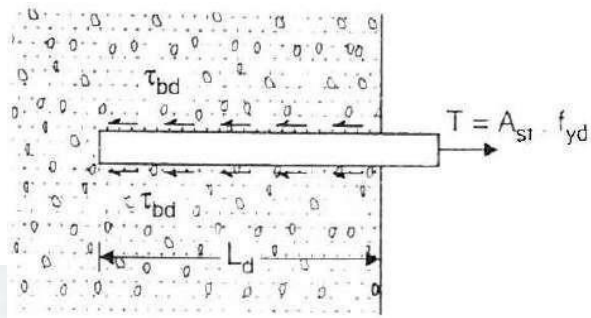


FIG- 5.2

Let us assume that average uniform bond stress is developed along the length. The required length necessary to develop full resisting force is called **Anchorage length** in case of axial tension or compression and **development length** in case of flexural tension and is denoted by  $L_d$ .

### DESIGN BOND STRESS:-

The design bond stress in limit state method for plain bars in tension shall be as given below (Table 6.1)

Table- 6.1

Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
Design bond stress $\tau_{bd}$ (N/mm <sup>2</sup> )	1.2	1.4	1.5	1.7	1.9

**Design bond stresses for deformed bars in tension :** For deformed bars conforming to IS 1786. These values shall be increased by 60%.

**Design bond stress for bars in compression:** For bars in compression, the values of bond stress for tension shall be increased by 25%.

### DEVELOPMENT LENGTH OF BARS (IS 456 : 2000)

The development length is defined as the length of the bar required on either side of the section under consideration, to develop the required stress in steel at that section through bond. The development length  $L_d$  given by

$$L_d = \frac{\sigma_s \phi}{4 \tau_{bd}} = k \phi \dots\dots\dots 5.4.1$$

Where  $\phi$  = nominal diameter of the bar

$\sigma_s$  = stress in bar at the section considered at design

$k$  or  $L_d/\phi$  = development length factor =  $\sigma_s / 4 \tau_{bd}$

**Note:** The development length includes the anchorage values of hooks in tension reinforcement. Taking

$$\sigma_s = 0.87 f_y \text{ at the collapse stage, } k_d = 0.87 f_y / 4 \tau_{bd}$$

#### 5.4.2

For bars in compression, the value of  $\tau_{bd}$  given in table 1.1 are to be increased by 25%. Hence developed length ( $L_{dc}$ ) for bars in compression is given by

$$L_{ds} = \phi \sigma_{sc} / 5 \tau_{bd} \dots\dots\dots 5.4.3$$



Hence the values of  $k_d$  for bars in compression will be  $= 0.87 f_y / 5 \tau_{bd}$

Table 6.2 gives the values of development length factor for various grades of concrete and the various grades of steel, both in tension as well as compression. The values have been rounded-off to the higher side.

**TABLE 6.2-VALUES OF DEVELOPMENT LENGTH FACTOR**

Grade of concrete	M 20			M 25		
Grade of steel	Fe250	Fe415	Fe500	Fe250	Fe415	Fe500
Bars in tension	46	47	57	39	41	49
Bars in comp.	37	38	46	31	33	39

Grade of concrete	M30			M35			M40		
Grade of steel	Fe250	Fe415	Fe500	Fe250	Fe415	Fe500	Fe250	Fe415	Fe500
Bars in tension	37	38	46	32	34	40	29	30	36
Bars in comp.	29	31	37	26	27	32	23	24	29

**Note :** When the actual reinforcement provided is more than that theoretically required, so that the actual stress ( $\sigma_s$ ) in steel is less than the full design stress ( $0.87 f_y$ ), the development length required may be reduced by the following relation:

$$\text{Reduced development length } L_{dr} = L_d (A_{st \text{ required}} \div A_{st \text{ provided}})$$

This principle is used in the design of footing and other short bending members where bond is critical. By providing more steel, the bond requirements are satisfied.

**Bars bundled in contact :** The development length of each bar bundled bars shall be that for the individual by 10% for two bars in contact, 20% for three bars in contact and 33% for four bars in contact.

#### **STANDARD HOOKS & BENDS FOR END ANCHORAGE ANCHORAGE LENGTH**

The development length required at the end of a bar is known as *anchorage length*. However, in the case of development length, the force in the bar is developed by transfer of force from concrete to steel, while in the case of anchorage length, there is dissipation of force from steel to concrete.

Quite often, space available at the end of beam is limited to accommodate the full development length  $L_d$ . In that case, hooks or bends are provided. The anchorage value ( $L_a$ ) of hooks or bend is accounted as contribution to the development length  $L_d$ .

