CHAPTER-1

ELECTRICAL MATERIAL

1.1 CONDUCTOR MATERIALS

Resistivity:

Resistivity or specific resistance of a material may be defined as the resistance offered between the opposite faces of a metre cube of that material. The unit of resistivity is ohm metre (Ω-m). We have according to law of resistance: The resistance of a material (R) depends-

- directly to its length (L)
- inversely to the X-sectional area (A)

So $R \propto L / A$

Or $R= \rho L/A$ (where ρ is known as resistivity of material)

Therefore $\rho = A R/L$

When R= Resistance in Ohms (Ω)

L= Length in m

A= Area of cross section in m^2

ρ = resistivity or Specific resistance in Ω-m

Temperature Coefficient of Resistance:

Based on temperature effect, electrical materials can be classified into two groups:

(i) *Positive temperature coefficient* means that the resistance of some of the metals and alloys increaseswhen their temperature is raised.

(ii) *Negative temperature coefficient* means that the resistance of some of the materials, i.e., carbon andinsulators and electrolytes, decreases when their temperature is raised.

If the resistance of a conductor is R_0 at 0^0C, then its resistance at t^0C is given by the equation

$$R_t = R_0 \alpha t$$

where α is the temperature coefficient of resistance at 0^0C and t is the difference in temperature.

1.1.1 Properties of Conductors

While selecting a material for a specific purpose in electrical engineering, its electrical, mechanical andeconomical properties are to be considered.

A. Electrical Properties

1. The conductivity must be good.
2. Electrical energy displayed in the form of heat must be low.
3. Resistivity must be low.
4. Temperature resistance ratio must be low.

B. **Mechanical Properties**

1. Ductility: It has that property of a material which allows it to be drawn into a wire.
2. Solderability: The joint should have minimum contact resistance.
3. Resistance to corrosion: Should not get rusted when used in outdoors.
4. Withstand stress and strain.
5. Easy to fabricate.

C. Economical Factors

1. Low Cost
2. Easily Available.

1.1.2 Properties and Uses of Different Conducting Material

1. COPPER PROPERTIES:

1. Pure copper is one of the best conductors of electricity and its conductivity is highly sensitive toimpurities.
2. It is reddish-brown in colour.
3. It is malleable and ductile.
4. It can be welded at red heat.
5. It is highly resistant to corrosion.
6. Melting point is $108^{4}0$C.
7. Specific gravity of copper is 8.9.
8. Electrical resistivity is 1.682 micro ohm cm.
9. Its tensile strength varies from 3 to 4.7 tonnes/cm^2.
10. It forms important alloys like bronze and gun-metal.

Uses: Wires, cables, windings of generators and transformers, overhead conductors, busbar etc.

Hard drawn (cold-drawn) copper conductor is mechanically strong with tensile strength of 40 Kg/mm^2. It is obtained by drawing cold copper bars into conductor length. It is used for overhead line conductors and busbars.

Annealed Copper (Soft Copper) Conductor. It is mechanically weak, tensile strength 20 Kg/mm^2, easily shaped into any form.

Low-resistivity Hard Copper. It is used in power cables, windings and coils as an insulated conductor. Ithas high flexibility and high conductivity.

2. SILVER PROPERTIES:

It is best known electrical conductor.

Properties

1. It is very costly.
2. It is not affected by weather changes.

3. It is highly ductile and malleable.
4. Its resistivity is 165 micro ohm cm.

Uses : Used in special contact, high rupturing capacity fuses, radio frequency conducting bodies, leads in valves and instruments.

3. ALUMINIUM PROPERTIES:

1. Pure aluminium has silvery colour and lustre. It offers high resistance to corrosion. Its electrical conductivity is next to that of copper.
2. It is ductile and malleable.
3. Its electrical resistivity is 2.669 micro ohms cm at 20^0C.
4. It is good conductor of heat and electricity.
5. Its specific gravity is 2.7.
6. Its melting point is 658^0C.
7. It forms useful alloys with iron, copper, zinc and other metals.
8. It cannot be soldered or welded easily.

Uses : Overhead transmission line conductor, busbars, ACSR conductors. Well suited for cold climate.

4. STEEL PROPERTIES:

Steel contains iron with a small percentage of carbon added to it. Iron itself is not strong but when carbon is added to it, it assumes very good mechanical properties. The tensile strength of steel is higher than that of iron. The resistivity of steel is 8-9 times higher than that of copper. Hence, steel is not generally used as conductor material. Galvanised steel wires are used as overhead telephone wires and as earth wires. Aluminium conductors are steel-reinforced to increase their tensile strength.

5. TUNGSTEN PROPERTIES:

1. It is grayish in colour when in metallic form.
2. It has a very high melting point (3300^0C)
3. It is a very hard metal and does not become brittle at high temperature.

4. It can be drawn into very thin wires for making filaments.
5. Its resistivity is about twice that of aluminium.
6. In its thinnest form, it has very high tensile strength.
7. It oxidizes very quickly in the presence of oxygen even at a temperature of a few hundred degrees centigrade.
8. In the atmosphere of an inert gas like nitrogen or argon, or in vacuum, it will reliably work up to 2000^0C.

Uses : It is used as filaments of electric lamps and as a heater in electron tubes. It is also used in thermionic valves, radars. Grids of electronic valves, sparking and contact points.

6. CARBON PROPERTIES:

Carbon is mostly available as graphite which contains about 90% of carbon. Amorphous carbon is found in the form of coal, coke, charcoal, petroleum, etc.

Electrical carbon is obtained by grinding the raw carbon materials, mixing with binding agents, moulding and baking it.

Properties :

1. Carbon has very high resistivity (about 4600 micro ohm cm).
2. It has negative temperature coefficient of resistance.
3. It has a pressure-sensitive resistance material and has low surface friction.
4. The current density is 55 to 65 A/cm^2.
5. This oxidizes at about 300^0C and is very weak.
6. It has very good abrasive resistance.
7. It withstands arcing and maintains its properties at high temperature.

7. PLATINUM PROPERTIES:

1. It is a grayish-white metal.
2. It is non-corroding.
3. It is resistant to most chemicals.
4. It can be drawn into thin wires and strips.
5. Its melting point is 1775^0C.
6. Its resistivity is 10.5 micro ohm cm.
7. It is not oxidized even at high temperature.

Applications:

1. It is used as heating element in laboratory ovens and furnaces.
2. It is used as electrical contact material and as a material for grids in special- purpose vacuumtubes.
3. Platinum-rhodium thermocouple is used for measurement of temperatures up to 1600^0C.

8. MERCURY PROPERTIES:

1. It is good conductor of heat and electricity.
2. It is a heavy silver-white metal.
3. It is the only metal which is liquid at room temperature.
4. Its electrical resistivity is 95.8 micro hom cm.
5. Oxidation takes place if heated beyond 300^0C in contact with air or oxygen.

It expands and contracts in regular degrees when temperature changes.

Uses : Mercury vapour lamps, mercury arc rectifiers, gas filled tubes; for making and breaking contacts;used in valves, tubes, liquid switch.

1.2 INSULATING MATERIALS

1.2.1 Insulating Materials for Electrical Engineering

The insulating materials used for various applications in electrical engineering are classified inthree categories:

- Insulating gases
- Liquid insulating material
- Solid insulating material

1.2.1.1 Insulating Gas: Properties and Applications

Air: Air provides insulation between the over-head transmission lines. It is the best insulating material when voltages are not very high. It is also used in air capacitor, switches and various electrical equipments.

It is easily available, non-inflammable, non-explosive, small dielectric strength (nearly 3to 5 kV/m) and reliable at low voltage.

Hydrogen: It is commonly used for cooling purpose in electrical machine due to its lightness. Its high thermal conductivityhelps to transmit heat from windings of high capacity alternator. Thus it reduces windage losses and increases efficiency.

Nitrogen: Nitrogen is used in place of air, to prevent oxidation due to its chemically inert property. It is generally used in transformers, gas pressure cable and capacitors.

Carbon Dioxide: Carbon dioxide is used in certain fixed type capacitor, and is used as a pre-impregnate for oil filled high voltage apparatus, such as cables and transformers. The relative permittivity of carbon oxide is 1.000985 at 0^0C.

Sulphur Hexafluoride (SF_6): The electromagnetic gases have high dielectric strength compared to other traditional dielectric gases like nitrogen and air. The dielectric strength of SF_6 is 2.35 times more than air. The electronegative gases are non-inflammable and non-explosive. The most important gas under this category is sulphur Hexafluoride, while others are Freon gases.

SF_6 is mostly used in high voltage application and its use is most satisfactory in dielectric machines, like X-ray apparatus, Van de Graff generators, voltage stabilizers, high-voltage switch gears, gas lasers etc. SF_6 bears some special properties as follows:

- SF_6 is colourless, nontoxic and non-inflammable gas. It is the heaviest gas and has low solubilityin water. The gas can be liquefied by compression. Its cooling characteristic is better than air and nitrogen.
- Under normal temperature conditions it is chemically inert and completely stable with high dielectric strength.
- This gas has very good electronegative property. Its relatively large molecules have a great affinity for free electrons, with which they combine making the gas-filled break much more resistant to dielectric breakdown.

1.2.1.2 Liquid Insulating Material: Properties and Applications

Mineral oils: The operating temperature range of mineral oil is 50-110^{0}C. These hydrocarbon oils areused as insulating oils in transformers, circuit breakers, switch gears, capacitors etc.

In transformers, light fraction oil, such as transil oil is used to allow convection cooling. Its high flash point is 130^{0}C, so it is able to prevent fire hazard. Highly purified oil has a dielectric strength of 180 kv/mm and if the oil contains polar and ionizing material its dielectric loss increases. The dielectric constant is about 2.3 and therefore it is capable of dissolving only very few substances in it and produce the conducting ions. The TRANSIL oil undergoes oxidation, particularly in the presence of catalysts suchas copper, to form sludge and acids.

Light oils having viscosity of 100 seconds at 40^0C, have been used under pressure in oil filled high voltage cables.

More viscous or tacky oils with viscosity of 2000 seconds at 40^0C, are generally to impregnate the paperin solid type cable.

Askarels: These are non-inflammable, synthetic insulating liquids, used in temperature range of 50 – 110^0C. Chlorinated hydrocarbons are the most widely used among the askarels because of high dielectric strength, low dielectric constant (4 to 6) and small dielectric loss. They do not decompose under the influence of electric arc and have good thermal, chemical and electrical stability.

Chlorinated hydrocarbons as askarels are used as transformer fluids to reduce fire hazards. Chlorinated diphenyl, penta chloro diphenyl, trichloro diphenyl, hexa chloro diphenyl, trichloro benzene, etc., are the most widely used hydrocarbons or askarels. Askarels are generally used to impregnate a cellulose insulating material, such as paper or press board etc., for its high breakdown strength.

Silicon Fluids: It is used in the temperature range of 90-220^0C and it is clear, water like liquid. It is available in wide range of viscosity and stable in high temperature. They are non-corrosive to metal upto 200^0C and bear excellent dielectric properties in wide range of temperature. So it is used as coolants in radio pulse and aircraft transformers.

Fluorinated Liquids: These are non inflammable, chemically stable oils used in temperature range of 50-200^0C. They provide efficient heat transfer from the winding and magnetic circuits in comparison to hydrocarbon oils and used in small electric and radio devices, transformers etc. In presence of moisture electrical properties are deteriorated.

Synthetic Hydrocarbon oils: Polybutylene, Polypropylene is the example of synthetic hydrocarbon oils. They have similar dielectric strength; thermal stability and susceptibility to oxidation properties are similar as that of mineral oils. The operating temperature range is 50-110^0C. These are used in high pressure gas filled cables and dc voltage capacitors.

Organic Esters: These organic fluids are used in the temperature range of 50-110^0C. They have dielectric constant and very low dielectric losses. The dielectric constant ranges from 2 to 3.5. The higher range of is obtained in tetra hydro-furyloxalate. These fluids are well suited for use in high frequency capacitors.

Vegetable Oils: These insulating liquids have temperature range of 20-100^{0}C. Drying oils are generally suitable in the formation of insulating varnishes, while non-drying oils are used as plasticisers in insulating resin compositions.

Varnish: It is the liquid form of resinous matter in oil or a volatile liquid. Hence by applying, it dries out by evaporation or chemical action to form hard, lustrous coating, which is resistant to air and water.

It is used to improve the insulation properties, mechanical strength and to reduce degradation caused by oxidation and adverse atmospheric condition.

1.2.1.3 Solid Insulating Materials: Properties and Applications

Mica: two kind of mica are used as neutral insulating material in electrical engineering. Those areMuscovite mica and Phlogophite mica.

1. ***Muscovite Mica:*** The chemical composition of muscovite mica is $KAI_3SiO_3O_{10}(OH)_2$. It is translucent green, ruby, silver or brown and is strong, tough and flexible. It exhibits good corrosion resistance and is notaffected by alkalis. It is used in capacitors and commutators.
2. ***Phlogophite Mica:*** The chemical composition of this is, $KMg_3AlSiO_3O_{10}(OH)_2$. It possesses less flexibility. It is amber, yellow, green or grey in colour. It is more stable, but electrical properties are poorer compared to Muscovite Mica. It is used in thermal stability requirements, such as in domestic appliances like iron, hotplates etc.

Polyethylene: It is obtained by polymerization of ethylene. The polymerization is performed in the presenceof catalyst at atmospheric temperature and pressure around 100^{0}C. To obtain heat resistance property polythene is subjected to ionizing radiation.

Polyethylene exhibits good electrical and mechanical properties, moisture resistant and not soluble in many solvents except benzene and petroleum at high temperature. The dielectric constant and power factor remainssteady over a wide range of temperature.

It is used as general purpose insulation, insulations of wires and cable conductors, in high frequency cables and television circuits, jacketing material of cables. Polyurethane films are also used as dielectric material incapacitors.

Teflon: The chemical name of Teflon is Polytetra fluoro-ethylene. This is synthesized by polymerization of tetra fluro ethylene. It bears good electrical, mechanical and thermal properties. Its dielectric constant is 2 to 2.2, which does not change with time, frequency and temperature. Its insulation resistance is very high and water resistant.

It is used as dielectric materials in capacitors, covering of conductors and cables, as base material for PCBs.

Polyvinyl Chloride (PVC): It is obtained by polymerization of vinyl chloride in the presence of a catalyst at 50^0C. PVC exhibits good electrical and mechanical properties. It is hard, brittle, and non-hygroscopic and can resist flame and sun light. PVC used as insulation material for dry batteries, jacketing material for wires and cables.

Epoxy Glass: Epoxy glass is made by bonding two or more layer of material. The layers used reinforcingglass fibers impregnated with an epoxy rasin. It is water resistant and not affected by alkalis and acids.

It is used as base material for copper-clad sheets used for PCBs, terminal port, instrument case etc.

Bakelite: It is hard, dark colored thermosetting material, which is a type of phenol formaldehyde. It is widely used for manufacture of lamp holders, switches, plug socket and bases and small panel boards.

1.3 MAGNETIC MATERIALS

Materials in which a state of magnetization can be induced are called magnetic materials when magnetized such materials create a magnetic field in the surrounding space.

The property of a material by virtue of which it allows itself to be magnetized is called permeability.

The permeability of free space is denoted by μ_o. Its value $\mu_o = 4\pi \times\times 10^{-7}$.

The material permeability $\mu = \mu_o \times \mu_r$
when μ_r = Relative permeability

1.3.1 Classification of Magnetic Materials

Magnetic materials classified as :

a. Diamagnetic material
b. Para-magnetic material
c. Ferro-magnetic material

1.3.1.1 Diamagnetic Material

The materials which are repelled by a magnet are known as diamagnetic materials. Eg. Zinc, Mercury, lead, Sulphur, Copper, Silver. Their permeability is slightly less than one. They are slightly magnetized when placed in a strong magnetic field and act in the direction opposite to that of applied magnetic field.

1.3.1.2 Paramagnetic Materials

The materials which are not strongly attracted by a magnet are known as paramagnetic materials. Eg. Aluminium, Tin, Platinum, Magnesium, Manganese, etc. Their relative permeability's is small but positive. Such materials are slightly magnetized when placed in a strong magnetic field and act in the direction of the magnetic field.

In paramagnetic materials the individual atomic dipoles are oriented in a random fashion. So the resultant magnetic field is negligible. When an external magnetic field is applied. The permanent magnetic dipoles orient themselves parallel to the applied magnetic field and give rise to a positive magnetization.

1.3.1.3 Ferro-Magnetic Materials

The materials which are strongly attracted by a magnet are known as ferro-magnetic materials. Their permeability is very high. eg. Iron, Nikel, Cobalt, etc.

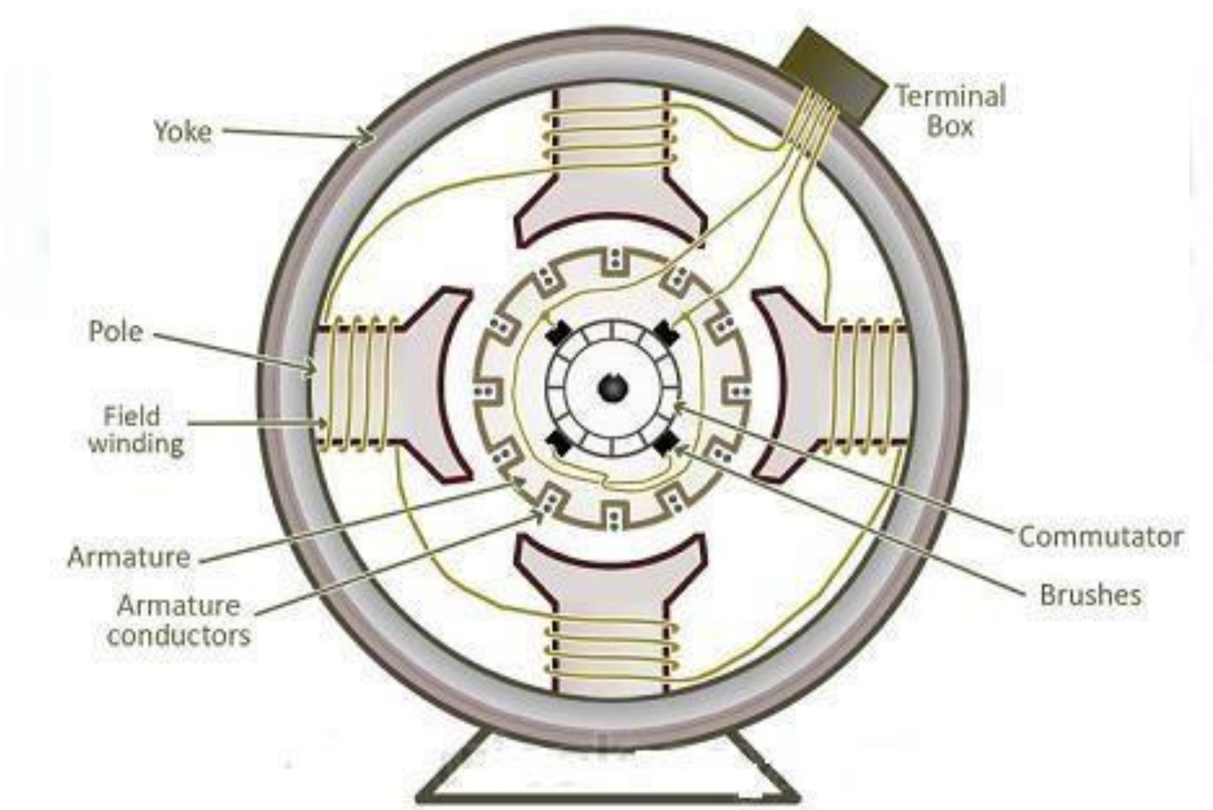
The opposing magnetic effects of electron orbital motion and electron spin do not eliminate each other in an atom of such a material.

