CHAPTER-1
REVIEW OF BASIC CONCEPTS

ENGINEERING MECHANICS

The subject of Engineering Mechanics is that branch of Applied Science, which deals with the laws and principles of Mechanics, along with their applications to engineering problems.

DIVISIONS OF ENGINEERING MECHANICS

The subject of Engineering Mechanics may be divided into the following two main groups:
1. Statics and
2. Dynamics

STATICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while

acting upon the bodies at rest.

DYNAMICS

It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches:

1. Kinetics and 2. Kinematics

KINETICS

It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

KINEMATICS

It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

1000 000 000 000	10^{12}	Tera T
1 000 000 000	10^{9}	giga G
1 000 000	10^{6}	mega M
1 000	10^{3}	kilo k
100	10^{2}	hecto* h
10	10^{1}	deca* da
0.1	10^{-1}	deci* d
0.01	10^{-2}	centi* c
0.001	10^{-3}	milli m
0.000 001	10^{-6}	micro µ
0.000 000 001	10^{-9}	nano n
0.000 000 000 001	10^{-12}	pico p

Factor by which the unit Standard forms Prefix Abbreviation is multiplied.

SCALAR QUANTITIES

The scalar quantities (or sometimes known as scalars) are those quantities which have magnitude only such as length, mass, time, distance, volume, density, temperature, speed etc.

VECTOR QUANTITIES

The vector quantities (or sometimes known as vectors) are those quantities which have both magnitude and direction such as force, displacement, velocity, acceleration, momentum etc.

FORCE

It is defined as an agent which produces or tends to produce, destroys or tends to destroy motion. *e.g.*, a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump.

EFFECTS OF A FORCE

A force may produce the following effects in a body, on which it acts :

1. It may change the motion of a body. *i.e.* if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.

2. It may retard the motion of a body.

3. It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium. We shall study this effect in chapter 5 of this book.

4. It may give rise to the internal stresses in the body, on which it acts. We shall study this effect in the chapters 'Analysis of Perfect Frames' of this book.

CHARACTERISTICS OF A FORCE

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force :

1. Magnitude of the force (*i.e.*, 100 N, 50 N, 20 kN, 5 kN, etc.)

2. The direction of the line, along which the force acts (*i.e.*, along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.

3. Nature of the force (*i.e.*, whether the force is push or pull). This is denoted by placing an arrow head on the line of action of the force.

4. The point at which (or through which) the force acts on the body.

PRINCIPLE OF PHYSICAL INDEPENDENCE OF FORCES

It states, "*If a number of forces are simultaneously acting on a *particle, then the resultant of these forces will have the same effect as produced by all the forces.*"

PRINCIPLE OF TRANSMISSIBILITY OF FORCES

It states, “*If a force acts at any point on a †rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body.*”

SYSTEM OF FORCES

When two or more forces act on a body, they are called to form a *system of forces*. Following systems of forces are important from the subject point of view :

1. ***Coplanar forces.*** The forces, whose lines of action lie on the same plane, are known as coplanar forces.

2. ***Collinear forces.*** The forces, whose lines of action lie on the same line, are known as collinear forces.

3. ***Concurrent forces.*** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.

4. ***Coplanar concurrent forces.*** The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.

5. ***Coplanar non-concurrent forces.*** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

6. ***Non-coplanar concurrent forces.*** The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

7. ***Non-coplanar non-concurrent forces.*** The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

MOMENT OF A FORCE

It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment,

$M = P \times l$

where P = Force acting on the body, and

l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

CLOCKWISE MOMENT

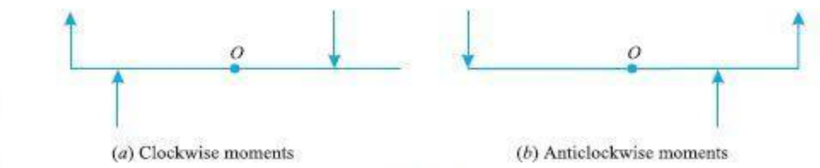


Fig. 1

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in Fig. 1 (*a*).

ANTICLOCKWISE MOMENT

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in Fig. 1 (*b*).

Note. The general convention is to take clockwise moment as positive and anticlockwise moment as negative

In the previous chapter, we have discussed the various methods of finding out resultant force, when a particle is acted upon by a number of forces. This resultant force will produce the same effect as produced by all the given forces. A little consideration will show, that if the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.