Fig. 5.5 (ai) shows a semi-circular hook, fully dimensioned, with respect to a factor  $K$ . The value of  $K$  is taken as 2 in the case of mild steel conforming to IS : 432-1966, (specifications for Mild-Steel and Medium Tensile Steel bars and Hard-Drawn steel wires for concrete reinforcement) or IS : 1139-1959. (specifications for 'Hot rolled mild steel and medium tensile steel deformed bars for concrete reinforcement'). The hook with  $K = 2$  is shown in Fig. 5.5 (aii) with equivalent horizontal length of the hook. For the case of Medium Tensile Steel conforming to IS : 432-1966 or IS : 1139-1959,  $K$  is taken as 3. In the case of cold worked steel conforming to IS : 1986-1961, (specifications for cold twisted steel bars for concrete reinforcement),  $K$  is taken as 4. In the case of bars above 25 mm, however, it is desirable to increase the value of  $K$  to 3, 4 and 6 respectively.

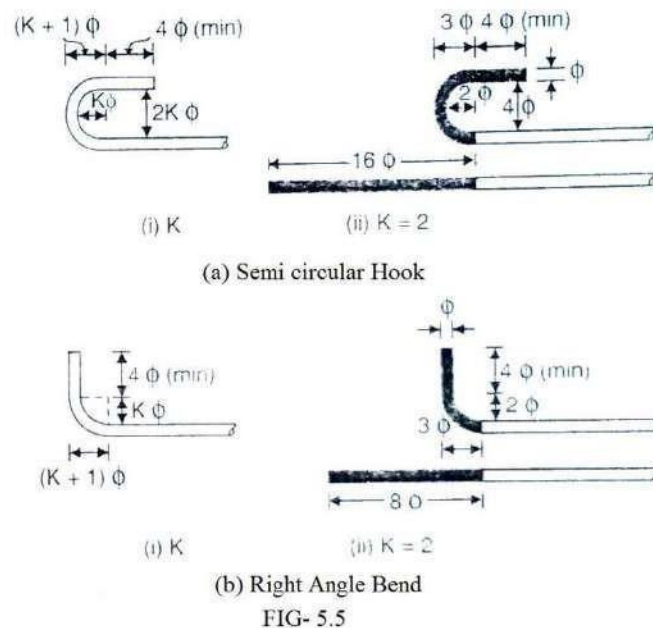


Fig-

5.5 shows a right angled bend, with dimensions in terms of  $K$ , the value of which may be taken as 2 for ordinary mild steel for diameters below 25 mm and 3 for diameters above 25 mm.

In the case of deformed bars, the value of bond stress for various grades of concrete is greater by 60% than the plain bars. Hence deformed bars may be used without hooks, provided anchorage requirements are adequately met with.

### CODER REQUIREMENTS FOR ANCHORING REINFORCING BARS (IS 456:2000)

- (i) **Anchoring Bars in Tension :-** Deformed bars may be used without end anchorages provided development length required is satisfied. Hooks should normally be provided for plain bars in tension. The anchorage value of bend shall be taken as 4 times the diameter of the bar for each  $45^\circ$  bend subject to a maximum of 16 times the diameter of the bar. The

anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

- (ii) **Anchoring Bars in Compression :-** The anchorage length of straight bar in compression shall be equal to the development length of bars in compression. The projected length of





hooks, bends and straight lengths beyond bends if provided for a bar in compression, shall be considered for development length.

(iii) **Anchoring Shear Reinforcement:-**

**Inclined bars :-** The development length shall be as for bars in tension ; this length shall be measured as under : (1) in tension zone from the end of the sloping or inclined portion of the bar and (2) in the compression zone, from mid depth of the beam.

**Stirrups :-** Notwithstanding any of the provisions of this standard, in case of secondary reinforcement , such as stirrups and traverse ties, complete development lengths and anchorages shall be deemed to have been provided when the bar is bent through an angle of at least  $90^\circ$  round a bar of at least its own diameter and is continued beyond the end of the curve for a length of at least eight diameters, or when the bar is bent through an angle of  $135^\circ$  and is continued beyond the end of curve for a length of at least six bar diameters or when the bar is bent through an angle of  $180^\circ$  and is continued beyond the end of the curve for a length at least four bar diameters.

**CHECKING DEVELOPMENT LENGTH OF TENSION BARS:-**

As stated earlier, the computed stress ( $\sigma_s$ ) in a reinforcing bar, at every section must be developed on both the sides of section. This is done by providing development length  $L_d$  to both sides of the section. Such a development length is usually available at mid-span location where positive (or sagging) B.M. is maximum for simply supported beams. Similarly, such a development length is usually available at the intermediate support of a continuous beam where negative (or hogging) B.M. is maximum. Hence no special checking may be necessary in such locations. However special checking for development length is essential at the following locations:

1. At simple supports
2. At cantilever supports
3. In flexural members that have relatively short spans
4. At points of contraflexure
5. At laps or splices
6. At points of bar cutoff
7. For stirrups and transverse ties.