CHAPTER - 2

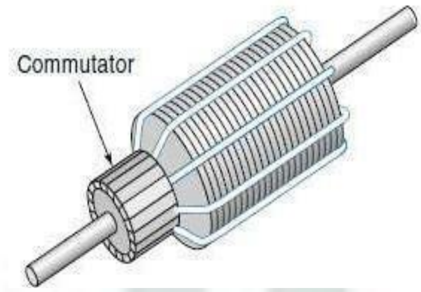
DC GENERATORS

2.1 CONSTRUCTIONAL FEATURES

A DC generator is an electrical machine which converts mechanical energy into direct current electricity**.**This energy conversion is based on the principle of production of dynamically induced emf. The figure shown below shows constructional details of a simple **4-pole DC machine**. A DC machine consists of two basic parts; stator and rotor. Basic constructional parts of a DC machine are described below.



Cross sectional view of DC Machine



Armature of DC Machine

1. **Yoke:** The outer frame of a dc machine is called as yoke. It is made up of cast iron or steel. It not only providesmechanical strength to the whole assembly but also carries the magnetic flux produced by the field winding.

2. **Poles and pole shoes:** Poles are joined to the yoke with the help of bolts or welding. They carry field winding and pole shoes are fastened to them. Pole shoes serve two purposes; (i) they support field coils and (ii) spread out theflux in air gap uniformly.

3. **Field winding:** They are usually made of copper. Field coils are former wound and placed on each pole and are connected in series. They are wound in such a way that, when energized, they form alternate North and South poles.

4. **Armature core:** Armature core is the rotor of a dc machine. It is cylindrical in shape with slots to carry armature winding. The armature is built up of thin laminated approximately 0.5 mm circular steel disks for reducing eddy current losses. It may be provided with air ducts for the axial air flow for cooling purposes. Armature is keyed to the shaft.

   It function is to provide a path of very low reluctance to the flux through the armature from a

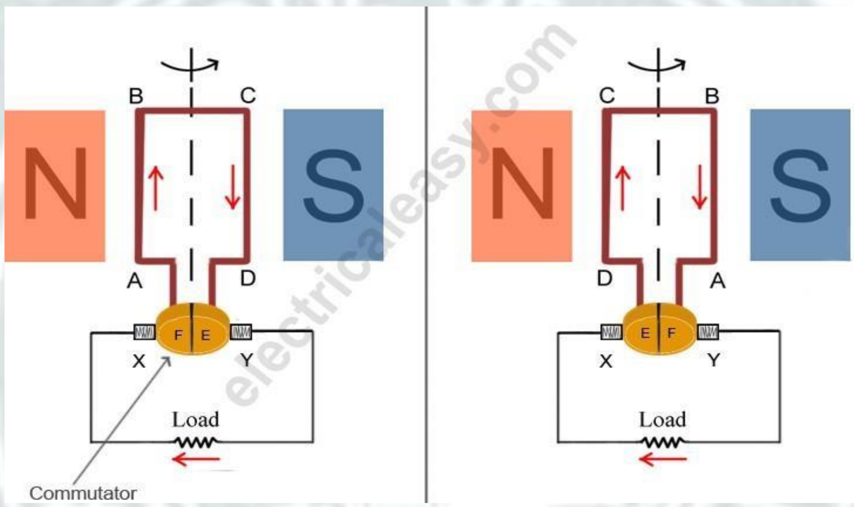
N- pole to a S-pole.

5. **Armature winding:** It is usually a former wound copper coil which rests in armature slots. The armature conductors are insulated from each other and also from the armature core. Armature winding can be wound by one of the two methods; lap winding or wave winding. Double layer lap or wave windings are generally used. A double layer winding means that each armature slot will carry two different coils.

6. **Commutator and brushes:** Physical connection to the armature winding is made through a commutator-brush arrangement. The function of a commutator, in a dc generator, is to collect the currentgenerated in armature conductors. Whereas, in case of a dc motor, commutator helps in providing current to the armature conductors. A commutator consists of a set of copper segments which are insulated from each other. The number of segments is equal to the number of armature coils. Each segment is connected to an armature coil and the commutator is keyed to the shaft. Brushes are usually made from carbon or graphite. They rest on commutator segments and slide on the segments when the commutator rotates keeping the physical contact to collect or supply the current.

2.2 WORKING PRINCIPLE OF A DC GENERATOR

According to Faraday's laws of electromagnetic induction, whenever a conductor is placed in a varying magnetic field (OR a conductor is moved in a magnetic field), an emf (electromotive force) gets induced in the conductor. The magnitude of induced emf can be calculated from the emf equation of dc generator. If the conductor is provided with a closed path, the induced current will circulate within the path. In a DC generator, field coils produce an electromagnetic field and the armature conductors are rotated into the field. Thus, an electromagnetically induced emf is generated in the armature conductors. The direction of induced current is given by Fleming's right hand rule.



Need of a Split ring commutator

According to Fleming's right hand rule, the direction of induced current changes whenever the direction of motion of the conductor changes. Let's consider an armature rotating clockwise and a conductorat the left is moving upward. When the armature completes a half rotation, the direction of motion of that particular conductor will be reversed to downward. Hence, the direction of current in every armature conductor will be alternating. If you look at the above figure, you will know how the direction of the induced current is alternating in an armature conductor. But with a split ring commutator, connections of the armature conductors also gets reversed when the current reversal occurs. And therefore, we getunidirectional current at the terminals.

2.3 CLASSIFICATION OF DC GENERATOR

DC generators are classified based on how their fields are excited (i.e. produced). There are two methods of excitation:

1. **Separately Excited DC Generators** – Field coils excited by some external source
2. **Self Excited DC Generators** – Field coils excited by the generator itself

Self-excited DC generators can further be classified depending on the position of their field coils. The threetypes of self-excited DC generators are:

1. Series Wound Generators
2. Shunt Wound Generators
3. Compound Wound Generators

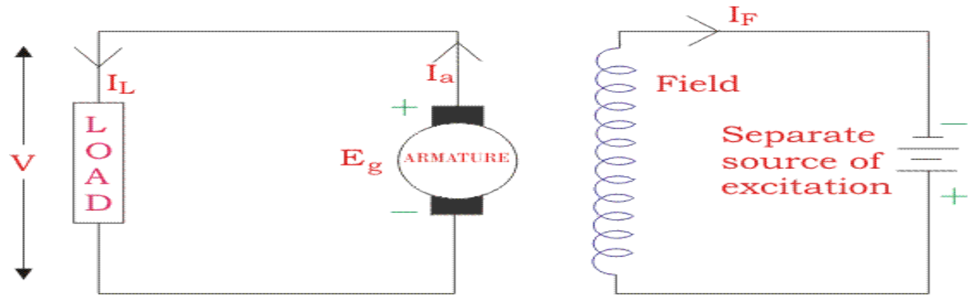
Compound Wound Generators is again classified in to two type

1. Long Shunt
2. Short shunt

2.3.1 Separately Excited DC Generator

These are the generators whose field magnets are energized by some external DC source, such as a battery.A circuit diagram of separately excited DC generator is shown in the figure below. The symbols below are:

- I_a = Armature current
- I_L = Load current
- V = Terminal voltage
- E_g = Generated EMF (Electromagnetic Force)



Separately Excited DC Generator

Voltage drop in the armature = $I_a \times R_a$ (Ra is the armature resistance)Let,

$$I_a = I_L = I\ (say)$$

Then,

$$voltage\ across\ the\ load, V = IR_a$$

Power generated is equal to

$$P_g = E_g \times I$$

And power delivered to the external load is equal to

$$P_L = V \times I$$

2.3.2 Self Excited DC Generators

Self-excited DC generators are generators whose field magnets are energized by the current supplied by themselves. In these type of machines, field coils are internally connected with the armature.

Due to residual magnetism, some flux is always present in the poles. When the armature is rotated, some EMF is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and thereby strengthening the pole flux.

As the pole flux strengthened, it will produce more armature EMF, which cause the further increase of current through the field. This increased field current further raises armature EMF, and this cumulative phenomenon continues until the excitation reaches the rated value.

According to the position of the field coils, self-excited DC generators may be classified as:

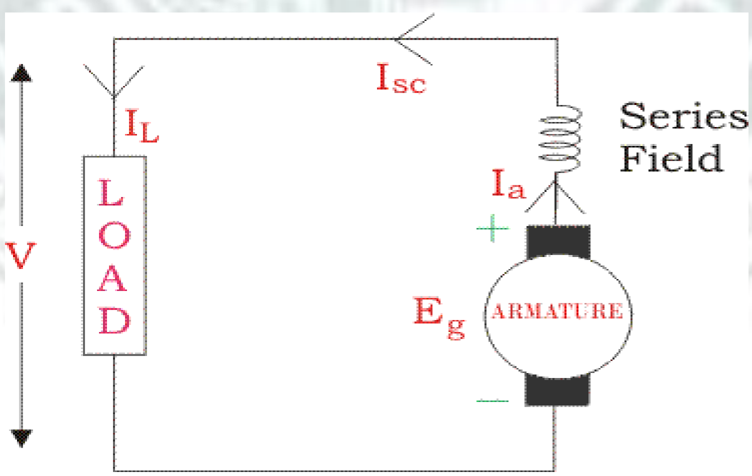
1. Series Wound Generators
2. Shunt Wound Generators
3. Compound Wound Generators

2.3.2.1 Series Wound Generator

In these type of generators, the field windings are connected in series with armature conductors, as shown in the figure below. Whole current flows through the field coils as well as the load. As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω).

Here:

- R_{sc} = Series winding resistance
- I_{sc} = Current flowing through the series field
- R_a = Armature resistance
- I_a = Armature current
- I_L = Load current
- V = Terminal voltage
- E_g = Generated EMF



Series Wound Generator

Then,

$$I_a = I_{sc} = I_L = I \ (say)$$

Voltage across the load is equal to,

$$V = E_g - I(I_a \times R_a)$$

Power generated is equal to,

$$P_g = E_g \times I$$

Power delivered to the load is equal to,

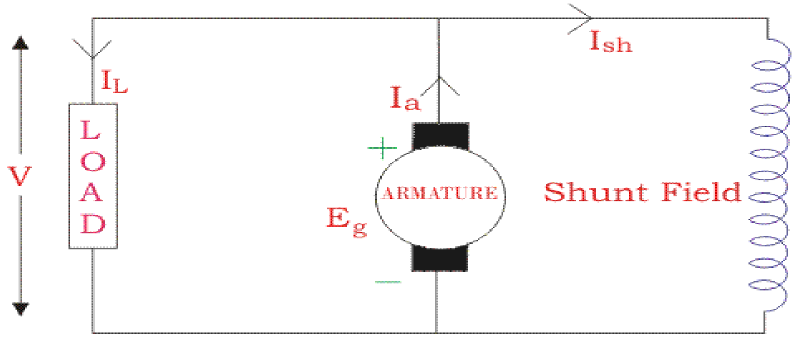
$$P_L = V \times I$$

2.3.2.2 Shunt Wound DC Generators

In these type of DC generators, the field windings are connected in parallel with armature conductors, as shown in the figure below. In shunt wound generators the voltage in the field winding is same as the voltage across the terminal.

Here:

- R_{sh} = Shunt winding resistance
- I_{sh} = Current flowing through the shunt field
- R_a = Armature resistance
- I_a = Armature current
- I_L = Load current
- V = Terminal voltage
- E_g = Generated EMF



Shunt Wound Generator

Here armature current I_a is dividing in two parts – one is shunt field current I_{sh}, and another is load current I_L.

So,

$$I_a = I_{sh} + I_L$$

The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired EMF.

Shunt field current is equal to,

$$I_{sh} = \frac{V}{R_{sh}}$$

Voltage across the load is equal to,

$$V = E_g - I_a R_a$$

Power generated is equal to,

$$P_g = E_g \times I_a$$

Power delivered to the load is equal to,

$$P_L = V \times I_L$$

2.3.2.3 Compound Wound DC Generator

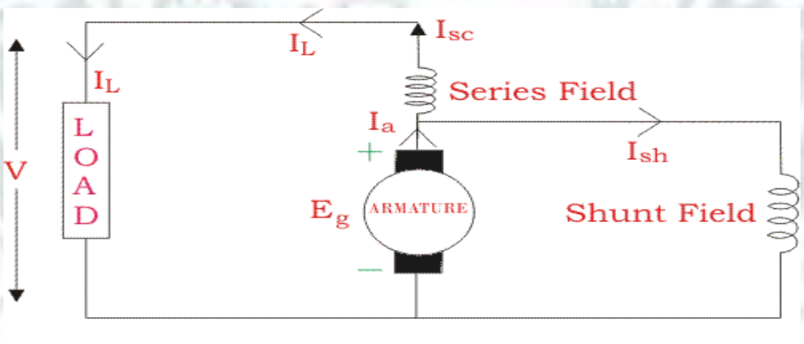
In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, the output voltage is inversely proportional with load current.

A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator.

Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature, and the other is placed in parallel with the armature. This type of DC generators may be of two types- short shunt compound-wound generator and long shunt compound-wound generator.

1. Short Shunt Compound Wound DC Generator

Short Shunt Compound Wound DC Generators are generators where only the shunt field winding is in parallel with the armature winding, as shown in the figure below.



Short Shunt Compound Wound Generator

Series field current is equal to,

$$I_{sc} = I_L$$

Shunt field current is equal to,

$$I_{sh} = \frac{(V + I_{sc}R_{sc})}{R_{sh}}$$

Armature current is equal to,

$$I_a = I_{sh} + I_L$$

Voltage across the load is equal to,

$$V = E_g - I_a R_a - I_{sc} R_{sc}$$

Power generated is equal to,

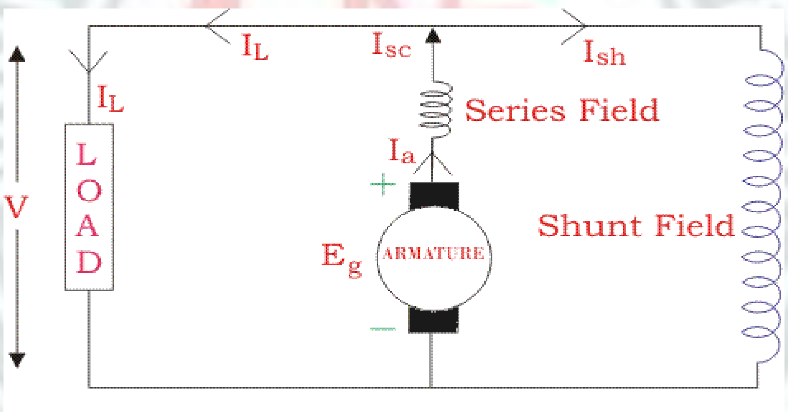
$$P_g = E_g \times I_a$$

Power delivered to the load is equal to,

$$P_L = V \times I_L$$

2. Long Shunt Compound Wound DC Generator

Long Shunt Compound Wound DC Generator are generators where the shunt field winding is in parallelwith both series field and armature winding, as shown in the figure below.



Long Shunt Compound Wound Generator

Shunt field current is equal to,

$$I_{sh} = \frac{V}{R_{sh}}$$

Armature current, I_a = series field current,

$$I_{sc} = I_L + I_{sh}$$

Voltage across the load is equal to,

$$V = E_g - I_a R_a - I_{sc} R_{sc} = E_g - I_a (R_a + R_{sc}) \ [\therefore I_a = I_{cs}]$$

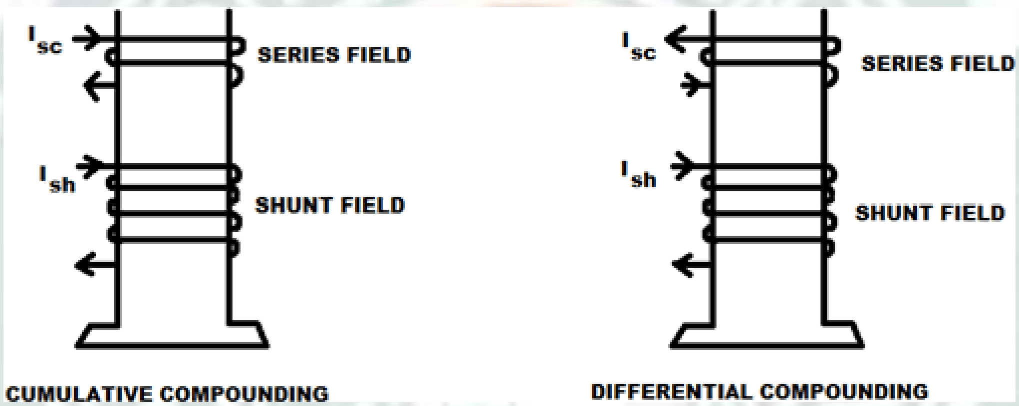
Power generated is equal to,

$$P_g = E_g \times I_a$$

Power delivered to the load is equal to,

$$P_L = V \times I_L$$

In a compound wound generator, the shunt field is stronger than the series field. When the series field assiststhe shunt field, generator is said to be **commutatively compound wound**.



On the other hand, if the series field opposes the shunt field, the generator is said to be **differentiallycompound wound**.

2.4 E.M.F. EQUATION OF A D.C. GENERATOR

We shall now derive an expression for the e.m.f. generated in a d.c. generator.

Let ϕ = flux/pole in Wb

Z = total number of armature conductors

P = number of poles

A = number of parallel paths = 2 .... for wave winding

= P .... for lap winding

N = speed of armature in r.p.m.