The force, which brings the set of forces in equilibrium is called an equilibrant. As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view:

1. *Two force principle.* As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. *Three force principle.* As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

3. *Four force principle.* As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

CONDITIONS OF EQUILIBRIUM

Consider a body acted upon by a number of coplaner non-concurrent forces. A little consideration will show, that as a result of these forces, the body may have any one of the following states:

1. The body may move in any one direction.

2. The body may rotate about itself without moving.

3. The body may move in any one direction and at the same time it may also rotate about itself.

4. The body may be completely at rest.

Now we shall study the above mentioned four states one by one.

1. If the body moves in any direction, it means that there is resultant force acting on it. A little consideration will show, that if the body is to be at rest or in equilibrium, the resultant force causing movement must be zero. Or in other words, the horizontal component of all the forces (ΣH) and vertical component of all the forces (ΣV) must be zero. Mathematically,

$$\Sigma H = 0 \text{ and } \Sigma V = 0$$

2. If the body rotates about itself, without moving, it means that there is a single resultant couple acting on it with no resultant force. A little consideration will show, that if the body is to be at rest or in equilibrium, the moment of the couple causing rotation must be zero. Or in other words, the resultant moment of all the forces (ΣM) must be zero. Mathematically,

$$\Sigma M = 0$$

3. If the body moves in any direction and at the same time it rotates about itself, if means that there is a resultant force and also a resultant couple acting on it. A little consideration will show that if the body is to be at rest or in equilibrium, the resultant force causing movement scand the resultant moment of the couple causing rotation must be zero. Or in other words, horizontal component of all the forces (ΣH), vertical component of all the forces (ΣV) and resultant moment of all the forces (Σ M) must be zero. Mathematically,

$$\Sigma H = 0 \; \Sigma V = 0 \text{ and } \Sigma M = 0$$

4. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show that in this case the following conditions are already satisfied:

$$\Sigma H = 0 \; \Sigma V = 0 \text{ and } \Sigma M = 0$$

The above mentioned three equations are known as the conditions of equilibrium.

CHAPTER 2

SIMPLE AND COMPLEX STRESS , STRAIN

INTRODUCTION

- ✓ various structure and machine members are such as building, bridge, cranes, aero plane, and ship etc. consist of a number of parts or member connected together in such a way that to perform useful function.
- ✓ External forces acting on individual structure or machines member of an engineering design is common.
- ✓ Engineering always endeavor to have such a design so that those are safe, durable and economical under the application of external forces and thus the load carrying capacity of the various members of structure and machines.
- ✓ It also considers the stability and rigidity of the members.
- ✓ Hence the study of the strength of material is aimed at predicting just how the geometric and physical properties of the structure influence the behavior under service condition.
- ✓ The theory of the structure involve the application of these principle of structures made up of beam, column, and slab etc.

Assumption in study of strength of materials:-

In elementary theory of analysis of a member or material subjected to external forces such assumption are made

The material is perfectly elastic i.e. the material should posses' elastic properties.

Elasticity:-

Whenever a force act on a body / member and it tends to deforms or under goes some deformation when the external force is removed, the body spring back its original configuration. Hence the properties of the materials regaining, it's original configuration.

The materials are **isotropic**; this means the materials should possess same properties in all direction.

The materials are **homogenous**; this means the materials should possess same properties anywhere in the body.

MECHANICAL PROPERTIES OF MATERIALS

The following are some of the important properties of materials related to the behaviour of a material under load. (a) As you know, when a body is subjected to external forces, it undergoes deformations, and develops internal resisting forces to balance the externally applied forces. When the external forces are removed, it comes back to its original shape and size. Materials which exhibit this property are known as elastic materials, and the property itself is called elasticity. Many metals exhibit this property up to a certain value of stress, known as the elastic limit of the material, beyond which permanent deformations remain in the body. A material is said to be homogeneous if it exhibits the same elastic properties at all points. It is said to be isotropic if it exhibits the same elastic properties in all directions at a point.

Plasticity is the property because of which a material subjected to forces undergoes deformations which do not disappear on the removal of external forces. Plasticity is important when a substance has to be molded into components. Many materials become plastic at large values of stress or at high temperatures.

Ductility is the property because of which it is possible to draw thin wires of a metal. Ductile materials can undergo large plastic deformations before breaking. Copper and mild steel exhibit this property.

Brittleness is the tendency of a material to shatter on receiving a shock. This happens due to lack of ductility. The material does not have the capacity to undergo large deformations before failure. Glass and certain high-strength steels are brittle.