**DEVELOPMENT LENGTH REQUIREMENTS AT SIMPLE SUPPORTS**



### **:DIAMETER OF REINFORCING BARS:-**

The code stipulates that at the simple supports (and at the point of inflection), the positive moment tension reinforcement shall be limited to a diameter such that

$$L_d \leq M_1 / V + L_o \dots\dots\dots 5.8.1$$

Where  $L_d$  = development length computed for design stress  $f_{yd}$  ( $= 0.87 f_y$ ) from Eq<sup>n</sup>

$M_1$  = Moments resistance of the section assuming all reinforcement at the section to be stressed to  $f_{yd}$  ( $= 0.87 f_y$ )

$V$  = Shear force at the section due to design loads

$L_o$  = sum of anchorage beyond the centre of supports and the equivalent anchorage value of any hook or mechanical anchorage at the simple support (At the point of inflexion,  $L_o$  is limited to  $d$  or  $12\phi$  whichever ever is greater).

The code further recommends that the value of  $M_1/V$  in eq<sup>n</sup> - 5.8.1 may be increased by

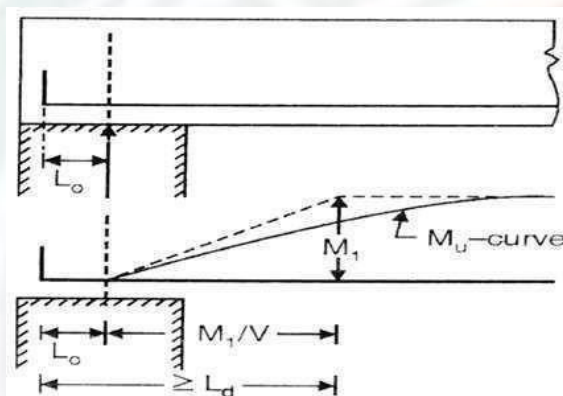


FIG-5.8.1

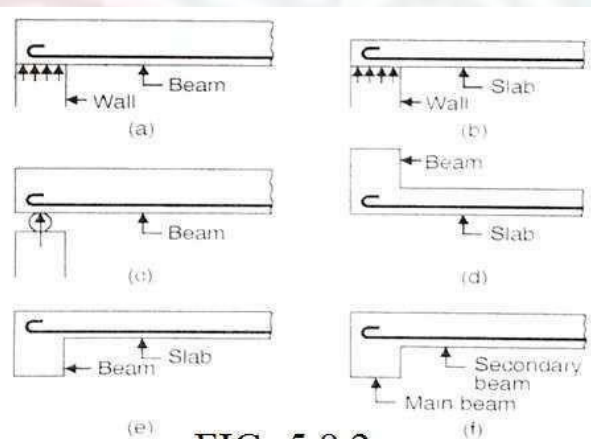


FIG- 5.8.2

30% when the ends of the reinforcement are confined by a compressive reaction. This condition of confinement of reinforcing bars may not be available at all the types of simple supports.

Four type of simple supports are shown in fig-5.8.2. In fig- 5.8.2 (a) , the beam is simply supported on a wall which offers a compressive reaction which confines the ends of reinforcement. Hence a factor 1.3 will be applicable. However in fig- 5.8.1 (c) and (d) though a simple support is available , the reaction does not confine the ends of the reinforcement, hence the factor 1.3 will not be applicable with  $M_1/V$  term. Similarly for the case of a slab connected to a beam Fig- 5.8.2 (e) or for the case of secondary beam connected to a main beam [Fig-5.8.2(f)]

Tensile reaction is induced and hence a factor 1.3 will not be available.

Thus at simple supports where the compressive reaction confines the ends of reinforcing bars we have  $L_d \leq 1.3M_1/V + L_0$  ..... 5.8.2

**Computation of the Moment of Resistance  $M_1$  of bars available at supports:**

In eqn 5.8.1 ,  $M_1$  = Moment of Resistance of the section corresponding to the area of steel ( $A_{st}$ ) continued into the support and stressed to design stress equal to design stress equal to  $0.87f_y$ .