E_g = e.m.f. of the generator = e.m.f./parallel path

Flux cut by one conductor in one revolution of the armature,

$$d\phi = P\phi \text{ webers}$$

Time taken to complete one revolution,

$$dt = 60/N \text{ second}$$

$$\text{e.m.f. generated/conductor} = \frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volts}$$

e.m.f. of generator, E_g = e.m.f. per parallel path

= (e.m.f./conductor) × No. of conductors in series per parallel path

$$= \frac{P\phi N}{60} \times \frac{Z}{A}$$

$$\therefore \quad E_g = \frac{P\phi Z N}{60\,A}$$

where $A = 2$ ... *for wave winding*

$= P$ ... *for lap winding*

2.5 PARALLEL OPERATION OF DC GENERATOR

2.5.1 Advantages of DC Generator Operating in Parallel

In a dc power plant, power is usually supplied from several generators of small ratings connected in parallel instead of from one large generator. This is due to the following reasons:

1. **Continuity of service:** If a single large generator is used in the power plant, then in case of its breakdown, the whole plant will be shut down. However, if power is supplied from a number of small units operating in parallel, then in case of failure of one unit, the continuity of supply can be maintained by other healthy units.

2. **Efficiency:** Generators run most efficiently when loaded to their rated capacity. Therefore, when load demand on power plant decreases, one or more generators can be shut down and the remaining units can be efficiently loaded.
3. **Maintenance and repair:** Generators generally require routine-maintenance and repair. Therefore, if generators are operated in parallel, the routine or emergency operations can be performed by isolating the affected generator while load is being supplied by other units. This leads to both safety and economy.
4. **Increasing plant capacity:** In the modern world of increasing population, the use of electricity is continuously increasing. When added capacity is required, the new unit can be simply paralleled with the old units.
5. **Non-availability of single large unit:** In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally a single large unit is more expensive.

2.5.2 Connection of Parallel DC Generators

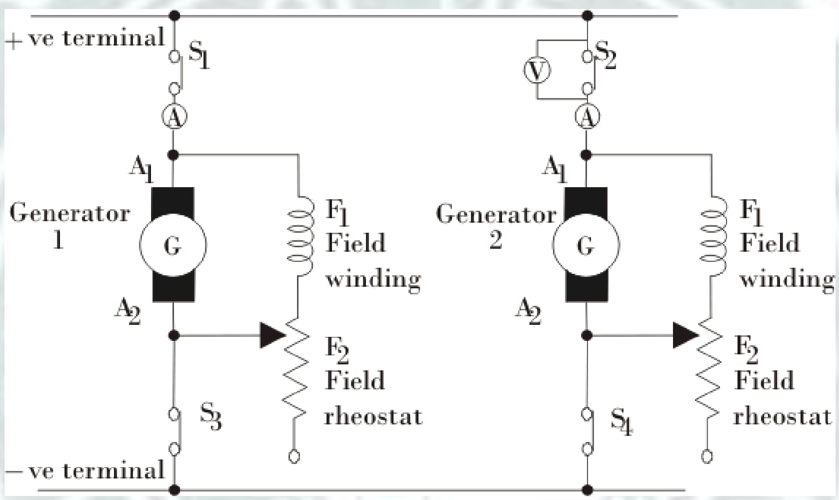
1. The generators in a power plant, connected by heavy thick copper bars, called bus-bars which act as positive and negative terminals. To connect the generators in parallel, Positive terminal of the generators are connected to the positive terminal of the bus-bars and negative terminals of generatorsare connected to negative terminal of the bus-bars, as shown in the figure.
2. To connect the 2 generators with the 1 existing working generators, first we have to bring the speed of the prime mover of the 2nd generator to the rated speed. At this point switch S_4 is closed.
3. The circuit breaker V_2 (voltmeter) connected across the open switch S_2 is closed to completethe circuit. The excitation of the generator 2 is increased with the help of field rheostat till it generates voltage equal to the voltage of bus-bars.
4. The main switch S_2 is then closed and the generator 2 is ready to be paralleled with existing generator. But at this point of time generator 2 is not taking any load as its induced e.m.f. is equal to bus-bar voltage. The present condition is called floating, that means ready for supply but not supplying current to the load.

5. In order to deliver current from generator 2, it is necessary that its induced e.m.f. E should be greater than the bus-bars voltage V. By strengthening the field current, the induced e.m.f. of generator 2 could be improved and the current supply will get started. To maintain bus-bar voltage, the field of generator 1 is weakened so that value remains constant.

Field current I given by

$$I = \frac{E - V}{R_a}$$

where, R_a is resistance of armature winding.



Example 1: A four pole generator, having lap wound armature winding has 51 slot containing 20 conductors. What will be the voltage generated in the machine when drivenat 1500 rpm assuming the flux per pole to be 7.0 mWb?

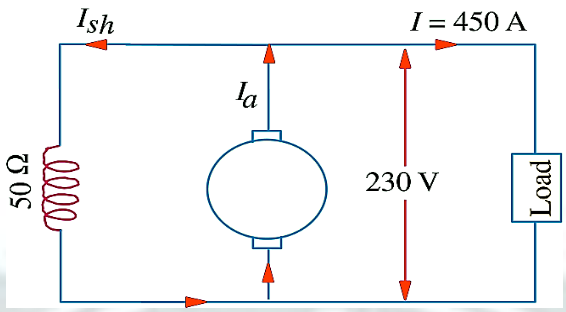
Solution:

$$Eg = \frac{P\phi ZN}{60A} \text{ volt}$$

$\Phi = 7 \times 10^{-3}$ Wb, Z = 51 × 20= 1020, A=P= 4, N= 1500 rpm

$$\therefore Eg = \frac{4 \times 7 \times 10^{-3} \times 1020 \times 1500}{60 \times 4} \text{volt} = 1785 \text{ volt}$$

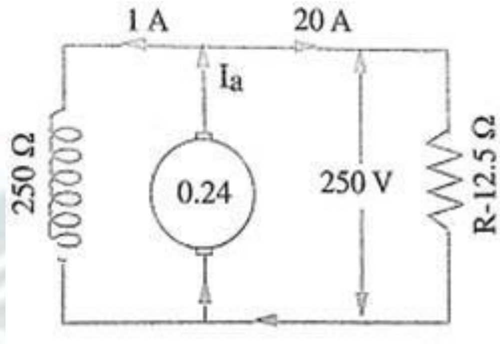
Example 2. A shunt generator delivers 450 A at 230 V and the resistance of the shunt field and armature are 50 Ω and 0.03 Ω respectively. Calculate the generated e.m.f?



Solution:

$$\begin{aligned}
&\textit{Current through shunt field winding is,}\\
I_{sh} &= 230/50 = 4.6\ A\\
\textit{Load current, } I &= 450\ A\\
\therefore \textit{Armature Current } I_a &= I + I_{sh}\\
&= 450 + 4.6 = 454.6\ A\\
\textit{Armature Voltage Drop, } I_aR_a &= 454.6 \times 0.03\\
&= 13.6\ V\\
\textit{Now, Generated Emf, } E_g &= \textit{terminal voltage} + \textit{armature drop}\\
&= V + I_aR_a\\
&\therefore \textit{Emf generated in armature}\\
E_g &= 230 + 13.6\\
&= 243.6\ V
\end{aligned}$$

<u>Example 3:</u> An 8-pole dc shunt generator with 778 wave-connected armature conductors and running at 500 rpm supplies a load of 12.5ohm resistance at terminal voltage of 250 V. The armature resistance is 0.24 ohm and the field resistance is 250ohm. Find the armature current, the induced emfand the flux per pole.



<u>Solution:</u>

The circuit is shown

Load current $= V/R = 250/12.5 = 20$ A

Shunt current $= 250/250 = 1$ A

Armature current $= 20 + 1 = 21$ A

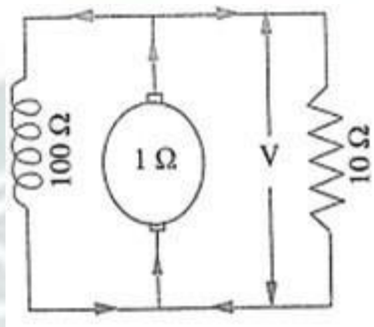
Induced e.m.f. $= 250 + (21 \times 0.24) = 255.04$ V

Now,
$$E_g = \frac{\Phi Z N}{60} \times \left(\frac{P}{A}\right)$$

$$\therefore \quad 255.04 = \frac{\Phi \times 778 \times 500}{60}\left(\frac{8}{2}\right)$$

$$\therefore \quad \Phi = 9.83 \text{ mWb}$$

Example 4: A 4- pole dc shunt generator with a shunt field resistance of 100 ohm and an armature resistance of 1 ohm has 378 wave connected conductors in its armature. The flux per pole is 0.02 Wb. If aload resistance of 10 ohm is connected across the armature terminals and the generator is driven at 1000 rpm, calculate the power absorbed by the load.



Solution:

Induced e.m.f. in the generator is

$$E_g = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ volt}$$

$$= \frac{0.02 \times 378 \times 1000}{60}\left(\frac{4}{2}\right) = 252 \text{ volt}$$

Now, let V be the terminal voltage *i.e.* the voltage available across the load as well as the shunt resistance

Load current = $V/10$ A and Shunt current = $V/100$ A

Armature current = $\frac{V}{10} + \frac{V}{100} = \frac{11\ V}{100}$

Now, $V = E_g$ – armature drop

$\therefore$ $$V = 252 - 1 \times \frac{11\text{V}}{100} \quad \therefore \quad \text{V} = 227 \text{ volt}$$

Load current = 227/10 = 22.7 A, Power absorbed by the load is = 227 × 22.7 = **5,153 W**

COURSE CONTENTS DC MOTOR

- Explain Principle of working of a DC motor.
- Explain concept of development of torque & back EMF in DC motor including simple problems.Derive equation relating to back EMF, Current, Speed and Torque equation
- Classify DC motors & explain characteristics, application.
- State & explain three point & four point stator/static of DC motor by solid State converter. Explain Speed of DC motor by field control and armature control method.
- Explain power stages of DC motor & derive Efficiency of a DC motor.

CHAPTER-3

DC MOTOR

3.1 PRINCIPLE OF OPERATION

DC motor operates on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force given by F = BIL newton. Where 'B' = flux density in wb, 'I' is the current and 'L' is the length of the conductor. The direction of force can be found by Fleming's left hand rule. From the point of construction, there is no difference between a DC generator and DC motor. Figure below shows a multipolar DC motor. Armature conductors are carrying current downwards under North Pole and upwards under South Pole. When the field coils are excited, with current carrying armature conductors, a force is experienced by each armature conductor whose direction can be found by Fleming's left hand rule. This is shown by arrows on top of the conductors. The collective force produces a driving torque which sets the armature into rotation. The function of a commutator in DC motor is to provide a continuous and unidirectional torque.

In DC generator the work done in overcoming the magnetic drag is converted into electrical energy. Conversion of energy from electrical form to mechanical form by a DC motor takes place by the work done in overcoming the opposition which is called the 'back emf'.

3.2 BACK EMF AND ITS SIGNIFICANCE

3.2.1 What is Back EMF

When the armature of a DC motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence emf is induced in them as in a generator.

The induced emf acts in opposite direction to the applied voltage V (Lenz's law) and is known as ***Back EMF or Counter EMF (Eb).***

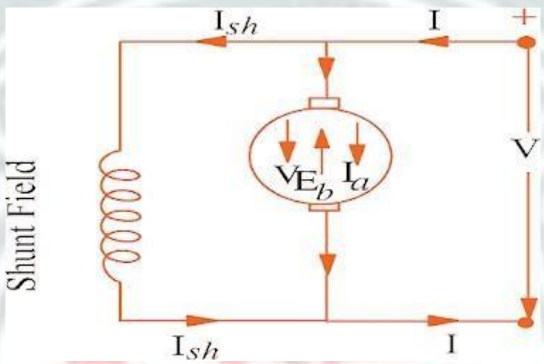
The equation for back emf in a DC motor is given below,

$$E_b = \frac{P\phi ZN}{60A}$$

The **back emf** E_b (= PΦZN/60 A) is always less than the applied voltage V, although this difference is small when the motor is running under normal conditions.

3.2.2 How Back EMF Occur in DC Motor

Consider a shunt wound DC motor



Therefore, driving torque acts on the armature which begins to rotate. As the armature rotates, back emf E_b is induced which opposes the applied voltage V. The applied voltage V has to force current through the armature against the back emf Eb. The electric work done in overcoming and causing the current to flow against E_b is converted into mechanical energy developed in the armature. It follows, therefore, that energyconversion in a dc motor is only possible due to the production of back emf E_b.

Net voltage across armature circuit = $V - E_b$

If Ra is the armature circuit resistance, then, $I_a = (V - E_b)/R_a$

Since V and R_a are usually fixed, the value of E_b will determine the current drawn by the motor.

If the speed of the motor is high, then back e.m.f. E_b (= PφZN/60 A) is large and hence the motor will drawless armature current and vice-versa.

3.2.3 The Significance of Back EMF

The presence of back emf makes the d.c. motor a *self-regulating machine* i.e., it makes the motor to draw asmuch armature current as is just sufficient to develop the torque required by the load.

Armature current (I_a),

$$I_a = \frac{V - E_b}{R_a}$$

When the motor is running on no load, small torque is required to overcome the friction and windage losses. Therefore, the armature current Ia is small and the back emf is nearly equal to the applied voltage.

If the motor is suddenly loaded, the first effect is to cause the armature to slow down. Therefore, the speed atwhich the armature conductors move through the field is reduced and hence the back emf Eb falls.

The decreased back emf allows a larger current to flow through the armature and larger current means increased driving torque.

Thus, the driving torque increases as the motor slows down. The motor will stop slowing down when the armature current is just sufficient to produce the increased torque required by the load.

If the load on the motor is decreased, the driving torque is momentarily in excess of the requirement so that armature is accelerated.

As the armature speed increases, the back emf Eb also increases and causes the armature current Ia to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load. Therefore, the back emf in a DC motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

3.3 TORQUE EQUATION

When armature conductors of a DC motor carry current in the presence of stator field flux, a mechanical torqueis developed between the armature and the stator. Torque is given by the product of the force and the radius at which this force acts.

Torque T = F × r (N-m) …where, F = force and r = radius of the armatureWork done by this force in once revolution = Force × distance = F × 2πr (where, 2πr = circumference of the armature)
Net power developed in the armature = word done / time
= (force × circumference × no. of revolutions) / time
= (F × 2πr × N) / 60 (Joules per second)

But, F × r = T and 2πN/60 = angular velocity ω in radians per second. Putting these in the above equation

Net power developed in the armature = P = T × ω (Joules per second) = $\frac{2\pi N}{60}$ × T = NT/9.55

3.3.1 Armature Torque (Ta)

The power developed in the armature can be given as P_a = Ta × ω = T_a × 2πN/60 rpm

The mechanical power developed in the armature is converted from the electrical power, Therefore, Mechanical Power = Electrical Power

That means, T_a × 2πN/60 = E_b . I_a

Or $$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} = 9.55\,\frac{E_b I_a}{N} N-m$$

The above equation shows the relationship between armature torque, back emf, flux and speed of a DC motor.

We know, $E_b = P\Phi NZ / 60A$

Therefore, $T_a \times 2\pi N/60 = (P\Phi NZ / 60A) \times I_a$

Rearranging the above equation,

$\mathbf{T_a = (PZ / 2\pi A) \times \Phi . I_a}$ (N-m) $= 0.159 \dfrac{P\emptyset ZIa}{A} N-m$

The term $(PZ / 2\pi A)$ is practically constant for a DC machine. Thus, armature torque is directly proportional to the product of the flux and the armature current i.e. $Ta \propto \Phi.\ Ia$

a. In case of DC series motor ϕ is directly promotional to Ia, therefore $Ta \propto Ia^2$

b. For shunt motor, ϕ is practically constant, hence $Ta \propto Ia$

3.3.2 Shaft Torque (Tsh)

Due to iron and friction losses in a dc machine, the total developed armature torque is not available at the shaft of the machine. Some torque is lost, and therefore, shaft torque is always less than the armature torque.

Shaft torque of a DC motor is given as

T_{sh} = output in watts / $(2\pi N/60)$ ….. (where, N is speed in RPM).

$= 9.55 \dfrac{Output}{N} N-m$

The difference $T_a - T_{sh}$ is known as lost torque.

3.4 SPEED OF A DC MOTOR

We know

$E_b = V - I_a R_a$ or $\frac{P\emptyset ZN}{60A} = V - I_a R_a$

$$\therefore N = \frac{V - I_a R_a}{\emptyset} \times \frac{60A}{ZP} rpm$$

$= \frac{E_b}{\emptyset}\frac{60A}{ZP} rpm$ or $N = k\frac{E_b}{\emptyset}$ or $N \propto \frac{E_b}{\emptyset}$

For Series Motor:

Let N_1, N_2 = speed in the 1st and 2nd case

I_{a1}, I_{a2} = armature current in the 1st and 2nd case

Φ_1, Φ_2 = flux / pole in the 1st and 2nd case

$N_1 \propto \frac{E_{b1}}{\emptyset_1}$ where $E_{b1} = V - I_{a1}R_a$ and $N_2 \propto \frac{E_{b2}}{\emptyset_2}$ where $E_{b2} = V - I_{a2}R_a$

$\therefore \frac{N_2}{N_1} = \frac{E_{b1}}{E_{b2}} \times \frac{\emptyset_1}{\emptyset_2}$ as in series motor $\emptyset \propto I_a$ $\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$

For Shunt Motor

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\emptyset_1}{\emptyset_2}$$

3.5 CLASSIFICATION OF DC MOTOR

The **types of DC motor** include:

1. Separately Excited DC Mtor
2. Self Excited DC Motorwrt4
    a. Shunt Wound DC Motor
    b. Series Wound DC Motor
    c. Compound Wound DC Motor
        i. Short shunt DC Motor
        ii. Long shunt DC Motor

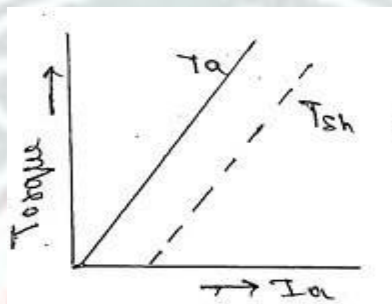
3.6 CHARACTERISTICS OF DC MOTOR

3.6.1 Characteristics of DC Shunt Motor

3.6.1.1 Armature Torque Vs Armature Current Ta vs Ia Characteristics

For a shunt motor flux can be assumed practically constant (through at heavy loads, ϕ decreases somewhat due to increased armature reaction), hence $T_a \propto I_a$.

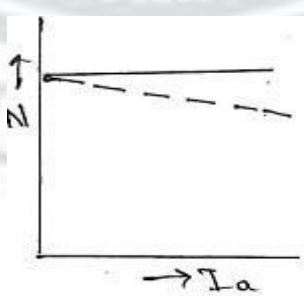
Therefore electrical characteristic is shown below, is practically a straight line through the origin. Shafttorque is shown as dotted line.