Toughness is the property which enables a material to absorb large amounts of energy by undergoing large plastic deformations, particularly due to shock loading.

Resilience is the ability of a material to recover its shape and size after deformation.

Hardness is the resistance of a material to indentation, scratching, cutting or wear by abrasion.

Fatigue is the phenomenon of a material failing under very little stress due to repeated cycles of loading. A fatigue failure is generally similar to brittle failure. This is important in case a component is likely to be subjected to cyclic or reversal loading, as in machine foundations or members subjected repeated dynamic load.

Creep is defined as the property by which a material under goes deformation to constant stress over a period of time.

FORCE

It is the action of the object, the object tends to move or move , and destroy or tends to destroy the motion of the object.

STRESS

It is the internal resisting capacity of the body in the unit cross sectional area. or it is defined as the force per unit area.

Mathematically, $\sigma = \frac{P}{A}$

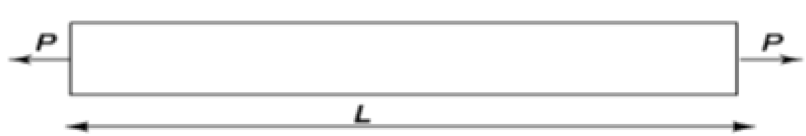
σ = stress

P= force

A= cross sectional area

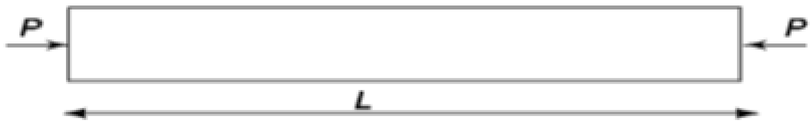
Tensile stress (Normal stress)

When the pull load is subjected the material it is called tensile load. Due to tensile load stress induced in the cross section area of the material is called tensile stress.



Compressive stress

When the push load is subjected the material, it is called compressive load. Due to compressive load, stress induced in the cross section area of the material is called compressive stress.



Shear stress

When two equal and opposite parallel forces not in the same line act on two parts of the body , then one part tends to slide over or shear from the other across any section and the stress developed is term as shear stress.

or

When a section is subjected to two equal and opposite forces acting tangential across the resisting section as a result of which the body tends to shear off across the resisting section, the stress induced is called tangential stress or shear stress.

Mathematically, Shear stress = $\frac{ShearLoad}{shearArea}$

$$\tau = \frac{P}{A}$$

τ = Shear stress

P= shear Load

A = Shear area

STRAIN

When, any external forces act on a body, the body under goes some elastic deformation (change in dimension). It is defined as the ratio of change in dimension to the original dimension.

Mathematically,

Strain = Change in dimension / original dimension

e = Change in dimension / original dimension

It has no units.

Types of strain

Liner or longitudinal strain

It is defined as the change in the liner dimension (length) to the original linear dimension (original length). Linear strain may be of tensile and compressive strain.

Tensile strain is the ratio of increase in length to original length and compressive strain is the ratio of decrease in length to original length.

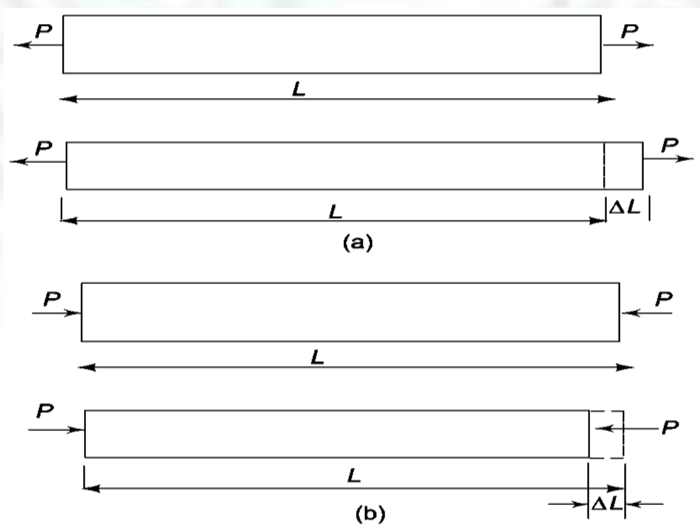
Mathematically,

$$e = \frac{\delta l}{l}$$

e=strain

δl = Change in length

l = Original length



Lateral strain

It is defined as the ratio of change in lateral dimension to original lateral dimension. Lateral dimension means the dimension of the member normal to line of action of load.