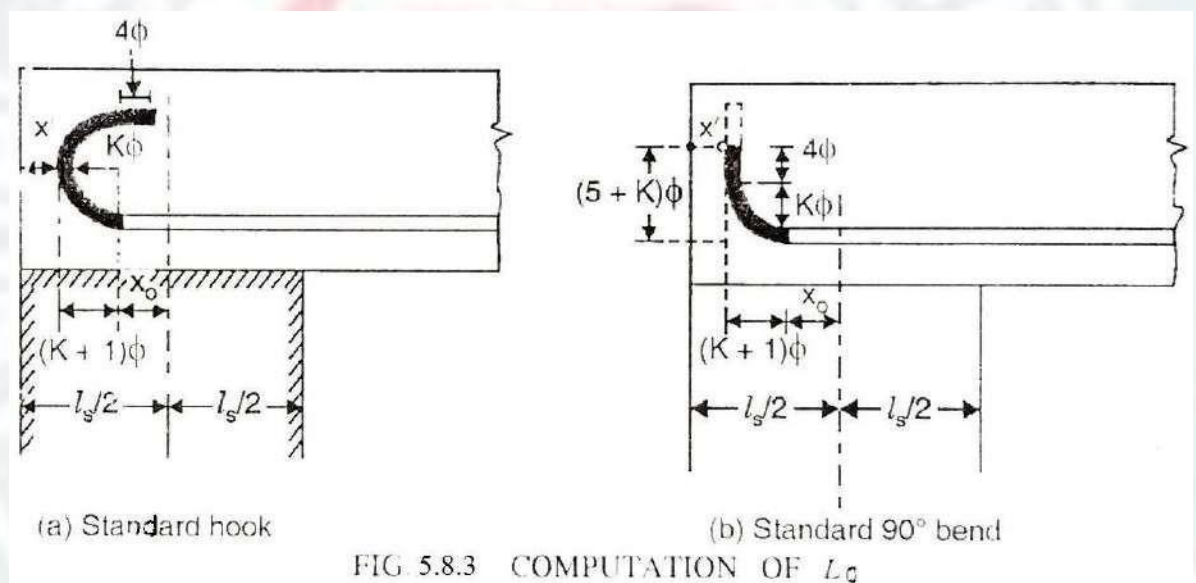
$$M_1 = 0.87 f_y A_{st} (d - 0.416 X_u) \dots\dots\dots 5.8.3$$

$$\text{Where } X_u = 0.87 f_y A_{st} / 0.36 f_{ck} b \dots\dots\dots 5.83(a)$$

### Computation of Length ( $L_0$ ) :

For the computation of  $L_0$ , the support width should be known. Fig- 5.8.3 (a) and (b) show a beam with end support with a standard hook and  $90^\circ$  bend respectively.

Let  $X$  be the side cover to the hook ( Or bend) and  $X_0$  be the distance of the beginning of the hook (Or Bend) from the centerline of the support.



(a) **Case-I : Standard Hook at the end [Fig-5.8.3(a)]:-** The dark portion shows the hook which has an anchorage value of 16 $\phi$  ( IS 456: 2000) for all types of steel. The distance of the beginning of the hook from its apex of the semi circle is equal to  $(K+1)3\phi$ . For mild steel bars  $K=2$  and for HYSD bars,  $K=4$ , Hence the distance  $3\phi$  for mild steel and  $12\phi$  for HYSD bars. Let  $l_s$  be the width of the support.





### CONDITIONS FOR CURTAILMENT OF REINFORCEMENT

In most of the cases, the B.M. varies appreciably along the span of the beam. From the point of view of economy, the moment of resistance of the beam should be reduced along the span according to the variation of B.M. This is effectively achieved by reducing the area of reinforcement, i.e. by curtailing the reinforcement provided for maximum B.M. In general, all steel, whether in tension or in compression, should extend  $d$  or  $12 \phi$  (whichever is greater) beyond the theoretical point of cut off (TPC).

### Conditions for termination of tension reinforcement in flexural members:

Curtailment of Flexural tension reinforcement results in the loss of shear strength in the \_\_\_\_\_ region of cut off and hence it is necessary to make provision to guard against such loss. Flexural reinforcement shall not be terminated in a \_\_\_\_\_ tension zone unless any one of the following condition is satisfied.

(a) The shear at the cutoff point does not exceed two thirds that permitted, including the shear strength of web reinforcement. In other words, the total *shear capacity* shall be atleast 1.5 times the applied shear at the point of curtailment, thus

$$V_u \leq \frac{2}{3} (V_{uc} + V_{us}) \quad \text{or} \quad V_{uc} + V_{us} \geq 1.5 V_u$$

Where  $V_{uc}$  = shear capacity of concrete, based on continuing reinforcement only.

$V_{us}$  = shear capacity of shear reinforcement

$V_u$  = applied shear at the point of curtailment.