Torque Current Characteristic of DC shunt motor

3.6.1.2 Speed vs Armature Current Na vs Ia Characteristics

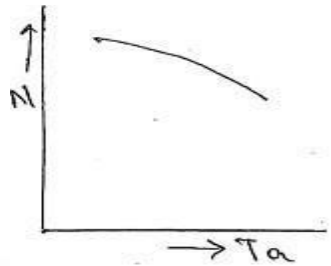
As flux ϕ is assumed to be constant, we can say N a Eb. But, as back emf is also almost constant, the speed should remain constant. But practically, ϕ as well as Eb decreases with increase in load. Back emf Eb decreases slightly more than ϕ, therefore, the speed decreases slightly. Generally, the speed decreases only by 5 to 15% offull load speed. Therefore, a shunt motor can be assumed as a constant speed motor. In speed vs. armature current characteristic in the following figure, the straight horizontal line represents the ideal characteristic and the actual characteristic is shown by the dotted line.



Speed vs armature current characteristics of DC shunt motor

3.6.1.3 Speed vs Armature Torque Na vs Ta Characteristics

From $T_a$vs I_a and $N_a$vs I_a with increase with torque the speed of DC shunt motor decreases. The nature of thecharacteristics is drooping in nature shown in figure as given below.



Speed vs armature torque characteristics of DC shunt motor

3.6.2 Characteristics of DC Series Motor

3.6.2.1 Armature Torque Vs Armature Current Characteristics

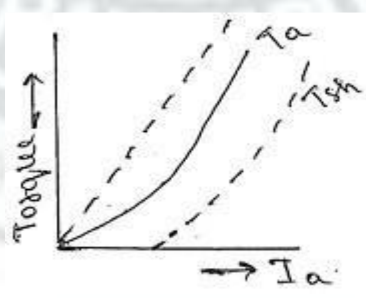
We know that $T_a \propto \phi I_a$.

In case of DC series motor as field windings also carry the armature current

$\phi \propto I_a$

$\therefore T \propto I^2$

At light loads, I_a and hence ϕ is small. But as I_a increases T_a increases as the square of the current up-to saturation. After saturation ϕ becomes constant, the characteristic becomes a straight line as shown in Figure below. Therefore a series motor develops a torque proportional to the square of the armature current. This characteristic is suited where huge starting torque is required for accelerating heavy masses.



Torque Current Characteristic of DC series motor

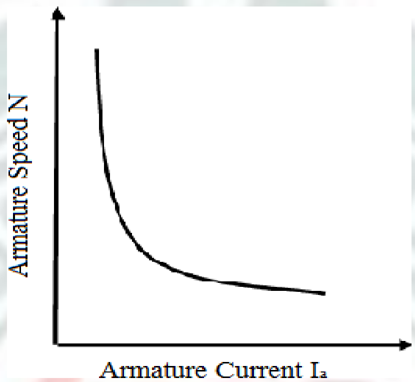
3.6.2.2 Speed vs Armature Current Characteristics

$N \propto E_b/\phi$

In DC series motor $I_a \propto \phi$

Therefore $N \propto 1/I_a$

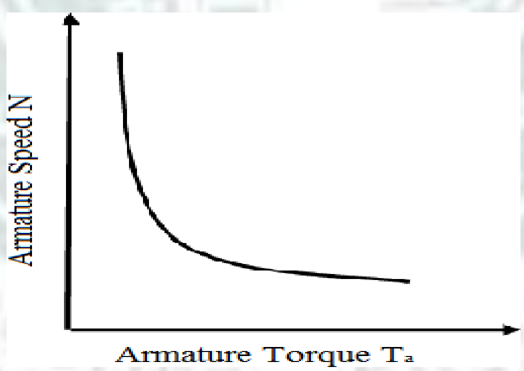
If I_a increases, speed decreases. This characteristic is shown in figure below. Therefore the speed is inversely proportional to armature current I_a. When load is heavy I_a ia heavy thus speed is low. When load is low I_a is low thus speed becomes dangerously high. Hence series motor should never start without load on it.



Speed vs armature current characteristics of DC series motor

3.6.2.3 Speed Vs Armature Torque Characteristics

From T_a vs I_a and N_a vs I_a characteristics Speed is inversely proportional to torque. The characteristic isshown in figure as given below.



Speed vs armature torque characteristics of DC series motor

3.6.3 Characteristics of DC Compound Motor

There are two different types of compound motors in common use, they are the cumulative compound motor and the differential compound motor. In the cumulative compound motor, the field produced by the series winding aids the field produced by the shunt winding. The speed of this motor falls more rapidly with increasing current than does that of the shunt motor because the field increases. In the differential compound motor, the flux from the series winding opposes the flux from the shunt winding. The field flux, therefore, decreases with increasing load current. Because the flux decreases, the speed may increases with increasing load. Depending on the ratio of the series-to-shunt field ampere-turns, the motor speed may increases very rapidly.

3.7 APPLICATION OF DC MOTORS

3.7.1 Application of DC Shunt Motor

The characteristics of a DC shunt motor give it a very good speed regulation, and it is classified as a constantspeed motor, even though the speed does slightly decrease as load is increased. Shunt wound motors are used in industrial and automotive applications where precise control of speed and torque are required.

3.7.2 Application of DC Series Motor

For a given input current, the starting torque developed by a DC series motor is greater than that developed by a shunt motor. Hence series motors are used where huge starting torques are necessary. Ewx. Cranes, hoists, electric traction etc. The DC series motor responds by decreasing its speed for the increased in load. The current drawn by the DC series motor for the given increase in load is lesser than DC shunt motor. The drop in speed with increased load is much more prominent in series motor than that in a shunt motor. Hence series motor is not suitable for applications requiring a constant speed.

3.7.3 Application of DC Compound Motor

Cumulative compound wound motors are virtually suitable for almost all applications like business machines, machine tools, agitators and mixers etc. Compound motors are used to drive loads such as shears, presses and reciprocating machines.

Differential compound motors are seldom used in practice (because of rising speed characteristics).

3.8 STARTING OF DC MOTOR

3.8.1 Necessity of Starter

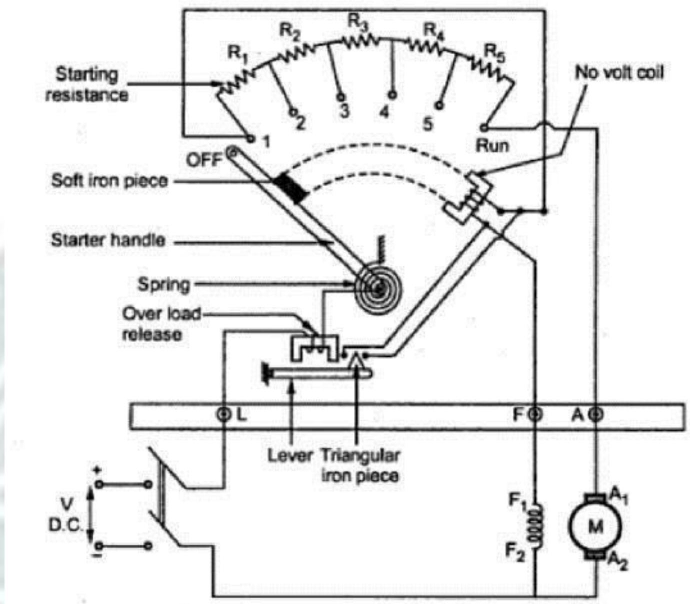
The current drawn by the armature is given by $I_a = \frac{Vt - Eb}{R_a}$

At starting, as N=0 so Eb = 0 thus $I_a = \frac{Vt}{R_a}$

Armature resistance will be very low. Therefore, the current drawn by the motor will be very high. In order to limit this high current, a starting resistance is connected in series with the armature. The starting resistance will be excluded from the circuit after the motor attains its rated speed. From there on back emf limits the current drawn by the motor.

3.8.2 Three Point Starter

The arrangement is shown in the figure below shows a three point starter for shunt motor.



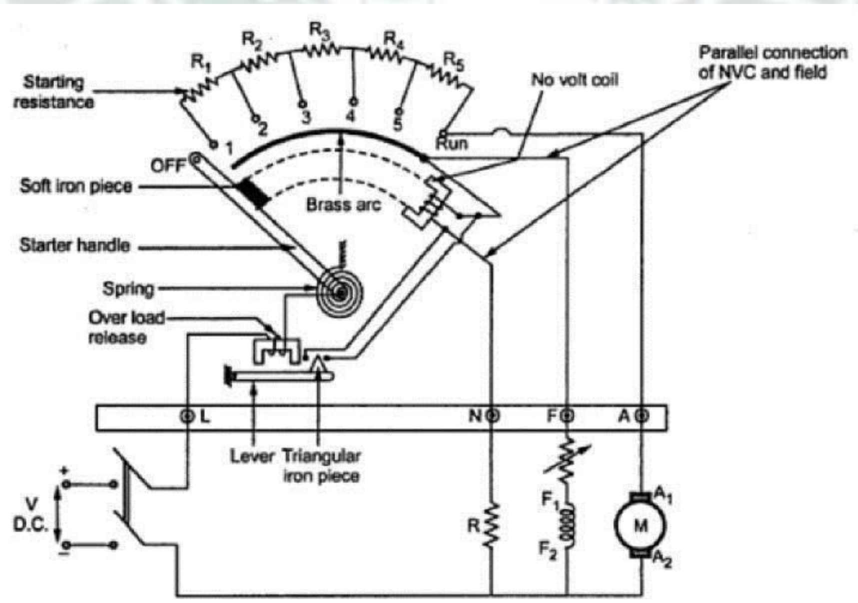
Internal view of three point starter

It consists of resistances arranged in steps, R_1 to R_5 connected in series with the armature of the shunt motor. Field winding is connected across the supply through a protective device called 'NO – Volt Coil'. Another protection given to the motor in this starter is 'over load release coil'. To start the motor the starter handle is moved from OFF position to Run position gradually against the tension of a hinged spring. An iron piece is attached to the starter handle which is kept hold by the No-volt coil at Run position. The function of No volt coil is to get de-energized and release the handle when there is failure or disconnection or a break in the fieldcircuit so that on restoration of supply, armature of the motor will not be connected across the lines without starter resistance. If the motor is over loaded beyond a certain predetermined value, then the electromagnet of overload release will exert a force enough to attract the lever which short circuits the electromagnet of Novolt coil. Short circuiting of No volt coil results in de-energisation of it and hence the starter handle will be released and return to its off position due to the tension of the spring.

If it is desire to control the speed of the motor in addition, then a field rheostat is connected in the field circuit. The motor speed can be increased by weakening the flux ($N \propto 1/\phi$). But there is one difficulty for control speed with this arrangement. If too much resistance is cut in by the field rheostat, then field current is reduced too much so that it is enable to create enough electromagnetic pull to overcome the spring tension. Hence the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-sped motor. This can be overcome with four-point starter.

3.8.3 Four Point Starter

One important change is the No Volt Coil has been taken out of the shunt field and has been connected directly across the line through a Protecting resistance 'R'. When the arm touches stud one. The current divides into three paths, 1. Through the starter resistance and the armature, 2. Through shunt field and the field rheostat and 3. Through No-volt Coil and the protecting resistance 'R'. With this arrangement, any change of current in shunt field circuit does not affect the current passing though the NO-volt coil because, the two circuits are independent of each other. Thus the starter handle will not be released to its off position due to changes in the field current which may happen when the field resistance is varied. Figure given below shows internal view of 4-point starter.



Internal view of four point starter

3.8.4 Losses and Efficiency of DC Machines

Efficiency of a DC motor: $\eta = \frac{Output}{Input} = \frac{Input - Losses}{Input}$

Various losses occurring in a DC machine are listed below- Total losses can be broadly divided into twotypes.

1) Constant losses
2) Variable losses

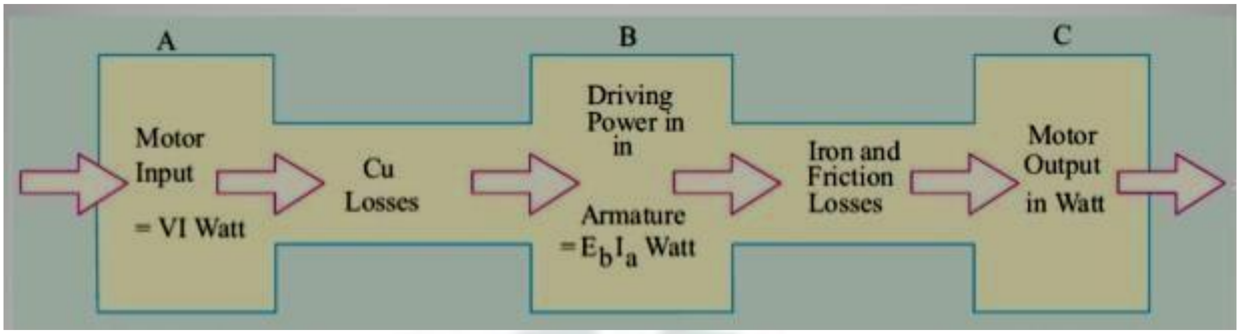
These losses can be further divided as

1) Constant losses –
   - i) Core loss or ironloss
     - a) Hysteresis loss
     - b) Eddy current loss
   - ii) Mechanical loss
     - a) Windage loss
     - b) Friction loss – brush friction loss and Bearing friction loss.
2) Variable losses –
   - i) copper loss ($I^2 r$)
     - a) Armature copper loss
     - b) Field copper loss
     - c) Brush contact loss
   - ii) Stray load loss
     - a) Copper stray load loss
     - b) Core stray load loss

Core loss or iron loss occurs in the armature core is due to the rotation of armature core in the magnetic fluxproduced by the field system. Iron loss consists of a) Hysteresis loss and b) Eddy current loss.

3.9 POWER STAGES OF DC MOTOR

Power flow in a DC generator is shown in figure



Power flow in a DC motor

Overall or commercial Efficiency= C/AElectrical Efficiency = B/A

Mechanical Efficiency = C/B

A-B = Cu Losses and B – C = iron and friction losses

CONDITION FOR MAXIMUM EFFICIENCY

Generator output = VI_L where V is the terminal voltage and I_L is load current.

Generator input = VI_L + losses = $VI_a + I_a^2r + P_c$

Where I_a^2r = Cu loss also called as variable loss; P_c= Iron loss also called as constant loss

If the shunt field current is negligible, then $I_a = I_L$Hence Generator input = $V I_L + I_L^2 r + P_c$

Generator Efficiency: $\eta = \frac{Output}{Input} = \frac{VIL}{VI_L + I_L^2 R_a + P_C}$

Efficiency will be maximum when $\frac{d}{dI_L}\eta = 0$ OR $I_L^2 R_a = P_C$

Hence efficiency is maximum when variable loss is equal to constant loss.

The load current corresponding to maximum efficiency is $I_L = \sqrt{\frac{P_c}{r_a}}$

3.10 SPEED CONTROL OF DC MOTOR

Speed of a DC motor can be controlled in a wide range.

$$N = \frac{Eb}{K\emptyset} = \frac{V - I_a R_a}{K\emptyset}$$

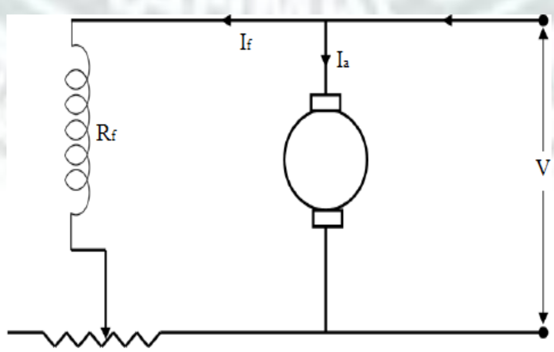
The speed equation shows that speed can be controlled by-

1. Variation of field current which varies the flux/pole and is known as field control.
2. Variation of armature resistance known as armature voltage control.
3. Variation of terminal voltage 'V' known as Ward Leonard method.

3.10.1 Speed Control of Shunt Motor

1. *Flux control method:* It is already explained above that the **speed of a dc motor** is inversely proportionalto the flux per pole. Thus by decreasing the flux, speed can be increased and vice versa.

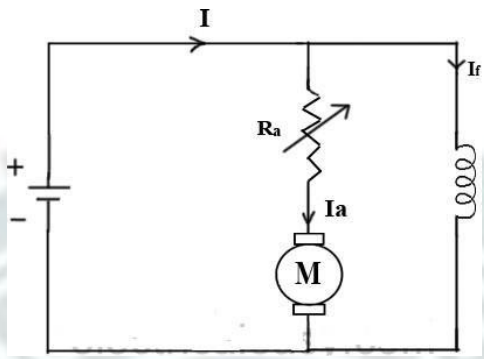
   To control the flux, a rheostat is added in series with the field winding, as shown in the circuit diagram. Adding more resistance in series with the field winding will increase the speed as it decreases the flux. In shunt motors, as field current is relatively very small, I_{sh}^2R loss is small. Therefore, this method is quite efficient. Though speed can be increased above the rated value by reducing flux with this method, it puts a limit to maximum speed as weakening of field flux beyond a limit will adversely affect the commutation.



Circuit diagram for speed control using field control method

2. *Armature control method*: **Speed of a dc motor** is directly proportional to the back emf E_b and

$E_b = V - I_aR_a$. That means, when supply voltage V and the armature resistance R_a are kept constant, then the speed is directly proportional to armature current I_a. Thus, if we add resistance in series with the armature, I_a decreases and, hence, the speed also decreases. Greater the resistance in series with the armature, greater the decrease in speed



Circuit diagram for speed control using armature control method

3.10.2 Speed Control of Series Motor

1. ***Flux control method:***

   a. *Field diverters:* variable resistance is connected parallel to the series field as shown in fig (a). This variable resistor is called as a diverter, as the desired amount of current can be diverted through this resistor and, hence, current through field coil can be decreased. Thus, flux can be decreased to the desired amount and speed can be increased.

   b. *Armature diverter*: Diverter is connected across the armature as shown in fig (b).

      For a given constant load torque, if armature current is reduced then the flux must increase, as $T_a \propto \text{Ø} I_a$

      This will result in an increase in current taken from the supply and hence flux Ø will increaseand subsequently speed of the motor will decrease.

   c. *Tapped field control:* As shown in fig (c) field coil is tapped dividing number of turns. Thus we can select different value of Ø by selecting different number of turns.

   d. *Paralleling field coils*: In this method, several speeds can be obtained by regrouping coils as shown in fig (d).