It is denoted as e_l.

Lateral strain $e_l = \frac{\delta b}{b} = \frac{\delta d}{d} = \frac{\delta t}{t}$

Poisson's ratio

It is defined as the ratio lateral strain to liner strain. It is denoted by $\frac{1}{m}$ or μ .

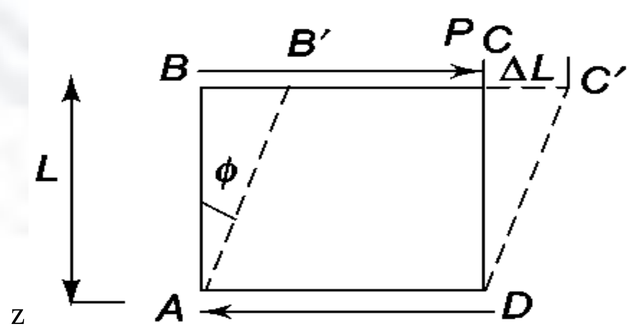
Mathematically, Poisson's ratio = Lateral strain / Linear strain

$$\frac{1}{m} = \frac{e_l}{e}$$

there fore $e_l = e \times \frac{1}{m}$

Shear strain

It is defined as angular displacement of a plane a fixed plane. It is denoted by ϕ



Volumetric strain

It is defined as change in volume to original volume. It is denoted as 'e_v.

If V is the original volume and δV is the change in volume,

Mathematically, Volumetric strain = Change in volume / original volume

$e_v = \frac{\delta V}{V}$

Hook's law

It states that "within elastic limit stress is direct proportional to strain.

i.e. stress $\propto$ strain

$$\frac{stress}{strain} = constant$$

This constant is known as Modulus of elasticity.

Modulus of elasticity

It is defined as the ratio of stress, which cause unit strain.

Mathematically, Modules of elasticity = $\frac{stress}{strain}$

Units: - N/mm^2, N/m^2 Pa

Types of modulus

There are three modulus of elasticity

Young modulus (E)

Rigid Modulus (G) or (C)

Bulk Modulus (K)

Young modulus (E)

It is the ratio of normal stress to longitudinal strain, It is denoted by E

Mathematically, Young Modulus = Normal stress / Longitudinal strain

or $E = \frac{\sigma}{e}$ or $E = \frac{\sigma}{\delta l / l} = \frac{P/A}{\delta l / l} = \frac{Pl}{\delta l A}$

$$\delta l = \frac{Pl}{AE}$$

Rigid modulus

It is defined as the ratio of shear stress to shear strain. It is denoted by G or C

Mathematically, Rigid Modulus = Shear stress /Shear strain

or $G = \frac{\tau}{\phi}$

Bulk Modulus

When a body subjected to three mutual perpendicular stress of equal intensity, the ratio of normal stress to volumetric strain is known as Bulk modulus. It is denoted by K

Mathematically, Bulk modulus = Normal stress / Volumetric strain

or $K = \frac{\sigma}{e_v}$

Relation between elastic constant

Relation between Young modulus *E* and rigid modulus *G*

$$E = 2G\left(1 + \frac{1}{m}\right)$$

Relation between Young modulus *E* and bulk modulus *K*

$$E = 3K\left(1 - \frac{2}{m}\right)$$

Relation between Young modulus *E* , rigid modulus *G* and bulk modulus *K*

$$E = \frac{9KG}{3K + G}$$

DEFORMATION OF A BODY DUE TO FORCE ACTION ON IT

Consider a body subjected to tensile or compressive load

let P= load or force acting on it

l = length of body

A = cross section area of the body

σ = stress induced in the body (tensile and compressive stress)

E = Modulus of elasticity of the material of the body

e = strain due to loading

δl = deformation of the body (extension or shorten)

We know from relation that

Stress $\sigma = \frac{P}{A}$

Strain $e = \frac{\sigma}{E} = \frac{P}{AE}$ ……(1)

Again we know $e = \frac{\delta l}{l}$ ……(2)

From Equation (1) and (2)

$$e = \frac{\delta l}{l} = \frac{P}{AE}$$

$$\delta l = \frac{Pl}{AE}$$

Deformation of body is due to force acting on it.

Bar of varying section

Some time a bar is made of different length and different cross section area are shown in figure.