(b) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from cutoff point equal to three fourth the effective depth of the member. Excess area of shear reinforcement is given by :

$$\text{Excess} \quad A_{sv} \geq \frac{0.4 b s_v}{f_y}$$

where

$$s_v \leq \frac{d}{8 \beta_b} \leq \frac{0.87 f_y A_{sv}}{0.4 b}$$

$$\beta_b = \frac{\text{area of bars cutoff at the section}}{\text{total area of bars at the section}}$$

(c) For 36 mm or smaller bars, the continuing bars provide double the area required for flexure at the cutoff point and the shear does not exceed three fourth that permitted.

Thus,

$$M_{ur} \geq 2 M_u$$

and

$$V_{uc} + V_{us} \geq 1.33 V_u$$

where

$M_{ur}$  = moment of resistance of remaining (or continued) bars

$M_u$  = B.M. at cutoff point ;  $V_u$  = S.F. at cutoff point

## 5.9 DEVELOPMENT LENGTH AT POINT OF INFLEXION

Fig. 8.8 shows the conditions at a point of inflection (P.I.) As already indicated in § 8.11, the Code states that the following condition be satisfied

$$\left( \frac{M_i}{V} + L_0 \right) \geq L_d \quad \dots\dots 5.9.1$$

where  $L_0$  should not be greater than  $d$  or  $12 \phi$  whichever is greater, and  $V$  is the shear force at the point of inflection.

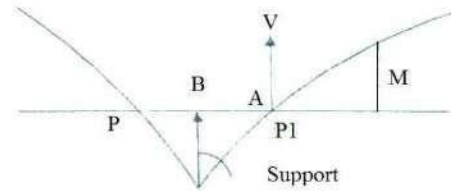


FIG. 5.9 DEVELOPMENT LENGTH AT A POINT OF INFLECTION

## SPLICING:

- (a) The purpose of splicing is to transfer effectively the axial force from the terminating bar to the connecting bar with the same line of action at the junction. [Fig- 5.10(a)].