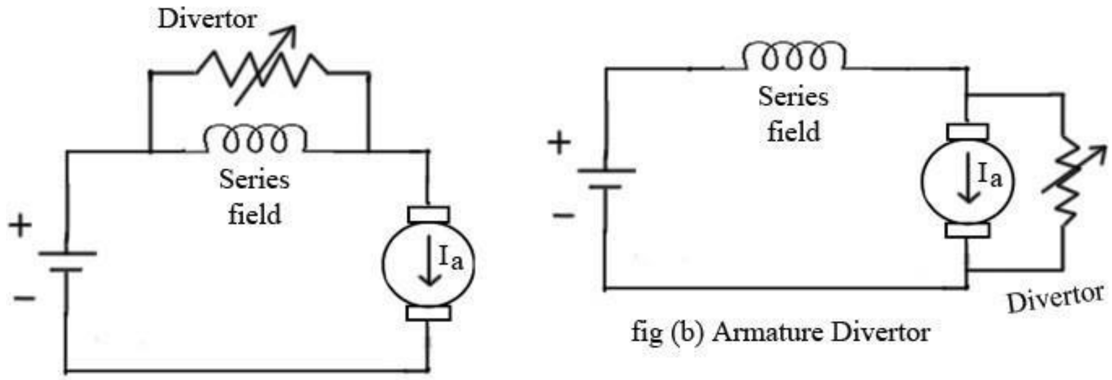


fig (a) Field Divertor

fig (b) Armature Divertor

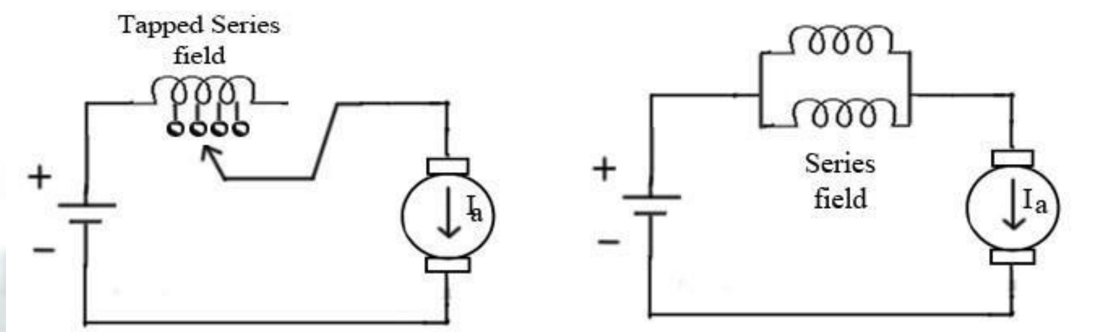


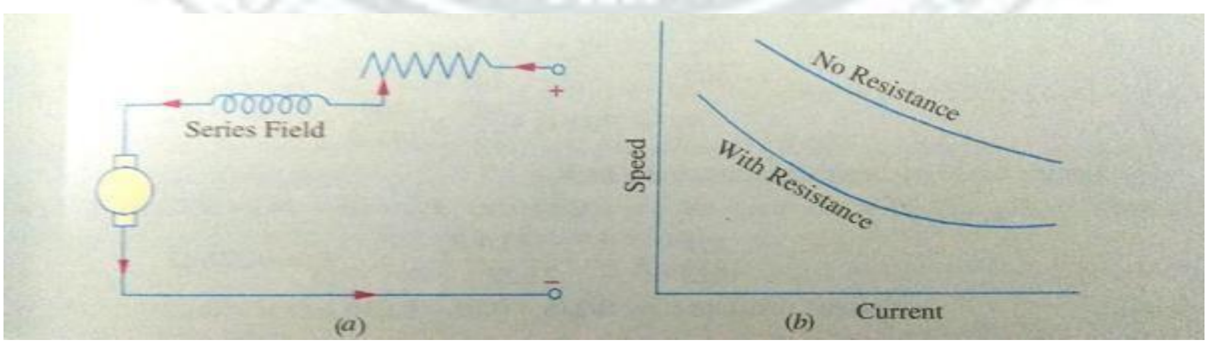
fig (c) Tapped field

fig (d) Paralleling Field coils

2. ***Variable Resistance in series with Motor:***

By increasing the resistance in series with the armature as shown in figure given below the voltage applied across the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted thatsince full motor current passes through this resistance, there is a considerable loss of power in it.



Example 1: A 4-pole wave wound dc series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m.

Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.

Solution:

$$T_a = 0.159\,\phi\,ZI_a\,(P/A)\text{ N-m}$$
$$209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a\,(4/2);\ I_a = 20.1\text{ A}$$
$$E_a = V - I_aR_a = 500 - 20.1 \times 3 = 439.7\text{ V}$$
$$E_b = \Phi\,ZN \times (P/A)\text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2$$
$$N = 6.73\text{ r.p.s. or } 382.2\text{ r.p.m.}$$

Example 2:

A 230V shunt motor delivers 30hp at the shaft at 1120rpm. If the motor has an efficiency of 87% at this load, determine:

a) The total input power.
b) The line current.

Solution

(a) $\eta = \frac{P_{o/p}}{P_{i/p}}$

$$P_{i/p} = \frac{P_{o/p}}{\eta}$$

$$= \frac{30 * 746}{0.87}$$ (output power= 30 Hp= 30×746 =22,380 W)

$$= 25.72\ Kw$$

$$P_{i/p} = 25.72\ Kw$$

(b) $P_{i/p} = V_t I_l$

Hence I_l = 25720/230 = 111.82 A

Example 3. A 250 volt DC shunt motor has armature resistance of 0.25 ohm on load it takes an armature current of 50A and runs at 750rpm. If the flux of the motor is reduced by 10% without changing the load torque, find the new speed of the motor.

Solution:

Given data $V = 250$

$R_a = 0.25$

$I_a = 50$

$N_1 = 750$ rpm$\Phi_2 = 90\%\Phi_1$

For shunt motor

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} = \frac{\Phi_1}{\Phi_2}$$

$E_{b1} = V - I_{a1}R_a = 250-(50x0.25) = 237.5V E_{b2} = V - I_{a2}R_a$

Load torque is constant

$Ta_1 = Ta_2$

Or $\Phi_1 \, Ia_1 = \Phi_2 \, Ia_2$

Or $\Phi_1 \times 50 = 0.9 \, \Phi_1 \, Ia_2$ hence $I_{a2} = 55.55A$ $E_{b2} = 250-55.55X0.25 = 236.12V$

N_2 = 828 rpm

Example 4. A 230-V d.c. shunt motor has an armature resistance of 0.5 Ω and field resistance of 115 Ω. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.

Solution.

$$N_1 = 1200 \text{ r.p.m.}, \ E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V}$$

$$N_2 = 1120 \text{ r.p.m.}, \ E_{b2} = 230 - 0.5\, I_{a2}$$

Now, $\dfrac{N_2}{N_1} = \dfrac{E_{b2}}{E_{b1}} \therefore \dfrac{1120}{1200} = \dfrac{230 - 0.5 I_{a2}}{228.75}$; $I_{a2} = \mathbf{33\ A}$

Line current drawn by motor $= I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35\ A}$

Power input at rated load $= 230 \times 35 = \mathbf{8{,}050\ W}$

Example 5. A dc motor takes an armature current of 110A at 480 V. The armature circuit resistance is 0.2 ohm. The machine has 6 poles and the armature is lap connected with 864 conductors. The flux per pole is 0.05 Wb . Calculate (i) the speed and (ii) gross torque developed by the armature?

Solution:

$E_b = 480 - 110 \times 0.2 = 458$ V, $\quad \Phi = 0.05$ W, $Z = 864$

$$E_b = \frac{\Phi ZN}{60}\left(\frac{P}{A}\right) \text{ or } 458 = \frac{0.05 \times 864 \times N}{60} \times \left(\frac{6}{6}\right)$$

$$N = 636 \text{ r.p.m.}$$

$$T_a = 0.159 \times 0.05 \times 864 \times 110\ (6/6) = 756.3 \text{ N-m}$$

Example 6:

A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.

Solution. As Generator [Fig. (*a*)]

Output current $= 25{,}000/250 = 100\ \text{A}\ ;\ I_{sh} = 250/100 = 2.5\ \text{A}\ ;\ I_a = 102.5\ \text{A}$

Generated e.m.f. $= 250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15\ \text{V}$

Power developed in armature $= E_b I_a = \dfrac{256.15 \times 102.5}{1000} = 26.25\ \text{kW}$

As Motor [Fig (*b*)]

Motor input current $= 100\ \text{A}\ ;\ I_{sh} = 2.5\ \text{A},\ I_a = 97.5\ \text{A}$

$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15\ \text{V}$

Power developed in armature $= E_b I_a = 244.15 \times 97.5/1000 = 23.8\ \text{kW}$

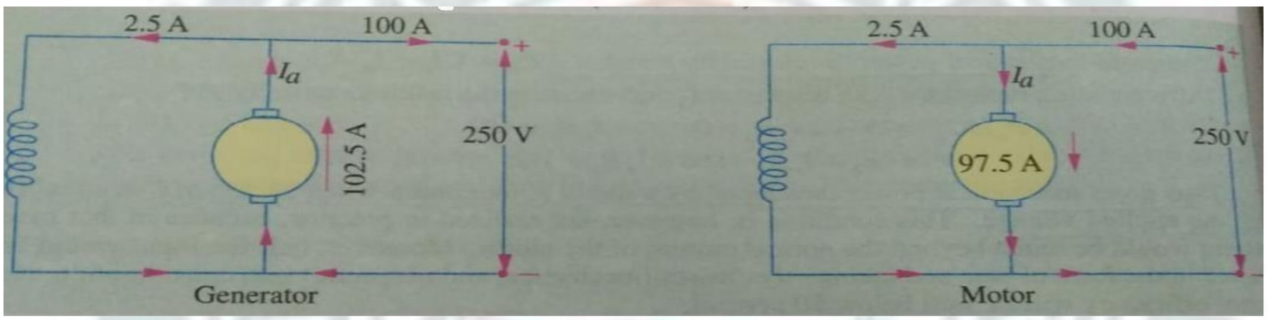


Figure (a) Figure (b)

Example 7:

Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

Solution. Developed torque or gross torque is the same thing as armature torque.

$\therefore\quad T_a = 0.159\,\Phi ZA\,(P/A)$

$= 0.159 \times 25 \times 10^{-3} \times 800 \times 45\ (4/2) = 286.2\ \text{N-m}$

$E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193\ \text{V}$

Now, $E_b = \Phi ZN\,(P/A)$ or $193 = 25 \times 10^{-3} \times 800 \times N \pi \times (4/2)$

$\therefore\quad N = 4.825\ \text{r.p.s.}$

Also, $2\pi N T_{sh} = \text{output}$ or $2\pi \times 4.825\ T_{sh} = 8200$ $\quad\therefore T_{sh} = 270.5\ \text{N-m}$

Example 8:

A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that $R_a = 0.2\ \Omega$.

Solution. As seen from Art 27.7, $T_a \propto \Phi I_a$. Since Φ is constant, $T_a \propto I_a$

$\therefore \quad T_{a1} \propto I_{a1}$ and $T_{a2} \propto I_{a2} \quad \therefore \quad T_{a2}/T_{a1} = I_{a2}/I_{a1}$

$\therefore \quad 2 = I_{a2}/50$ or $I_{a2} = 100$ A

Now, $N_2/N_1 = E_{b2}/E_{b1}$ – since Φ remains constant.

$E_{b1} = 220 - (50 \times 0.2) = 210$ V $\qquad E_{b2} = 220 - (100 \times 0.2) = 200$ V

$\therefore \quad N_2/500 = 200/210 \qquad \therefore N_2 = $ **476 r.p.m.**

Example 9:

A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 Ω. Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) rotational losses and (e) efficiency.

Solution.

$$E_b = V - I_a R_a - \text{brush drop} = 240 - (50 \times 0.1) - 2 = 233 \text{ V}$$

Also

$$I_a = 50 \text{ A}$$

(a) Armature torque $T_a = 9.55 \dfrac{E_b I_a}{N}$ N-m $= 9.55 \times \dfrac{233 \times 50}{1000} =$ **111 N-m**

(b) $T_{sh} = 9.55 \dfrac{\text{output}}{N} = 9.55 \times \dfrac{11,190}{1000} =$ **106.9 N-m**

(c)
$$E_b = \frac{\Phi Z N}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

$$\therefore \quad 233 = \frac{\Phi \times 540 \times 1000}{60} \times \left(\frac{4}{2}\right) \quad \therefore \quad \mathbf{\Phi = 12.9\ mWb}$$

(d) Armature input $= V I_a = 240 \times 50 =$ **12,000 W**

Armature Cu loss $= I_a^2 R_a = 50^2 \times 0.1 = 250$ W ; Brush contact loss $= 50 \times 2 = 100$ W

$\therefore$ Power developed $= 12,000 - 350 = 11,650$ W ; Output $= 11.19$ kW $= 11,190$ W

$\therefore$ Rotational losses $= 11,650 - 11,190 =$ **460 W**

(e) Total motor input $= VI = 240 \times 51 = 12,340$ W ; Motor output $= 11,190$ W

$$\therefore \quad \text{Efficiency} = \frac{11,190}{12,240} \times 100 = \mathbf{91.4\ \%}$$

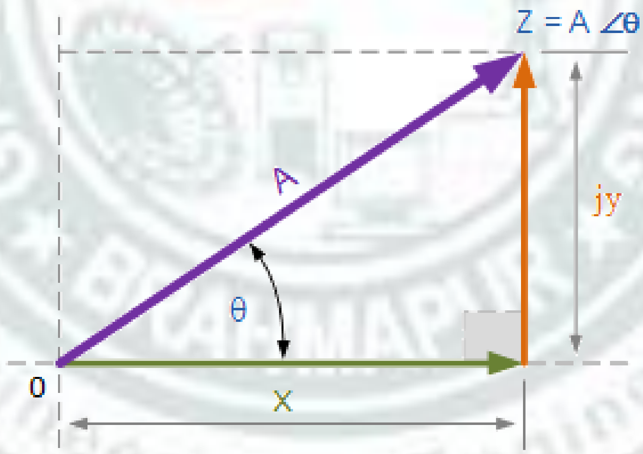
CHAPTER - 4

AC CIRCUITS

- State Mathematical representation of phasors, significant of operator "J"
- Discuss Addition, Subtraction, Multiplication and Division of phasor quantities.
- Explain AC series circuits containing resistance, capacitances, Conception of active,reactive and apparent power and Q-factor of series circuits & solve related problems.
- Find the relation of AC Parallel circuits containing Resistances, Inductance andCapacitances Q-factor of parallel circuits.

4.1 PHASOR ALGEBRA

The 'Phasor' is defined as "The complex number in the polar form with which we can analyze the circuit". It is a vector quantity. In this vector representation we use Cartesian plane.



A vector quantity can be expressed in terms of

(i) Rectangular or Cartesian form

(ii) Trigonometric form

(iii) Exponential form

(iv) Polar form

(i) Rectangular or Cartesian form

In the rectangular form, the phasor is divided up into a real part, x and an imaginary part, y forming the generalised expression Z= x + jy

Where $x = A\cos\theta$ is the active part and $y = A\sin\theta$ is the reactive part

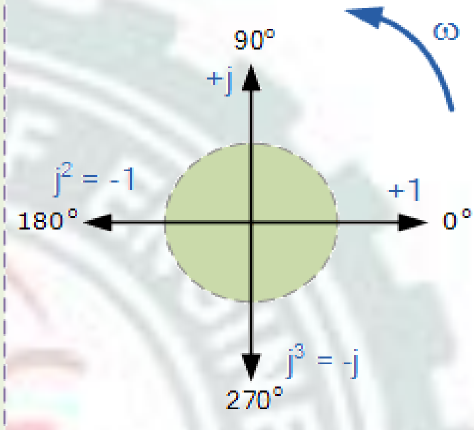
j is an operator which shift the phasor by an angle of 90° in counter-clock wise direction.

$$90^0 \text{ rotation: } \quad j^1 = \sqrt{-1} = +j$$

$$180^0 \text{ rotation: } \quad j^2 = \left(\sqrt{-1}\right)^2 = -1$$

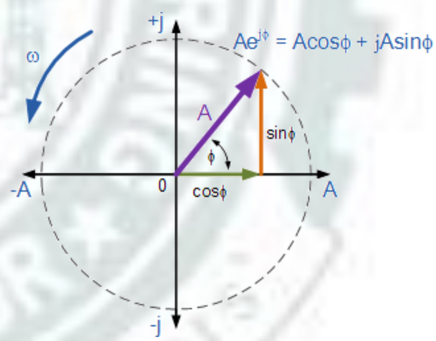
$$270^0 \text{ rotation: } \quad j^3 = \left(\sqrt{-1}\right)^3 = -j$$

$$360^0 \text{ rotation: } \quad j^4 = \left(\sqrt{-1}\right)^4 = +1$$



(ii) Trigonometric form

$Z = A(\cos\theta + j\sin\theta)$



(iii) Exponential form :-

$Z = A\,e^{j\theta}$ where $\theta = \tan^{-1} y/\underline{x}$ and $A=\sqrt{(x^2 + y^2)}$

(iv) Polar form :- Z= A∠θ

$$A^2 = x^2 + y^2$$

$$A = \sqrt{x^2 + y^2}$$

Also, $x = A.\cos\theta$, $y = A.\sin\theta$

Complex Addition and Subtraction

$A = x + jy \qquad B = w + jz$

$$A + B = (x + w) + j(y + z)$$

$$A - B = (x - w) + j(y - z)$$

Multiplication and Division of Complex Numbers

$$A \times B = (4 + j1)(2 + j3)$$

$$= 8 + j12 + j2 + j^2 3$$

but $j^2 = -1$,

$$= 8 + j14 - 3$$

$$A \times B = 5 + j14$$

Multiplication in Polar Form

$$Z_1 \times Z_2 = A_1 \times A_2 \angle \theta_1 + \theta_2$$

Multiplying together $6 \angle 30^o$ and $8 \angle -45^o$ in polar form gives us.

$$Z_1 \times Z_2 = 6 \times 8 \angle 30^o + (-45^o) = 48 \angle -15^o$$

Division in Polar Form

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right) \angle \theta_1 - \theta_2$$

$$\frac{Z_1}{Z_2} = \left(\frac{6}{8}\right) \angle 30^o - (-45^o) = 0.75 \angle 75^o$$

Example 1: Find $|-1 + 4i|$.

Sol: $|-1 + 4i| = \sqrt{1 + 16} = \sqrt{17}$

Examples 2. Write the following complex numbers in trigonometric form:

(a) $-4 + 4i$

To write the number in trigonometric form, we need A and θ.