In such case the stress and strain has changes in length for each section is works out separately. The total change in length is equal to the sum of the change of all individual length. It may be noted that each section is subjected to same load.

Lets P= load or force acting on it

l = length of body

A = cross section area of the body

σ = stress induced in the body (tensile and compressive stress)

E = Modulus of elasticity of the material of the body

e = strain due to loading

δl = deformation of the body (extension or shorten)

Change in length $\delta l = \frac{Pl}{AE}$

$$\delta l_1 = \frac{Pl_1}{A_1E}$$

$$\delta l_2 = \frac{Pl_2}{A_2E}$$

$$\delta l_3 = \frac{Pl_3}{A_3E}$$

Total change in length

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l = \frac{Pl_1}{A_1E} + \frac{Pl_2}{A_2E} + \frac{Pl_3}{A_3E}$$

$$\delta l = \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

Total change in length = $\frac{P}{E}$ (total length / total area)

Bar of composite section

Composite section means the material consist of more than two materials. The marital properties are not loss when they combined act a single material, and also it can be separate from the composite section. It should be maintain its mother properties. Example: - concrete, RCC.

Suppose the cross section of a member consist of different material the load applied on the member sharing by various components of the section.

let's, a column consist of an outer tube of area A_1 and young modulus E_1 and inner tube having A_2 and young modulus E_2 ,lets l is the length of the column and P be the load applied on the column

Lets σ_1 and σ_2 be the stress in the outer and inner tube of the section.

δl is the decrease in length of column

Therefore strain in outer tube = strain in inner tube

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Total load = load of outer tube + load of inner tube

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

From the equation we can find σ_1 and σ_2 value.

Modular ratio

It is defined ratio of young modulus between materials of a composite section.

it is denoted by m.

$$m = \frac{E_1}{E_2}$$

E_1= Modulus of elasticity of section 1

E_2 = Modulus of elasticity of section 2

Temperature stress

When temperature of a material changes there will be corresponding changes in its dimension. When a member is free to expand or contract due to rise or fall of temperature stress will induced in the member But if the natural change in the length occur due to rise on temperature be prevented. Stress will be offered known as temperature stress.

Solid are expand due to rise in temperature. The linear expansion of the solid is propositional to rise in temperature and propositional constant of material is called coefficient of linier thermal expansion and it is denoted by α . If α is the linier expansion of a solid .Its linier dimension will increase by α meter per every meter of its original length for every 1·/c rise in temperature.

Thermal expunction $\delta l = \alpha t l$

Temperature strain = Expunction or contraction prevention / Original length

$$e = \frac{\alpha t l}{l} = \alpha t$$

Temperature stress = $e \times E = \propto tE$

Force = stress × area

P = $\propto tE \times A$

Stress – Strain diagram

Stress – Strain diagram for ductile material

A= proportional limit

B = Elastic limit

C = upper yield point

D = lower yield point

E = Maximum or Ultimate stress point

F = Breaking or Failure point

In design various part of a machine or structure it os necessary to know how the materials will function in its service. For this certain characteristics or properties of the material should be known.

The mechanical properties mostly used in mechanical engineering practice are commonly determined for a standard tensile test.

This test consist of a gradually loaded of a standard specimen and nothing the corresponding values of load and elongation until the specimen fraction.

The load is applied and meandered by a testing machine. The stress is determined by dividing the load values by original cross section area of specimen. The elongation is measured by the amount that the two reference points of the specimen are moved apart by the acting of the machine.

The original reference between the points is known as gauge length. The strain are determined by dividing the elongation value by the gauge length.

The value of stress and corresponding strain are used to draw the stress strain diagram for mild steel under tensile test.

The various properties of the material discuss below

We see from the diagram that from o to A is a straight line, which represents that, the stress is direct proportional to strain so the point A is known as proportional limit.

Beyond the strain rapidly increases and point C is reached known as a yield point

At this point some appreciable elongation up to D, then increase the load. At this stage the plastic stage will reach i.e. on removal of the load, the material not to able to recover its original shape and size.

In between A and C there is a point B known as elastic limit, up to material shows elastic properties. Now on for their increase in load, strain also increases, there is point ultimate strength or maximum stress point reached.

After that a local extension takes place and cross section area reduces and failure of backing takes place as shown in figure.

Proportional limit

The value of the point up to which the stress and strain remain proportional is called proportional limit.