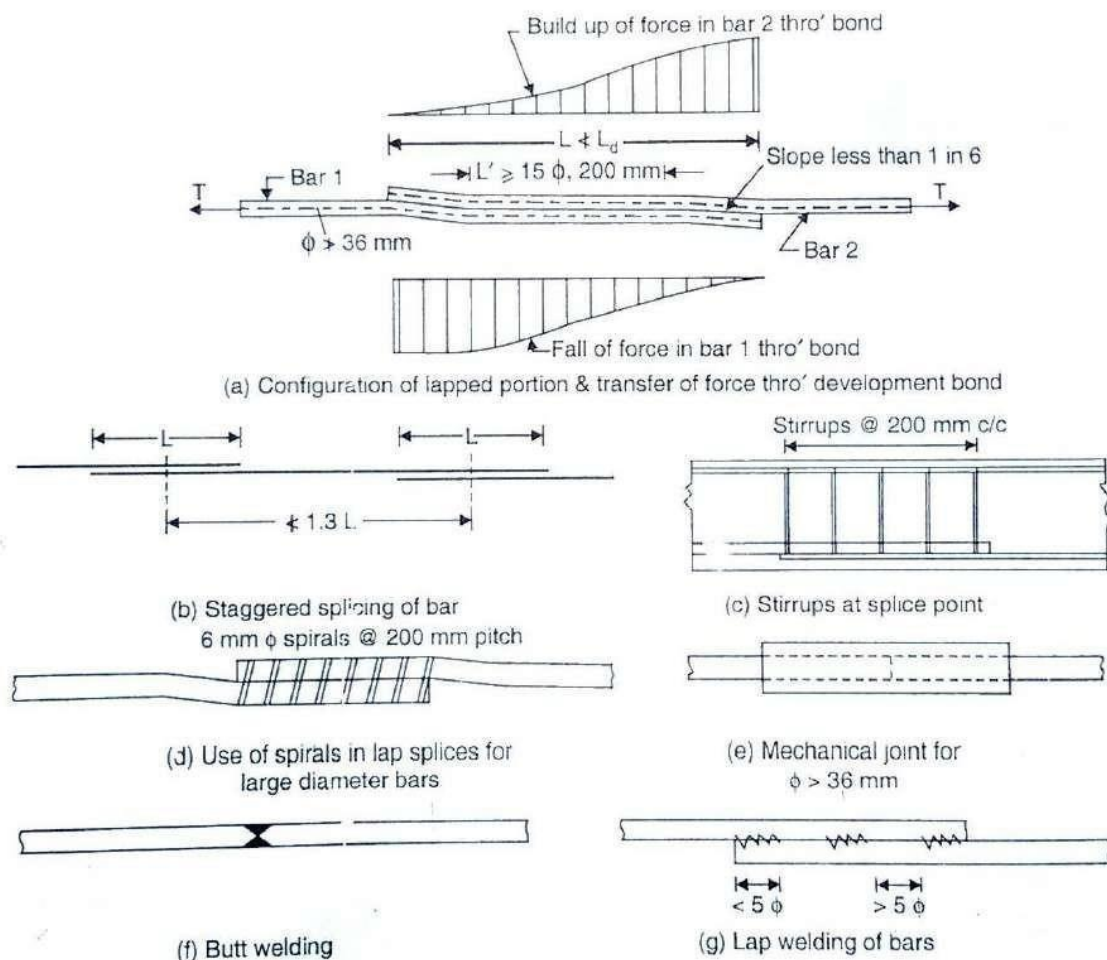


FIG- 5.10 REINFORCEMENT SPLICING



Slicing of a bar is essential in the field due to either the requirements of construction or non-availability of bars of desired length. The Figures given are as per the recommendation of the IS 456 :2000.

- (a) Lap splices shall not be used for bars larger than 36 mm. For larger diameters bars may be welded. In case where welding is not practicable, lapping of bars larger than 36 mm may be permitted, in which case additional spiral should be provided around the lapped bars [Fig-5.10(d)].

(b) Lap splices shall be considered as staggered if the centre to centre distance of the splices is not less than 1.3 times the lap length calculated as described in (c).

(c) The lap length including anchorage value of hooks for bars in *flexural tension* shall be  $L_d$  or  $30 \phi$  whichever is greater and for *direct tension* shall be  $2 L_d$  or  $30 \phi$  whichever is greater. The *straight length* ( $L'$ ) of the lap shall not be less than  $15 \phi$  or 200 mm (Fig. 5.10 [a]) The following provisions shall also apply :

(1) Top of a section as cast and the minimum cover is less than twice the diameter of the lapped bar, the lapped length shall be increased by a factor of 1.4.

(2) Corner of a section and minimum cover to either face is less than twice the diameter of the lapped bar or where the clear distance between adjacent laps is less than 75 mm or 6 times the diameter of lapped bar, whichever is greater, the lap length should be increased by a factor of 1.4.

Where both conditions (1) and (2) apply, the lap length should be increased by a factor of 2.0.

**Note :** Splices in tension members shall be enclosed in spirals made of bars not less than 6 mm diameter with pitch not more than 100 mm.

(d) The lap length in compression shall be equal to the development length in compression, but not less than  $24 \phi$ .

(e) When bars of two different diameters are to be spliced, the lap length shall be calculated on the basis of diameter of the smaller bar.

(f) When splicing of welded wire fabric is to be carried out, lap splices of wires shall be made so that overlap measured between the extreme cross wires shall be not less than spacing of cross wires plus 100 mm.

(g) In case of bundled bars, lapped splices of bundled bars shall be made by splicing one bar at a time : such individual splices within a bundle shall be staggered.

### Strength of Welds:

The following values may be used where the strength of weld has been proved by tests to be at least as great as that of the parent bars.

#### (a) Splices in compression:

For welded splices and mechanical connection, 100 percent of the design strength of joined bars.

#### (b) Splices in tension:

- (1) 80% of the design strength of welded bars (100% if welding is strictly supervised and if at any cross-section of the member not more than 20% of the tensile reinforcement is welded)

(2) 100% of the design strength of mechanical connection.

**End Bearing Splices:** End bearing splices should be used only for bars in compression. These are of square cut and concentric bearing ensured by suitable devices.



### EXAMPLE-6.1

A SIMPLY SUPPORTED IS 25 cm X50cm and has 2 – 20 mm TOR bars going into the support. If the shear force at the center of the support is 110 KN at working loads, determine the anchorage length. assume M20 mix and Fe 415 grade TOR steel.

#### Solution:-

For a load factor equal to 1.5, the factored SF =  $1.5 \times 110 = 165$  kN.

Assuming 25 mm clear cover to the longitudinal bars

Effective depth =  $500 - 25 - 20/2 = 465$  mm.