$A = \sqrt{-4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = \frac{4}{-4} = -1$

Hence $\theta = 3\pi/4$ Then, $-4 + 4i = 4\sqrt{2}\,(\cos 3\pi/4 + j\sin 3\pi/4)$

(b) $2 - j\,\frac{2\sqrt{3}}{3}$

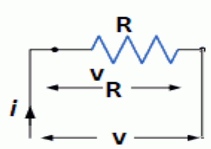
$$E = \sqrt{\left[2^2 + \left(\frac{2\sqrt{3}}{3}\right)\right]} = \frac{4\sqrt{3}}{3}$$

$$\tan\theta = \frac{-\frac{2\sqrt{3}}{3}}{2} = -\sqrt{3}/3 \text{ hence } \theta = 11\pi/6,$$

Then, the trigonometric form is $\frac{4\sqrt{3}}{3}(\cos 11\pi/6 + j\sin 11\pi/6)$

Purely Resistive Circuit

In a purely resistive circuit whole of the applied voltage is utilized in overcoming the ohmicresistance of the circuit. A *purely resistive circuit* is also known as the non-inductive circuit.



Pure Resistive Circuit

Applied Voltage, $v = V_m \sin\omega t$

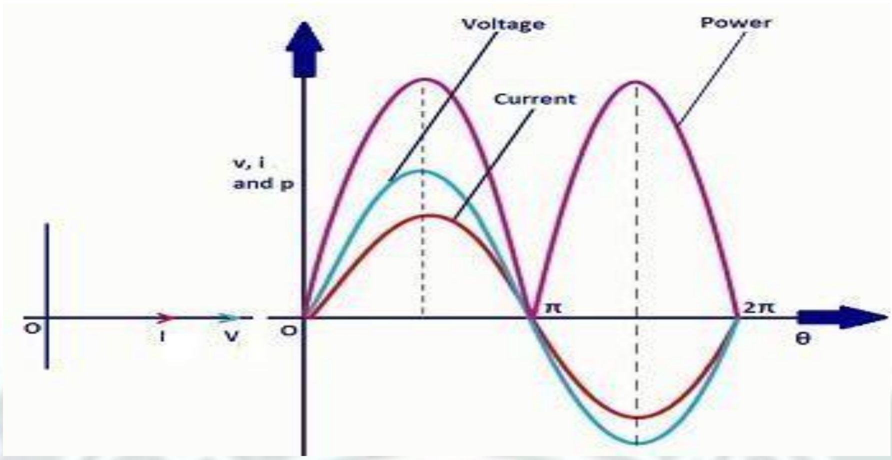
Resultant Current, $i = I_m \sin\omega t$

where, $I_m = V_m / R$

Power = VI watts

Power factor, $\cos\phi = 1$

From the expression of instantaneous applied voltage and instantaneous current it is evident that in a **purely resistive circuit**, the applied voltage and current are in phase with each other.

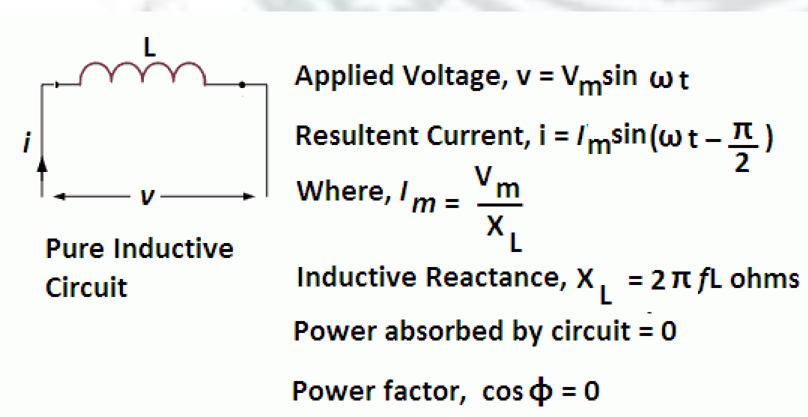


It is seen from the **power curve for purely resistive circuit** no part of power cycle becomes negative at any time i.e. in the purely resistive circuit power is never zero. This is so because instantaneous values of voltage and current are always either positive or negative and hence the product is always positive. The frequency of power cycle is double that of the voltage andcurrent waves.

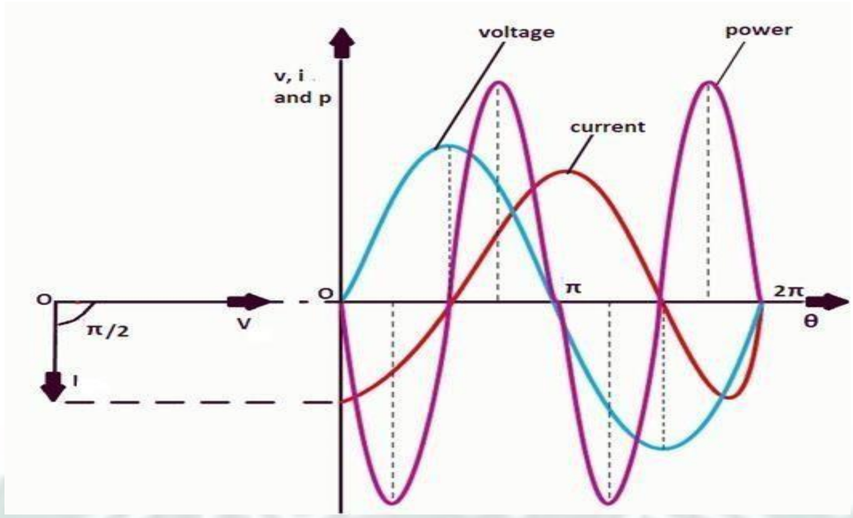
The power factor of the purely resistive circuit (cos φ) is 1.

Purely Inductive Circuit

A pure inductive coil is that which has no ohmic resistance and hence no I^2R loss.



From the expression of instantaneous applied voltage and instantaneous current flowing through the purely inductive circuit, it is observed that the current lags behind the voltage by π/2.

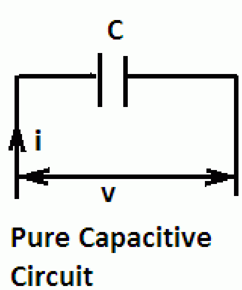


The power curve for the purely inductive circuit is shown in above figure. It is clear thataverage power in a half cycle is zero as the negative and positive loop area under power curve is the same.

In a purely inductive circuit, during the first quarter cycle, what so ever energy (or power) issupplied by the source that is stored in the magnetic field set-up around the coil. In the next quarter cycle, the magnetic field collapses and the energy (or power) stored in the magnetic field is returned to the source. Hence, no power is consumed in a purely inductive circuit.

Purely Capacitive Circuit

When an alternating voltage is applied to a purely capacitive circuit, the capacitor is chargedfirst in one direction and then in the opposite direction.



Pure Capacitive Circuit

Applied Voltage, $v = V_m \sin \omega t$

Resultent Current, $i = I_m \sin(\omega t + \frac{\pi}{2})$

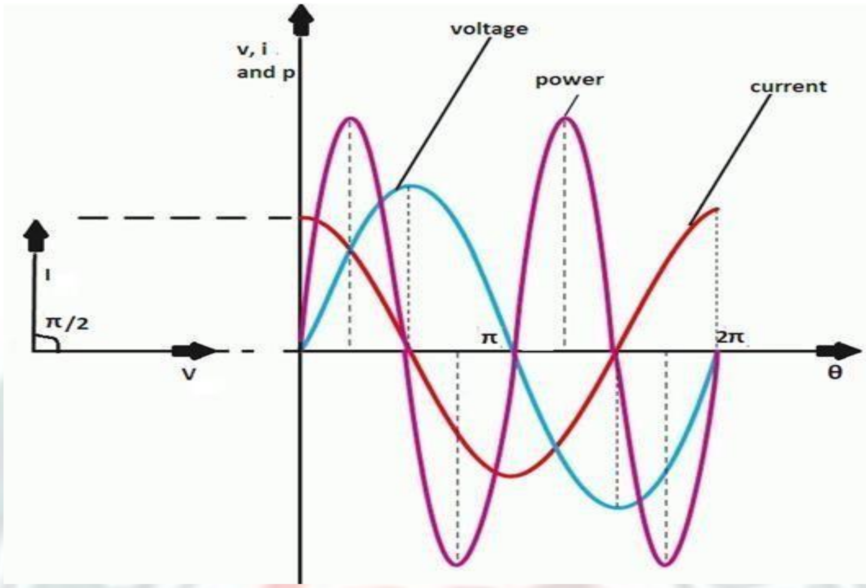
Where, $I_m = \frac{V_m}{X_C}$

Capacitve Reactance, $X_C = 1/\,2\pi f C$ ohms

Power absorbed by circuit = 0

Power factor, $\cos\phi = 0$

From the expression of instantaneous applied voltage and instantaneous current flowing through the purely capacitive circuit, it is observed that the current leads the voltage by π/2.

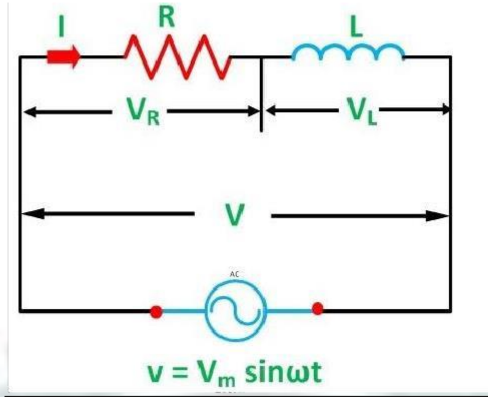


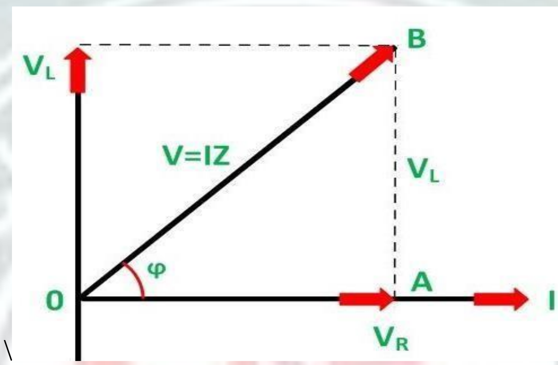
The power curve for the purely capacitive circuit is shown in the figure. It is clear that average power in a half cycle is zero as the negative and positive loop area under powercurve is the same.

In the purely capacitive circuit, during the first quarter cycle, what so ever energy (or power) is supplied by the source is stored in the electric field set-up between the capacitor plates. In the next quarter cycle, the electric field collapses and the energy (or power) stored in the electric field is returned to the source. This process is repeated in every alternation. Hence, no power is consumed in the purely capacitive circuit.

4.2 AC THROUGH RL SERIES CIRCUIT

When an AC supply voltage V is applied the current, I flows in the circuit. IR and IL will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other.





Where,

- VR – voltage across the resistor R
- VL – voltage across the inductor L
- V – Total voltage of the circuit

VR = I R= RMS value of voltage across resistor

VL = IXL = RMS value of voltage across inductor

where XL = 2πfL Ω= inductive reactance

From the phasor diagram:

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit andis called impedance of the circuit. It is measured in ohms (Ω).

From the phasor diagram it is clear that current lags behind the voltage by an angle (ϕ) lessthan 90°.

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

The equation for current is $i = I_m \sin(\omega t - \varphi)$

The instantaneous power is given by the equation

P = vi

$$P = (V_m \sin\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the power factor of the circuit.

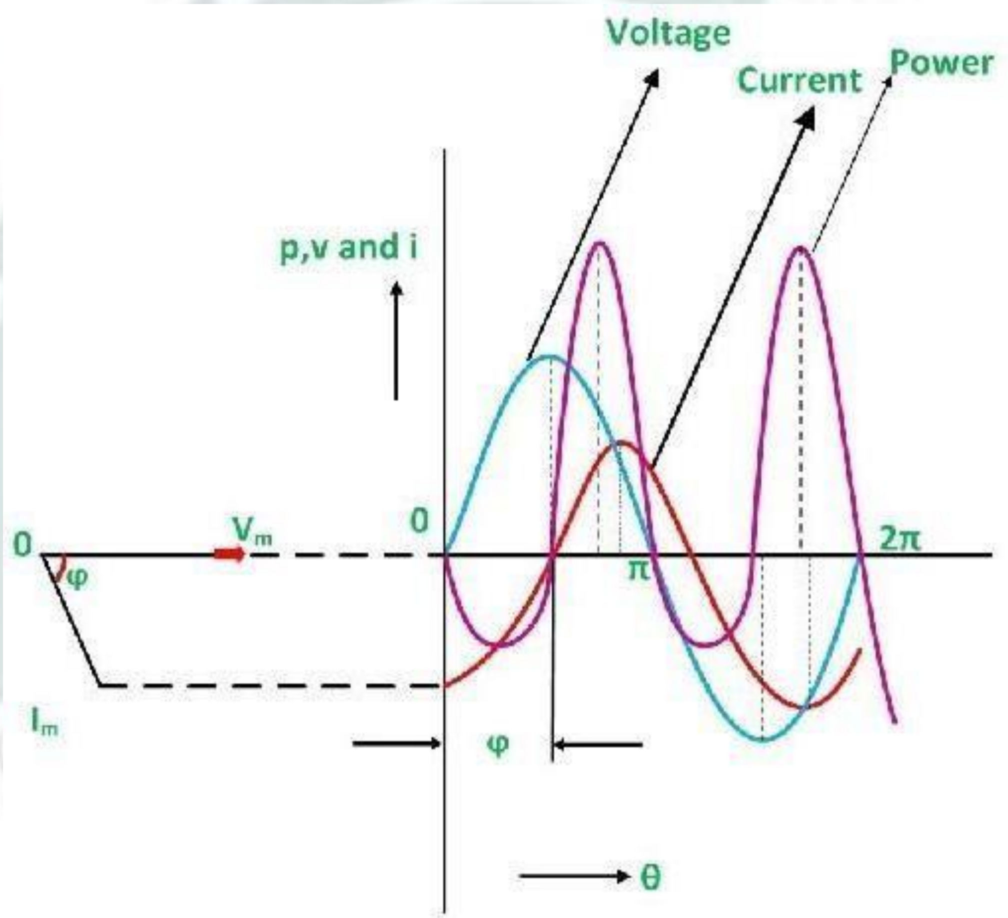
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Putting the value of V and cosϕ from the above equation the value of power will be

$P = (IZ)(I)(R/Z) = I^2 R$

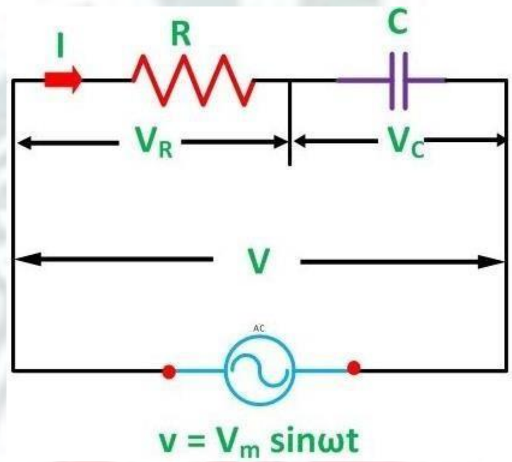
From the above equation it can be concluded that the inductor does not consume any power in the circuit.

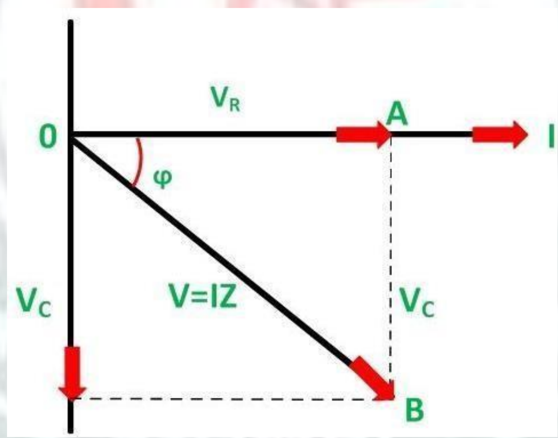
The **waveform** and **power curve** of the RL Series Circuit is shown below



4.3 RC SERIES CIRCUIT

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied to and current I flows through the resistance (R) and the capacitance (C) of the circuit. The RC Seriescircuit is shown in the figure below





Where,

- VR – voltage across the resistance R
- VC – voltage across the capacitor C
- V – total voltage across the RC Series circuit

Now VR = I R= RMS value of voltage across resistor

VL = IXL = RMS value of voltage across inductor

where XL = 2πfL Ω= inductive reactance

From the phasor diagram of a RC series circuit

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \qquad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where $Z = \sqrt{R^2 + X_C^2}$

Z is called impedance of the circuit. It is measured in ohms (Ω).

From the phasor diagram shown above it is clear that the current in the circuit leads theapplied voltage by an angle ϕ and this angle is called the phase angle.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \qquad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Thus the expression of instantaneous value of current through RC series circuit is given by

$$i = I_m \sin(\omega t + \varphi)$$

The instantaneous power is given by the equation P = vi

By putting the value of v and i

$$P = (V_m \operatorname{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t + \varphi)$$

The Average power consumed in the circuit over a complete cycle is given by

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{ average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

Where, cosϕ is called the power factor of the circuit.

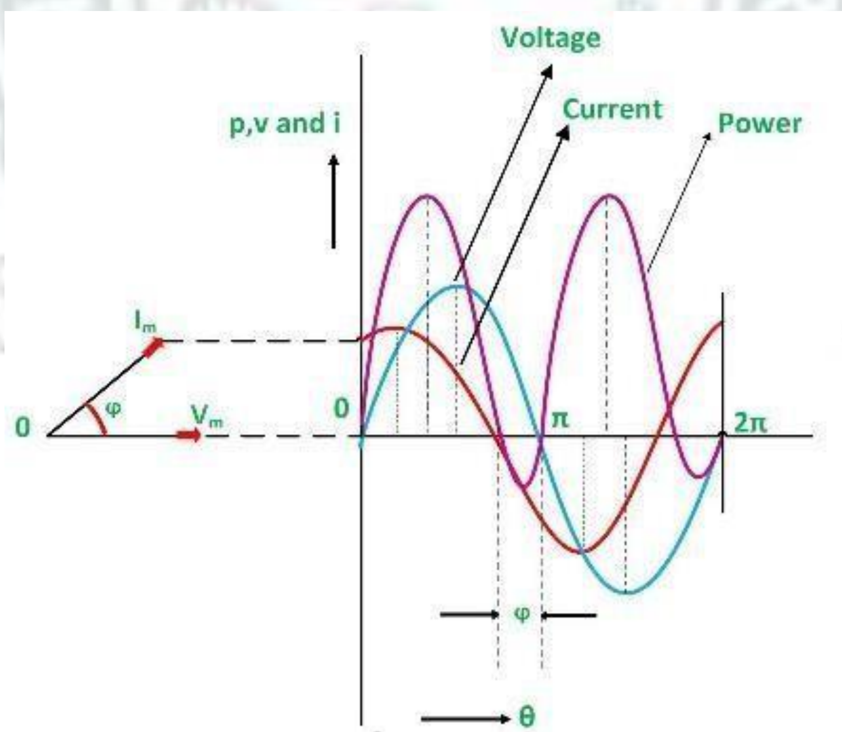
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Putting the value of V and cosϕ from the above equation the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R$$

From the above equation it is clear that the power is actually consumed by the resistanceonly and the capacitor does not consumes any power in the circuit.