Elastic Limit

It represents the maximum stress that may be developed such that there may be no permanent deformation after the removal of load the material recovery its original configuration. It is also greater than the proportional limit. But some material elastic limit and proportional limit maybe same

Yield stress

It is the point at which there is an appreciable elongation of the material without any corresponding increase in load and the material said to be plastic.

Stress at this point is called yield stress or yield point stress.

Maximum or ultimate stress

It is the maximum value of stress up to which the specimen can with stand. It is the maximum load per cross sectional area.

i.e. Maximum or ultimate stress = Maximum load / original cross sectional area

Working or safe or permissible stress

It is the magnitude of the stress which may be consider as safe for working condition. It is the stress at working condition

Mathematically, it is the ratio of ultimate stress to factor of safety.

i.e. Working stress = Ultimate stress / Factor of safety

Factor of safety

It is defined as the ratio of ultimate stress to working stress.

i.e. F.O.S = Ultimate stress / Working stress

CHAPTER - 3

BENDING STRESS IN SIMPLE BEAM

Bending stress

A bending moment bends a member stress induced by the bending moment are called bending stress.

Simple bending or pure bending

The portion of beam which is free from shear force (which means shear force is equal to zero) but only subjected only bending moment are called simple bending or pure bending.

Assumption of pure bending

- ✓ The material of the beam is perfectly homogeneous and isotropic (same material properties in all direction).
- ✓ The material is stressed in stressed with in elastic limits and obey **Hook's law.**
- ✓ The value of modulus of elasticity (*E*) for material is same in tension and compression.
- ✓ The beam is subjected pure bending and therefore bends in the forms an arch of a circle.
- ✓ Each layer of the beam is free to expand and contract independently.
- ✓ The neutral axis of the beam is not changing its position before and after bending.

Equation of pure bending

$$\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$$

Where, M = moment

I= moment of inertia of given beam

σ = stress induced in beam

Y = Distance from neutral axis

E= young's modulus / modulus of elasticity

R= radius

Neutral axis

It is the imaginary line, which divided the cross section beam into the tension and compression zone on the opposite side of the plane.

Section modulus / modulus of sections (Z)

It is the ratio of moment of inertia about neutral axis to the distance of the outer most layer of the beam.

Mathematically, $Z = \frac{I}{Y_{max}}$

Section modulus for rectangle section

Consider a rectangle section having width (b) and depth (d).

Moment of inertia of rectangle section is about an axis through its center of gravity (neutral axis).

Therefore, $I = \frac{bd^3}{12}, Y = \frac{d}{2}$

so, section modulus $Z = \frac{I}{Y_{max}}$

$$Z = \frac{\frac{bd^3}{12}}{\frac{d}{2}},$$

$$Z = \frac{bd^3}{12} \times \frac{2}{d}$$

$$Z = \frac{bd^3}{6}$$

Section modulus for circular section

Consider a circular section having diameter‘d’

Moment of inertia about neutral axis

$$I = \frac{\pi}{64} d^4$$

$$Y = \frac{d}{2}$$

i.e.

$$Z = \frac{I}{Y_{max}} = \frac{\frac{\pi}{64} d^4}{\frac{d}{2}} = \frac{\pi}{64} d^4 \times \frac{2}{d} = \frac{\pi d^3}{32}$$

$$Z = \frac{\pi d^3}{32}$$

Section modules of hollow circular section

Consider a circular section having external diameter ‘D’ and internal diameter‘d’

Moment of inertia about neutral axis

$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$Y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{Y_{max}} = \frac{\frac{\pi}{64}\left(D^4 - d^4\right)}{\frac{D}{2}} = \frac{\pi}{64}\left(D^4 - d^4\right) \times \frac{2}{D} = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

$$Z = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

Note

Maximum bending moment

For simple supported beam

When point load acting on the simple supported beam $M = \frac{wl}{4}$

When uniform distributed load acting entire span of the simple supported beam $M = \frac{wl^2}{8}$

For cantilever beam

When point load acting on the cantilever beam $M = wl$

When uniform distributed load acting entire span of the cantilever beam $M = \frac{wl^2}{2}$

Q. A steel table is bent into a circular arch of radius 10m, if the plate section is 120 mm wide and 20 mm thick. Find the maximum stress induced and the bending moment which can produce the stress .Take E = $E = 2 \times 10^5$ N/mm^{2}.

CHAPTER - 4

COLUMN AND STRUCT

Column

A structural member subjected to an axial compressive force is called strut. A strut may be horizontal, inclined and vertical. But a vertical strut used in building framed is called a column.

or

Column or strut is a compressive member in which the length is consider longer than the cross sectional dimensions.