Characteristic strength of TOR steel  $\sigma_y = 415 \text{ N/mm}^2$

Moment of resistance  $M_1 = 0.87 \sigma_y A_t (d - 0.42 x)$

$$x = \frac{0.87 \sigma_y A_t}{0.36 \sigma_{ck} b} = \frac{0.87 \times 415 \times 628}{0.36 \times 20 \times 250} = 126 \text{ mm} < x_m \quad \text{OK}$$

or  $M_1 = 0.87 \times 415 \times 2 \times \pi/4 \times 20^2 (465 - 0.42 \times 126) = 93.45 \times 10^6 \text{ Nmm}$

Bond stress  $\tau_{bd} = 1.2 \text{ N/mm}^2$  for M20 mix. It can be increased by 60% in case of TOR bars.

$$\text{Development length } L_d = \frac{\phi \sigma_s}{4 \tau_{bd}} = \frac{0.87 \times 415 \phi}{4 \times (1.6 \times 1.2)} = 47 \phi$$

If the bar is given a 90° bend at the centre of support, its anchorage value

$$L_o = 8 \phi = 8 \times 20 = 160 \text{ mm}$$

$$L_d \leq 1.3 M_1/V + L_o$$

$$47 \phi \leq \left[ \frac{1.3 \times 93.45 \times 10^6}{165 \times 1000} \right] + 160$$

or,  $\phi \leq 19 \text{ mm}$



Since actual bar diameter of 20 mm is greater than 19 mm, there is a need to increase the anchorage length. Let us increase the anchorage length  $L_o$  to 240 mm. It gives

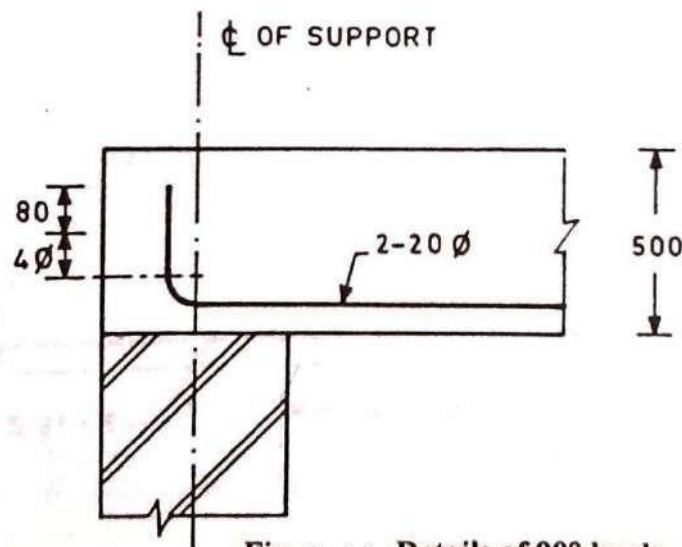
$$\phi \leq 20.8 \text{ mm} \quad \text{OK}$$

The arrangement of 90° bend is shown in Fig. 8.19a.

*Alternatively*

Provide a U bend at the centre of support, its anchorage value,

$$L_o = 16 \phi = 320 \text{ mm}$$



**Fig. Ex 1.1 Details of 90° hook**

$$L_d \leq 1.3 M_1 / V + L_o.$$

$$47 \phi \leq \left[ \frac{1.3 \times 93.45 \times 10^6}{1.6 \times 200} \right] + 320$$

$$\text{Or. } \phi \leq 22.47 \text{ mm}$$

Actual bar diameter provided is 20 mm <

22.47 mm. The arrangement of U-

Bend is shown in Fig-Ex1.2.

In High strength reinforced bars U-Bend should be avoided as far as possible since they may be brittle and may fracture with bending.