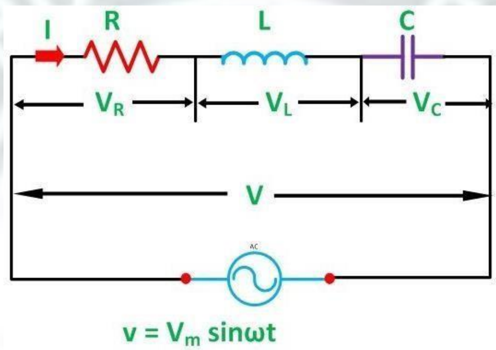
The waveform and power curve of the RC Circuit is shown below



The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is positive.

4.4 RLC SERIES CIRCUIT

The **RLC Series Circuit** is defined as when a pure resistance of R ohms, a pure inductanceof L Henry and a pure capacitance of C farads are connected together in series combinationwith each other. As all the three elements are connected in series so, the current flowing in each element of the circuit will be same. RLC series circuit is shown below:

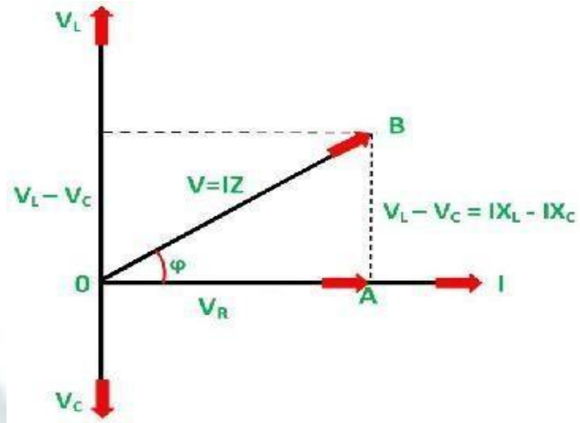


In the RLC Series Circuit $X_L = 2\pi fL$ and $X_C = 1/2\pi fC$

When the AC voltage is applied through the RLC Series Circuit the resulting current I flowsthrough the circuit, and thus the voltage across each element will be

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across the capacitor C and it lags the current I by an angle of 90 degrees.

The phasor diagram of the RLC Series Circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as acapacitive circuit.



Hence from phasor diagram

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

The product of voltage and current is defined as power (since power only consumed byresister)

$$P = VI\cos\varphi = I^2R$$

Where cosϕ is the power factor of the circuit and is expressed as

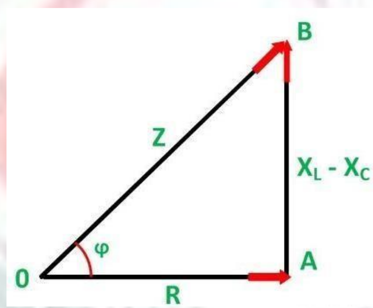
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When XL > XC, the phase angle ϕ is positive. The circuit behaves as a RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When XL < XC, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When XL = XC, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other.The value of power factor is unity.

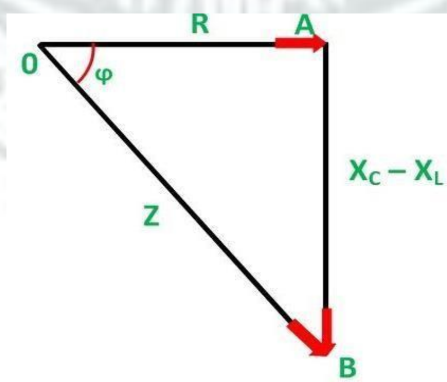
4.5 IMPEDANCE TRIANGLE OF RLC SERIES CIRCUIT

The impedance triangle of the RLC series circuit, when (XL > XC) is shown below



If the inductive reactance is greater than the capacitive reactance then the circuit reactance is inductive giving a lagging phase angle.

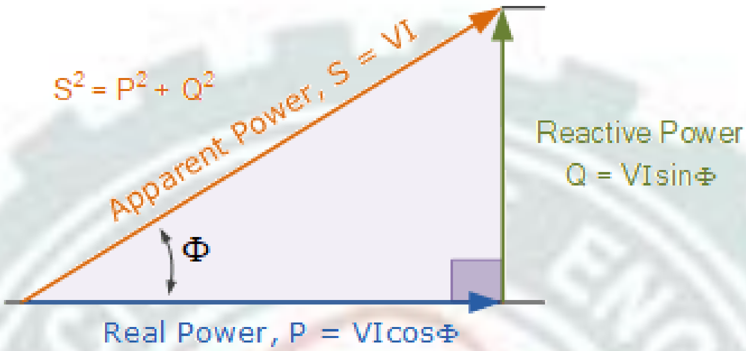
The impedance triangle of the RLC series circuit, when (XL< XC) is shown below:



When the capacitive reactance is greater than the inductive reactance the overall circuitreactance acts as a capacitive and the phase angle will be leading.

Power Triangle :

Power triangle for RLC series circuit is shown below when (XL > XC)



Where:

P = I^2R or Real power that performs work measured in watts (W).

Q = I^2X or Reactive power measured in volt-amperes reactive(VAr)

S = I^2Z or Apparent power measured in volt-amperes (VA)

Φ is the phase angle in degrees.

The larger the phase angle, the greater the reactivepower

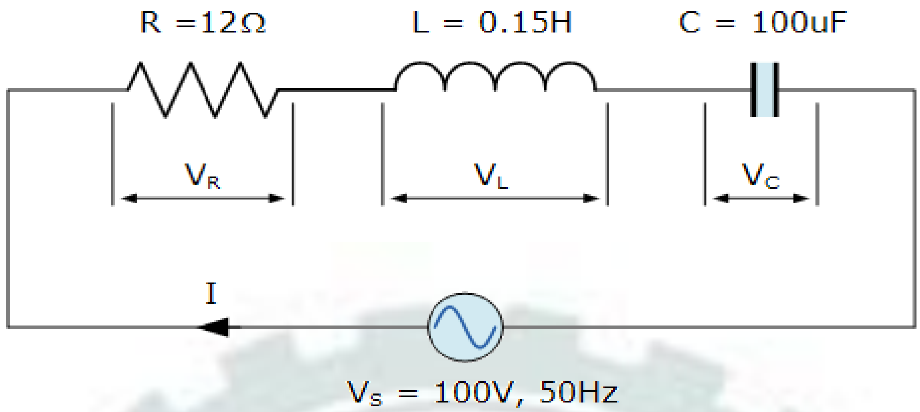
$\cos(\Phi) = P/S = W/VA$ = power factor, p.f.$\sin(\Phi) = Q/S = VAr/VA$

$\tan(\Phi) = Q/P = VAr/W$

Hence the power factor is calculated as the ratio of the real power to the apparent power.

RLC series circuit Example No1

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.



Solution: Inductive reactance:

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current:

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\text{Amps}$$

Voltages across the Series RLC Circuit, VR, VL, VC.

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

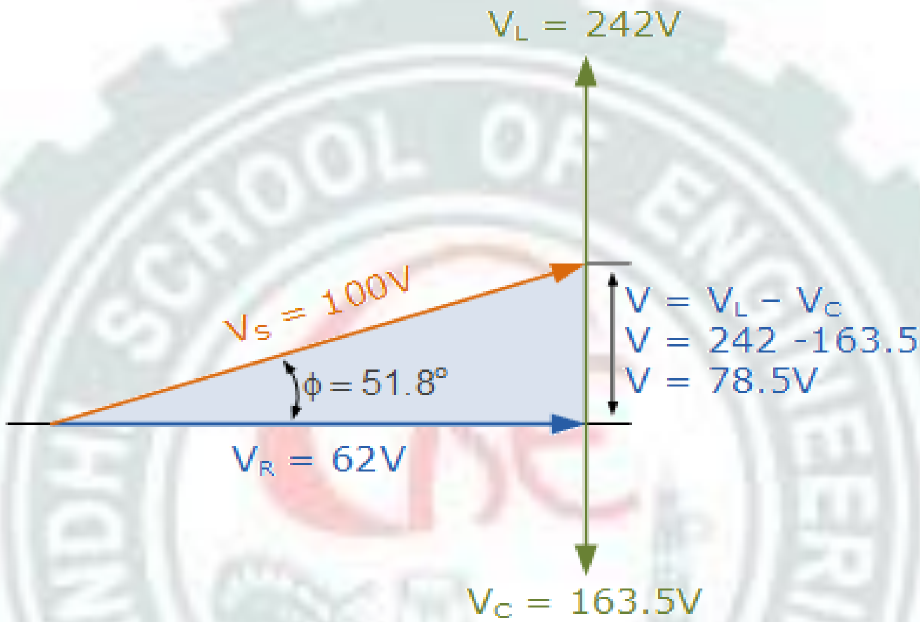
$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

Circuits Power factor and Phase Angle, θ.

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^{0} \text{ lagging}$$

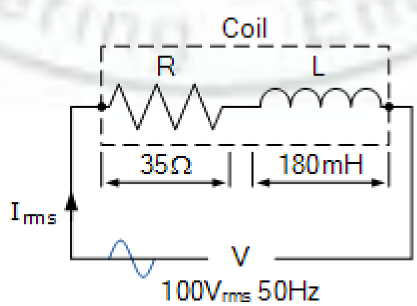
Phasor Diagram.



Since the phase angle θ is calculated as a positive value of 51.8° the overall reactance of the circuit must be inductive. Hence the current "lags" the source voltage by 51.8°

RLC series circuit Example No2

A wound coil that has an inductance of 180mH and a resistance of 35Ω is connected to a 100V 50Hz supply. Calculate: a) the impedance of the coil, b) the current, c) the power factor, and d) the apparent power consumed.



Also draw the resulting power triangle for the above coil. Data given: R = 35Ω, L = 180mH, V = 100V and f = 50Hz.

(a) Impedance (Z) of the coil:

$$R = 35\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.18 = 56.6\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{35^2 + 56.6^2} = 66.5\Omega$$

(b) Current (I) consumed by the coil:

$$V = I \times Z$$

$$\therefore I = \frac{V}{Z} = \frac{100}{66.5} = 1.5\,A_{(rms)}$$

(c) The power factor and phase angle, Φ:

$$\cos\phi = \frac{R}{Z}, \quad \text{or} \quad \sin\phi = \frac{X_L}{Z}, \quad \text{or} \quad \tan\phi = \frac{X_L}{R}$$

$$\therefore \cos\phi = \frac{R}{Z} = \frac{35}{66.5} = 0.5263$$

$$\cos^{-1}(0.5263) = 58.2^o \ (\text{lagging})$$

(d) Apparent power (S) consumed by the coil:

$$P = V \times I\cos\phi = 100 \times 1.5 \times \cos\left(58.2^o\right) = 79W$$

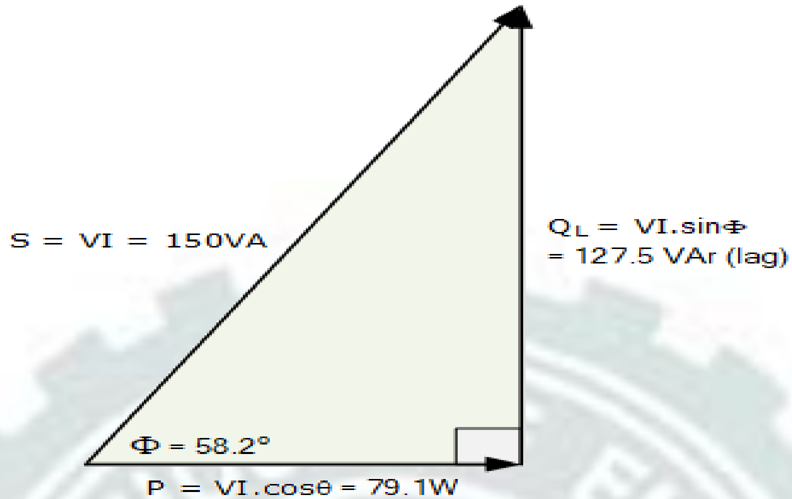
$$Q = V \times I\sin\phi = 100 \times 1.5 \times \sin\left(58.2^o\right) = 127.5VAr$$

$$S = V \times I = 100 \times 1.5 = 150VA$$

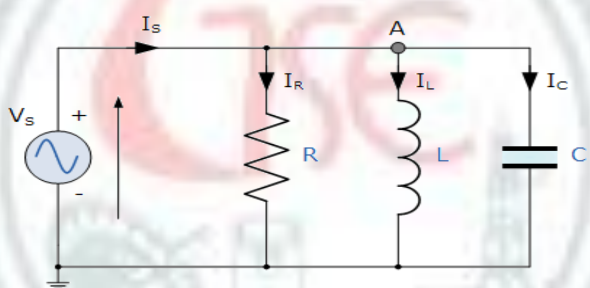
$$\text{or} \quad S^2 = P^2 + Q^2$$

$$\therefore S = \sqrt{P^2 + Q^2} = \sqrt{79^2 + 127.5^2} = 150VA$$

(e) Power triangle for the coil:



4.6 PARALLEL RLC CIRCUIT



All the variable shown in the diagram are RMS quantities.

Here:

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore\ I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

where: $I_R = \frac{V}{R},\quad I_L = \frac{V}{X_L},\quad I_C = \frac{V}{X_C}$

Impedance of a Parallel RLC Circuit

$$R = \frac{V}{I_R} \quad X_L = \frac{V}{I_L} \quad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Where

$\frac{1}{Z} = \text{Y} = \text{admittance},$

$\frac{1}{R} = \text{G} = \text{conductance},$

$\frac{1}{X_L} = B_L = \text{inductive susceptance},$

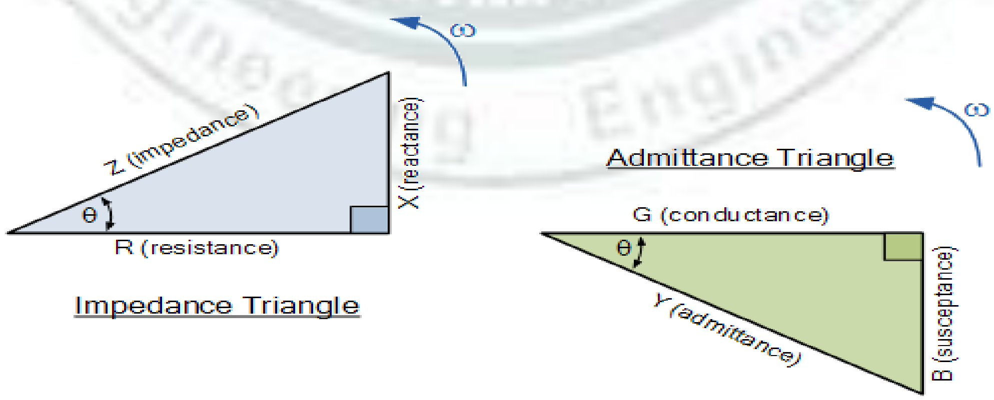
$\frac{1}{X_C} = B_C = \text{capacitive susceptance}.$

$$Y = \sqrt{G^2 + \left(B_L - B_C\right)^2}$$

where: $Y = \frac{1}{Z} \qquad G = \frac{1}{R}$

$B_L = \frac{1}{\omega L} \qquad B_C = \omega C$

Admittance Triangle for a Parallel RLC Circuit is shown below:



Giving us a power factor angle of:

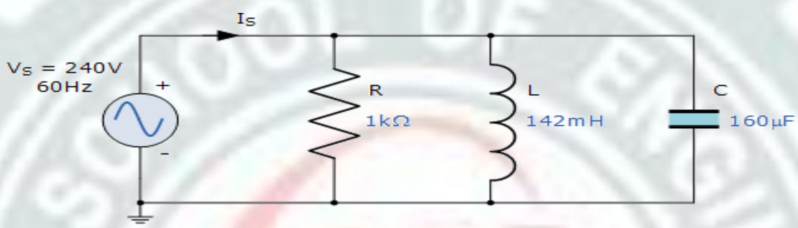
$$\cos\phi = \frac{G}{Y} \qquad \phi = \cos^{-1}\left(\frac{G}{Y}\right)$$

or

$$\tan\phi = \frac{B}{G} \qquad \phi = \tan^{-1}\left(\frac{B}{G}\right)$$

Parallel RLC Circuit Example No1

A 1kΩ resistor, a 142mH coil and a 160uF capacitor are all connected in parallel across a240V, 60Hz supply. Calculate the impedance of the parallel RLC circuit and the current drawn from the supply.



Solution: R = 1kΩ

Inductive Reactance, (XL):

$$X_L = \omega L = 2\pi f L = 2\pi.60.142\times10^{-3} = 53.54\Omega$$

Capacitive Reactance, (XC):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.60.160\times10^{-6}} = 16.58\Omega$$

Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{1000}\right)^2 + \left(\frac{1}{53.54} - \frac{1}{16.58}\right)^2}}$$

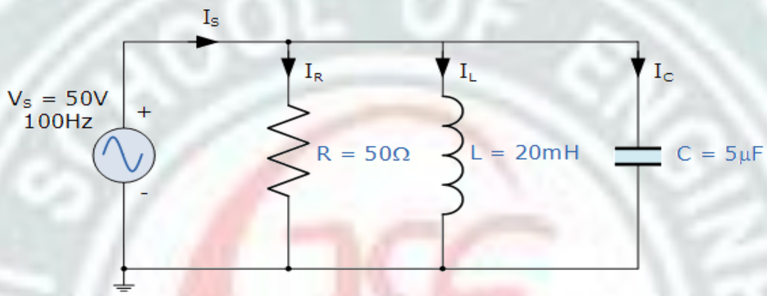
$$Z = \frac{1}{\sqrt{1.0\times10^{-6} + 1.734\times10^{-3}}} = \frac{1}{0.0417} = 24.0\Omega$$

Supply Current, (Is):

$$I_S = \frac{V_S}{Z} = \frac{240}{24} = 10\ \text{Amperes}$$

Parallel RLC Circuit Example No2

A 50Ω resistor, a 20mH coil and a 5uF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current and admittance triangles representing the circuit.