Long column

It is defined as the ratio of effective length of a column to the least cross sectional dimensions is greater than 12. Or also the slenderness ratio is greater than 40

Short column

It is defined as the ratio of effective length of a column to the least cross sectional dimensions is less than 12. Or also the slenderness ratio is less than 40

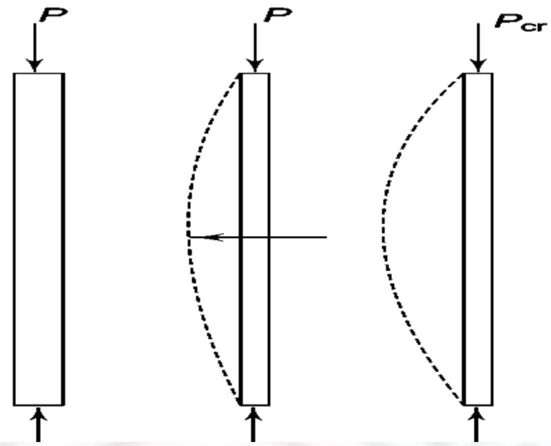
Failure of a column or strut

It has been observed that when a column or strut it subjected to some compressive force then the stress induced, $\sigma = \frac{P}{A}$

Where, P= compressive force

A= cross sectional of the column

If a long column is subjected to a compressive load, it is subjected to a compressive stress, if the load is gradually increased the column will reach a stage when it will start buckling. The load at which the column just buckler's is called buckling load or critical load or crippling load.



Buckling of column

Euler's buckling and crippling formula

$$P = \frac{\pi^2 EI}{l^2}$$

Assumptions in the Euler's column theory

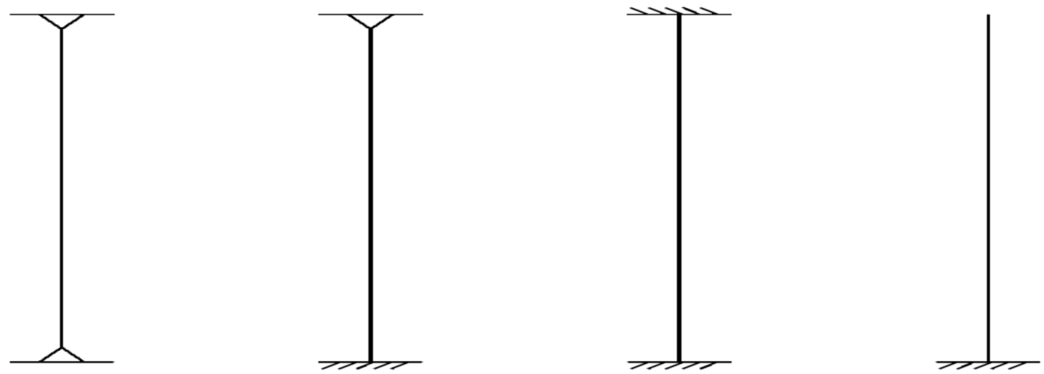
- ✓ The column is initially straight and it's axial loaded.
- ✓ The section of the column is uniform.
- ✓ The column material is perfectly homogenous, isotropic and elastic and obeys hooks law
- ✓ The direct stress is very small compare with the bending moment.
- ✓ The joints of column are friction less.
- ✓ The column is fail by buckling

End condition

Different end conditions of column are

- ✓ Both end hinged or pinned
- ✓ one end fixed and other end pinned

- ✓ Both end fixed
- ✓ one end fixed and other end free



End condition of column

End condition of column	Effective length	Euler's crippling load/ buckling load
✓ Both end hinged or pinned	$l_{eff} = 1l$	$P = \frac{\pi^2 EI}{l^2}$
✓ one end fixed and other end pinned	$l_{eff} = \frac{1}{\sqrt{2}} l$	$P = \frac{2\pi^2 EI}{l^2}$
✓ Both end fixed	$l_{eff} = \frac{1}{2} l$	$P = \frac{4\pi^2 EI}{l^2}$
✓ one end fixed and other end free	$l_{eff} = 2l$	$P = \frac{\pi^2 EI}{4l^2}$

Slenderness ratio

We know that general equation of crippling load is $P = \frac{\pi^2 EI}{l^2}$

Again we know $I = Ar^2$

Where I = moment of inertia

A = area of the cross section

r = least radius of gyration

Then, $r = \sqrt{\frac{I}{A}}$, $\Rightarrow$ $P = \frac{\pi^2 EI}{l^2}$

$$\Rightarrow P = \frac{\pi^2 EAr^2}{l^2}$$

$$\Rightarrow P = \frac{\pi^2 EA}{\frac{l^2}{r^2}}$$

Where, $\frac{l}{r}$ is known as slenderness ratio.

l = length of the column and r = radius of gyration.