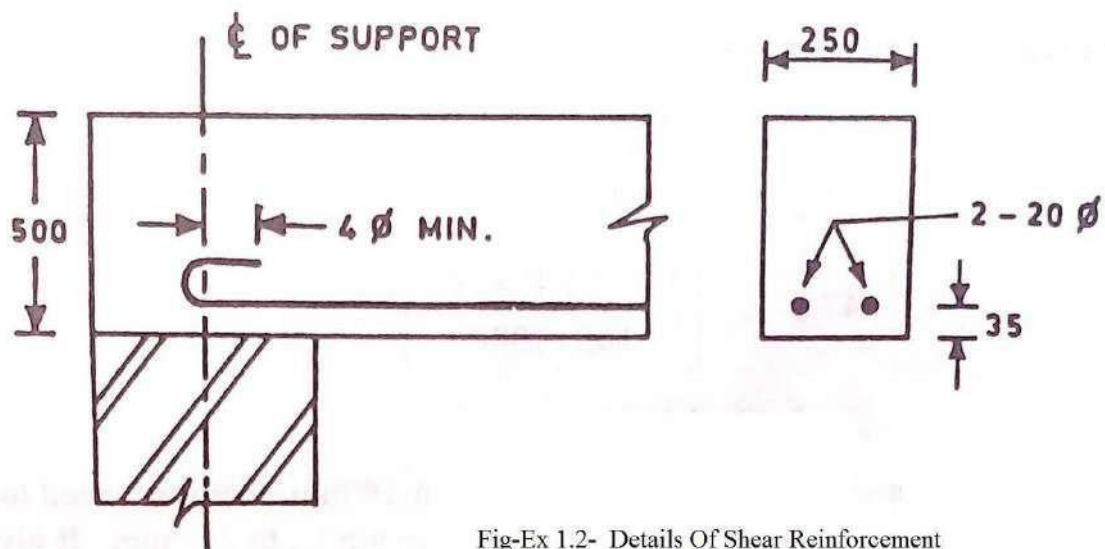


Fig-Ex 1.2- Details Of Shear Reinforcement

### Example 5.2:

A continuous beam 25 cm X 40 cm carries 3-16 mm longitudinal bars beyond the point of inflection in the sagging moment region as shown in Fig. Ex 1.3. If the factored SF at the point of inflection is 150 kN,  $\sigma_{ck} = 20 \text{ N/mm}^2$  and  $\sigma_y = 415 \text{ N/mm}^2$ , check if the beam is safe in bond?

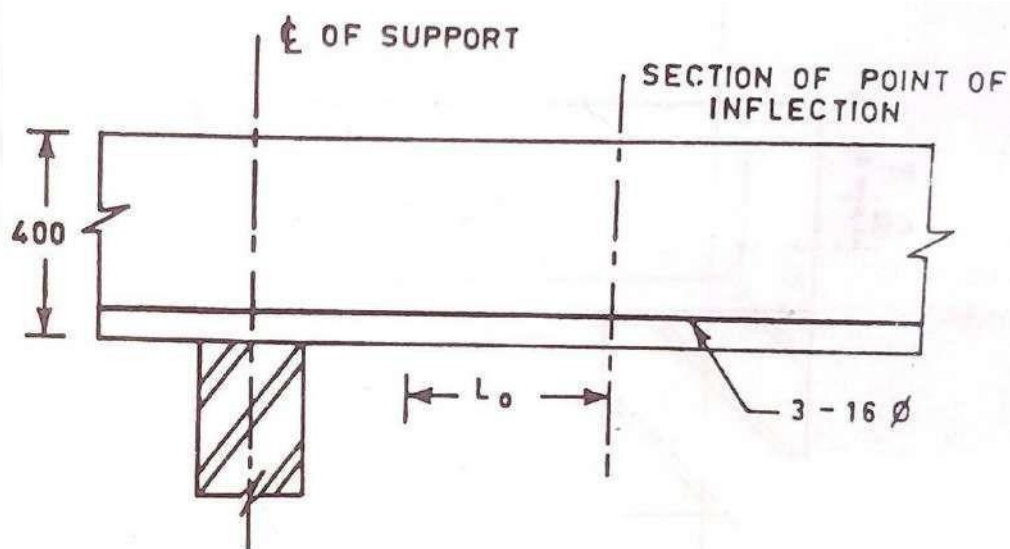


Fig- Ex 1.3 - Section of Continuous Beam

$$\text{Depth of neutral axis } x = \frac{0.87 \sigma_y A_t}{0.36 \sigma_{ck} b} = \frac{0.87 \times 415 \times 3 \times \pi / 4 \times 16^2}{0.36 \times 20 \times 250}$$

$$= 120 \text{ mm} < x_m (= 0.48 d) \quad \text{OK}$$

$$\text{Moment of resistance } M_1 = 0.87 \sigma_y A_t (d - 0.42 x)$$

$$= 0.87 \times 415 \times 603 (367 - 0.42 \times 120) = 68.90 \times 10^6 \text{ Nmm}$$

$$\text{Development length } L_d = \frac{\sigma_s \phi}{4 \tau_{bd}}$$

$$\text{Bond stress } \tau_{bd} = 1.6 \times 1.2 \text{ N/mm}^2 \text{ for M20 mix and HSD steel}$$

$$\text{or } L_d = \frac{0.87 \times 415 \phi}{4 \times 1.6 \times 1.2} = 47 \phi$$

$$\text{Anchorage length } L_o = \text{greater of } d \text{ or } 12 \phi$$

$$= \text{greater of } 367 \text{ mm, or } 12 \times 16 = 192 \text{ mm}$$

$$= 367 \text{ mm}$$

$$L_d \leq \frac{M_1}{V} + L_o$$

$$\text{or } 47 \phi \leq \frac{68.9 \times 10^6}{150 \times 1000} + 367 \quad \text{or, } \phi \leq 17.6 \text{ mm}$$

Thus, 16 mm bars are safe in bond at the point of inflection.