CHAPTER - 5

SHEAR FORCE AND BENDING MOMENT

Beam

A beam is a structural member subjected to a system of external force at right angle to its axis.

Types of beam

Various type of beams are

1. Cantilever Beam
2. Simple supported beam
3. over hanging beam
4. Fixed beam
5. Continues beam

Cantilever Beam

When a beam fixed at one end and free at other end that beam is called cantilever beam

Simple supported beam

A beam support or resting freely on the wall of column at its both end’s known as simple supported beam.

Over hanging beam

A beam having its end portion extended in the form of cantilever beyond the support is known as overhanging beam

Fixed beam

When a beam rigidly fixed with both end that type of beam is called fixed beam

Continues beam

A beam which provided with more than two support is called continues beam

Types of loading

A beam may be subjected the following types of load

1. Point load/ concentrated load

2. Uniformly distributed load (udl)

3. Varying load

1. Point load/ concentrated load

Load acting at a point of a beam is known as point load/ concentrated load.

2. **Uniformly distributed load (udl)**

A load which is spread over a beam in such a way that each unit length is loaded to the same extend is known as u.d.l.

For all calculation per portion the total udl is assume act the center of gravity of the load.

Uniformly varying load

A load spread over a beam in such a way that its exciter various uniformly on each unit length is known as UVL. Some time the load is 0 at one end and an increase uniformly to the other end. Such load is known as triangle load.

Types of supports

Roller support

It is a support on which a beam rest freely. A roller support is a simplest and gives only one reaction either X or Y direction.

Hinged or pinned support

In this case the beam is hinged or pinned to the support. it is gives only two reaction forces, one against vertical moment and another against horizontal moment.

Fixed support

This support gives three reaction i.e. reaction in X& Y direction and fixing moments

Shear force

Shear force at the cross section of a beam may be defined as the algebraic sum of unbalanced vertical load to the right or left of the section.

Bending moments

Bending moments at the cross section of a beam may be defined as the algebraic sum of moments of the force to the right or left of the section.

Sign convection

In case of Shear force from the right hand side the down word force and from left hand side upward force are positive otherwise all are negative sign.

In case of bending moments the right hand side the anti clock wise moment and from left hand side clock wise moments are positive otherwise all are negative sign.

Shear force diagram

A shear force diagram for a structural member is a diagram which shows the value of shear force at various section of the member.

Bending moment diagram

Bending moment for a member is a diagram which shows the value of the bending moment at various section of the member.

Procedure for drawing SFD and BMD

- ✓ Find the reaction of any one of the support by taking moment about the hinged or pinned support and equating to zero.
- ✓ Find the reaction of other support by means of static equilibrium equation i.e. summation of X equal to zero and summation of Y equal to zero.
- ✓ Draw a base line of length equal to the length of beam to same scale.
- ✓ Starting from one end, calculation the shear force and at all points. If there is a vertical load including reaction force at the section calculate the shear force just left or as well as right of the vertical load and reaction
- ✓ Calculate the bending moment at different points.
- ✓ Plot the positive values of the sheer force and bending moments above the base line and the negative value below the base line, SFD and BMD can be obtained by joining the ordinaries.

The following points should be kept on the mind for drawing SFD and BMD.

- ✓ IF there is no loading on the portion of the beam the shear force diagram will be horizontal. The bending moment diagram will be inclining. The SFD will remain on change between any two vertical loads provided there is no loading between the vertical loads.
- ✓ If there is UDL between two sections the shear force diagram will be inclining or straight line and BMD will be a curve.
- ✓ The bending moment will be zero at the free end of the cantilever and at two support of simple supported beam.

Point of contra flexure

For the purpose of shear force and bending moment the overhanging beam will be analyzed as a combination of simple supported beam and a cantilever beam. The bending moment is negative for cantilever beam and positive for simple supported beam. That implies in an overhanging beam there is a point at which the bending moment change sign from positive to negative or negative to positive such a point , where the bending moment change sign is known as point of contra flexure. There may be one or more point of contra flexure**.**