1). Inductive Reactance, (XL):

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2). Capacitive Reactance, (XC):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5\times10^{-6}} = 318.3\Omega$$

3). Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

4). Current through resistance, R (IR):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0(A)$$

5). Current through inductor, L (IL):

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

6). Current through capacitor, C (IC):

$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

7). Total supply current, (IS):

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2} = \sqrt{1^2 + \left(3.9 - 0.16\right)^2} = 3.87\,(A)$$

8). Conductance, (G):

$$G = \frac{1}{R} = \frac{1}{50} = 0.02S \text{ or } 20mS$$

9). Inductive Susceptance, (BL):

$$B_L = \frac{1}{X_L} = \frac{1}{12.6} = 0.08S \text{ or } 80mS$$

10). Capacitive Susceptance, (BC):

$$B_C = \frac{1}{X_C} = \frac{1}{318.3} = 0.003S \text{ or } 3mS$$

11). Admittance, (Y):

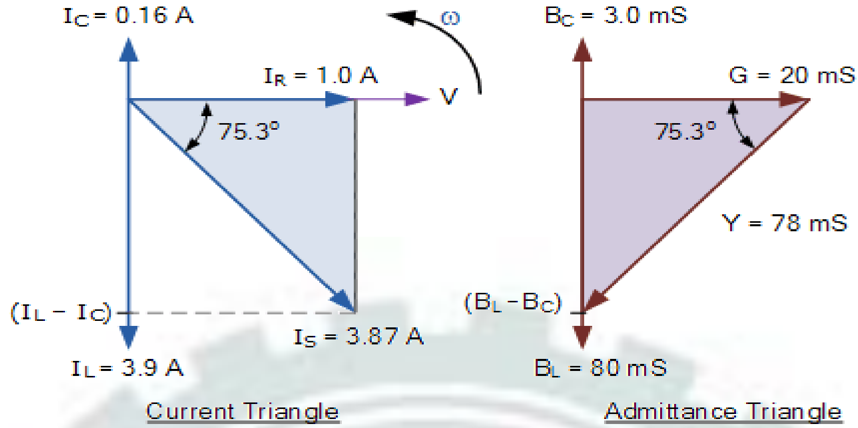
$$Y = \frac{1}{Z} = \frac{1}{12.7} = 0.078S \text{ or } 78mS$$

12). Phase Angle, (φ) between the resultant current and the supply voltage:

$$\cos\phi = \frac{G}{Y} = \frac{20mS}{78mS} = 0.256$$

$$\phi = \cos^{-1} 0.256 = 75.3^0 \quad (\text{lag})$$

Current and Admittance Triangles



Q- Factor: Quality factor is defined as the reciprocal of power factor i.e

$$\text{Q-factor} = \frac{1}{\cos\phi}$$

TRANSFORMER

- State construction & working principle of transformer & define connection of Ideal transformer. Derive of EMF equation of transformer, voltage transformation ratio.
- Discuss Flux, Current, EMF components of transformer and their phasor diagram underno load
- condition.
- Discuss Phasor representation of transformer flux, current EMF primary andsecondary voltages under loaded condition.
- Explain types of losses in Single Phase (1-ø) Transformer. Explain open circuit & short-circuit test (simple problems)Explain Parallel operation of Transformer.

INDUCTION MOTOR

- Explain construction feature, types of three-phase induction motor. State principle of development of rotating magnetic field in the stator.
- Establish relationship between synchronous speed, actual speed and slip of induction motor. Establish relation between torque, rotor current and power factor.
- Explain starting of an induction motor by using DOL and Star-Delta stator. State industrial use of induction motor.
-

SINGLE PHASE INDUCTION MOTOR

- Explain construction features and principle of operation of capacitor type and shaded pole type of single-phase induction motor.
- Explain construction & operation of AC series motor.Concept of alternator & its application.

CHAPTER - 5

TRANSFORMERS

5.1 INTRODUCTION

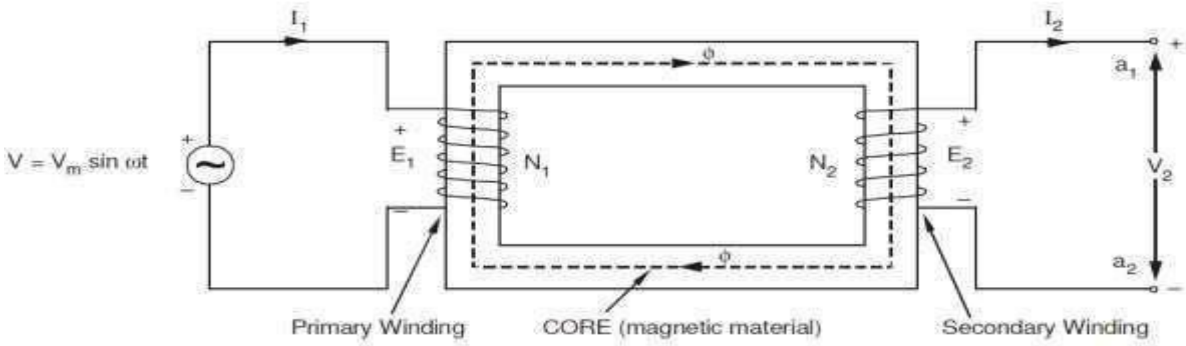
The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

5.2 SINGLE PHASE TRANSFORMER

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig 1. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage V_1 whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating e.m.f. E_2 is induced in the secondary. This induced e.m.f. E_2 in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load.

If $V_2 > V_1$, it is called a step up-transformer.
If $V_2 < V_1$, it is called a step-down transformer.



Schematic diagram of single phase transformer

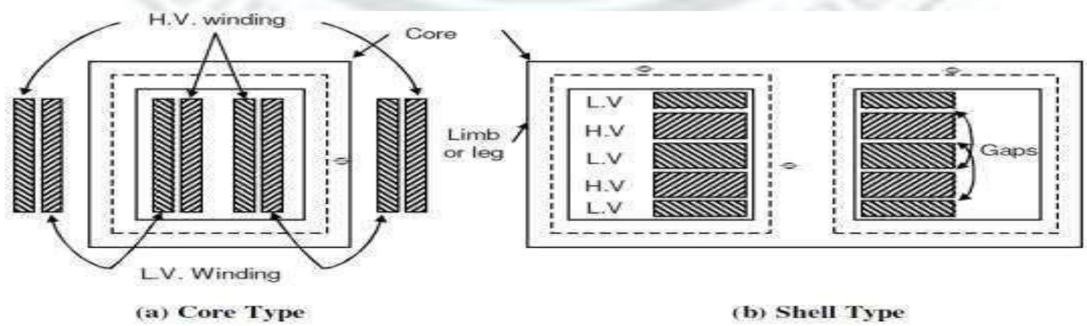
5.3 CONSTRUCTIONAL DETAILS

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called (a) core type, and (b) shell type.

5.3.1 Core-type and Shell-type Construction

In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high-voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure below (a) shows the cross-section of the arrangement.

In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in figure below (b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.



Core type & shell type transformer

5.3.2 Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminium is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

5.4 PRINCIPLE OF OPERATION

When an alternating voltage V1 is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E1 and E2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E1 is termed as primary e.m.f. and e.m.f. E2 is termed as secondary e.m.f.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E2 and E1 depend upon the number of turns on the secondary and primary respectively.

If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. If $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer.

If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit toanother with a change in voltage level.

The following points may be noted carefully:

(a) The transformer action is based on the laws of electromagnetic induction.

(b) There is no electrical connection between the primary and secondary.

(c) The a.c. power is transferred from primary to secondary through magnetic flux.

(d) There is no change in frequency i.e., output power has the same frequency as theinput power.

(e) The losses that occur in a transformer are:

   (i) ***Core Losses*** — Eddy Current and Hysteresis Losses

   (ii) ***copper losses***—in the resistance of the windings

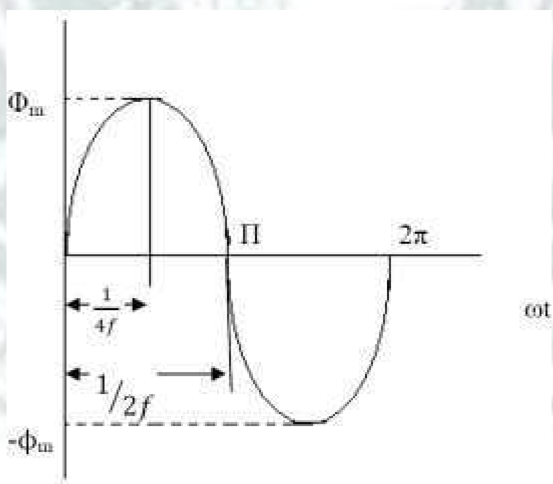
In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

5.5 E.M.F. EQUATION OF A TRANSFORMER

When the primary winding is excited by an alternating voltage V_1, it is circulating alternatingcurrent, producing an alternating flux ϕ.

Let ϕ_m - maximum value of flux

f - Frequency of the supply voltage



The flux increases from zero value to maximum value ϕ_m in one fourth of a cycle i.e in 1/4fseconds.

So average rate of change of flux: $\frac{d\emptyset}{dt} = \frac{\emptyset_{max}}{1/4f} = 4f\emptyset_{max}$

Since average emf induced per turn in volt is equal to the average rate of change of flux (according to Faraday's law of electromagnetic induction)

Average emf induced per turn= $4f\emptyset_{max}$ volts

Since flux varies sinusoidally, emf induced will be sinusoidal and form factor for sinusoidal wave is 1.11 i.e the rms value is 1.11 times the average value.

∴ RMS value of emf induced per turn = 1.11× $4f\emptyset_{max}$ volts.

If N_1, N_2 - Number of primary turn and secondary turns, then RMS value of emf induced in primary,

E_1 = emf induced per turn× Number of primary turn

= $4.44f\emptyset_{max} \times N_1$= $4.44fN_1\emptyset_{max}$ volt.

Similarly, R.M.S. value of the secondary induced e.m.f, $E_2 = 4.44fN_2\emptyset_{max}$ volt.

Alternatively

The sinusoidal flux ϕ produced by the primary can be represented as:

$\phi=\phi_m \sin\omega t$

∴Instantaneous value of emf induced per turn

$$= \frac{-d\emptyset}{dt} \text{ volt} = -\omega \Phi_{max} \cos \omega t = \omega\emptyset_{max} \sin\left(\omega t - \frac{\pi}{2}\right) \text{ volts}$$

It is clear from the above equation that the maximum value of emf induced per turn

$$= \omega\emptyset_{max} = 2\pi f\emptyset_{max} \text{ volts}$$

And rms value of emf induced per turn = $\frac{1}{\sqrt{2}} 2\pi f\emptyset_{max} = 4.44 f\emptyset_{max}$ volts

Hence rms value of emf induced in primary = $E_1 = 4.44fN_1\emptyset_{max}$ volts

And rms value of emf induced in secondary = $E_2 = 4.44fN_2\emptyset_{max}$ volts

In an ideal transformer the voltage drops in primary and secondary windings are negligible, so

EMF induced in primary winding E_1= Applied voltage to primary V_1

And terminal voltage V_2 = EMF induced in primary winding E_2

5.5.1 Voltage Ratio

Voltage transformation ratio is the ratio of e.m.f induced in the secondary winding to the e.m.f induced in the primary winding.

$$\frac{E2}{E1} = \frac{4.44\phi mf\, N2}{4.44\phi mf\, N1}$$

$$\boxed{\frac{E2}{E1} = \frac{N2}{N1} = K}$$

This ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio

$$E_2 = KE_1 \qquad \text{where } K = \frac{N2}{N1}$$

1. If $N_2 > N_1$ i.e. $K>1$ we get $E_2 > E_1$ then the transformer is called step up transformer.
2. If $N_2 < N_1$ i.e. $K<1$ we get $E_2 < E_1$ then the transformer is called step down transformer.
3. If $N_2 = N_1$ i.e. $K=1$ we get $E_2 = E_2$ then the transformer is called isolation transformer or 1:1 transformer.

5.5.2 Current Ratio

Current ratio is the ratio of current flow through the primary winding (I_1) to the current flowingthrough the secondary winding (I_2). In an ideal transformer -

Apparent input power = Apparent output power.

$V_1 I_1 = V_2 I_2$

OR $\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$

i) The transformer rating is specified as the products of voltage and current (VA rating).

ii) On both sides, primary and secondary VA rating remains same. This rating is generally expressed inKVA (Kilo Volts Amperes rating).

$$\text{KVA Rating of transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000} \text{ (1000 is to convert KVA to VA)}$$

$I_1 = (\boldsymbol{KAV\ Rating} \times \mathbf{1000})/\mathbf{V_1}$

$I_2 = (KAV\ Rating \times 1000)/V_2$

Example 1: It is desired to have a 4.13 mWb maximum core flux in a transformer at 110V and 50 Hz. Determine the required number of turns in the primary.

Solution: EMF induced in primary, E_1 = 110V Supply frequency, f = 50Hz

Maximum core flux, ϕ_{max} = 4.13 mWb

= 4.13 × 10^{-3} Wb

Required number of turns on primary,

$$N_1 = \frac{E_1}{4.44 f \phi_{max}}$$

$$= \frac{110}{4.44 \times 50 \times 4.13 \times 10^{-3}} = 120$$

Example 2: The emf per turn of a single phase 10 kVA, 2200/220V, 50 Hz transformer is 10V. Calculate (i) thenumber of primary and secondary turns, (ii) the net cross-sectional area of core for a maximum flux density of 1.5T.

Solution: EMF per turn = 10V

Primary induced emf, $E_1 = V_1$ = 2,200 V

Secondary induced emf, $E_2 = V_2$ = 220V

Supply frequency, f = 50 Hz

Maximum flux density, B_{max} = 1.5 T

For (i)

Number of primary turns,

$$N_1 = \frac{E_1}{EMF\ per\ turn} = \frac{2,200}{10} = 220$$

Number of secondary turns,

$$N_2 = \frac{E_2}{EMF\ per\ turn} = \frac{220}{10} = 22$$

Maximum value of flux,

$$\phi_{max} = \frac{EMF\ per\ turn}{4.44 \times f} = \frac{10}{4.44 \times 50} = 0.045Wb$$

For (ii)

Net cross-sectional area of core,

$$a = \frac{\phi_{max}}{B_{max}} = \frac{0.045}{1.5} = 0.03m^2$$

Example 3: A single phase transformer has 350 primary and 1,050 secondary turns. The net cross-sectional areaof the core is 55 cm^2. If the primary winding be connected to a 400 V, 50 Hz single phase supply, calculate (i) maximum value of the flux density in the core and (ii) the voltage induced in the secondary winding.

Solution: Net cross-section area of core,

$$a = 55^2 = 0.0055m^2$$

Maximum value of flux,

$$\phi_{max} = \frac{E_1}{4.44 \times f \times N_1} = \frac{400}{4.44 \times 50 \times 350} = 5.148 \times 10^{-3}Wb$$

For (i)

Peak value of flux density in the core,

$$B_{max} = \frac{\phi_{max}}{a} = \frac{5.148 \times 10^{-3}}{0.0055} = 0.936T$$

For (ii)

Voltage induced in the secondary winding,

$$E_2 = E_1 \times \frac{N_2}{N_1} = 400 \times \frac{1050}{350} = 1200V$$

Example 4: A 25 kVA, single phase transformer has 250 turns on the primary and 40 turns on the secondarywinding. The primary is connected to 1500 V, 50 Hz mains calculate (i) secondary emf (ii) primary and secondary current on full load (iii) maximum flux in the core.

Solution:

Supply voltage V_i = 1500 V

Primary induced emf, $E_1 = V_i$ = 1500 V

For (i)

Secondary emf,

$$E_2 = \frac{E_1 \times N_2}{N_1} = \frac{1500 \times 40}{250} = 240V$$

For (ii)

Appropriate value of primary current on full load,

$$I_1 = \frac{Rated\ kVA\ 1000}{V_i} = \frac{24 \times 1000}{1500} = 16,667A$$

Appropriate value of secondary current on full load,

$$I_2 = \frac{Rated\ kVA\ \ \times 1000}{E_2\ or\ V_2} = \frac{25 \times 1000}{240} = 104.167A$$

For (iii)

Maximum value of flux in the core,

$$\phi_{max} = \frac{E_1}{4.44fN_1} = \frac{1500}{4.44 \times 50 \times 250} = 0.027Wb or 27mWb$$

5.6 TRANSFORMER ON NO-LOAD

a) Ideal transformer

b) Practical transformer

a) Ideal Transformer

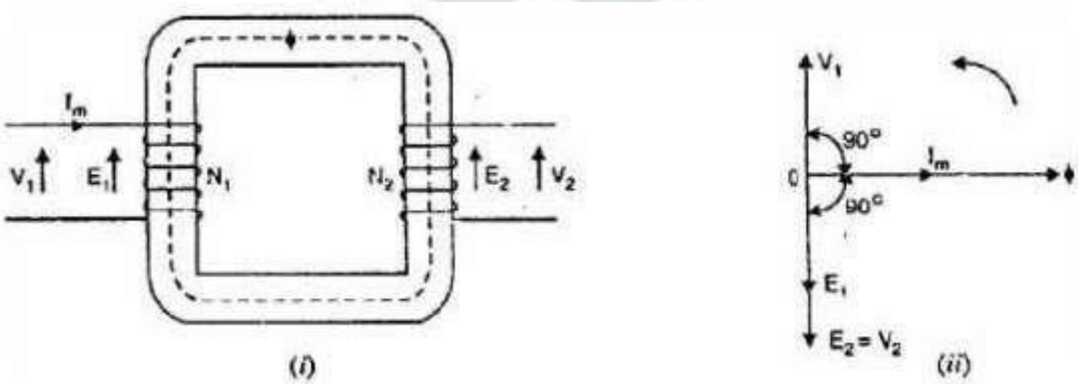
An ideal transformer is one that has

(i) No winding resistance

(ii) No leakage flux i.e., the same flux links both the windings

(iii) No iron losses (i.e., eddy current and hysteresis losses) in the core

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.



Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in *above figure (i)* under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage V1 is applied to the primary, it draws a small magnetizing current Im which lags behind the applied voltage by 90°. This alternating current Im produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces e.m.f. E1 in the primary and e.m.f. E2 in the secondary. The primary e.m.f. E1 is, at every instant, equal to and in opposition to V1 (Lenz's law). Both e.m.f.s E1 and E2 lag behind flux ϕ by 90°.However, their magnitudes depend upon the number of primary and secondary turns. *Above figure (ii)* shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor.

$$\boxed{\frac{E2}{E1} = \frac{V2}{V1} = K}$$

The primary e.m.f. E1 and secondary e.m.f. E2 lag behind the flux ϕ by 90°. Note that E1 andE2 are in phase. But E1 is equal to V1 and 180° out of phase with it.

5.7 PHASOR DIAGRAM

1. Φ (flux) is reference
2. Im produce ϕ and it is in phase with ϕ, V_1 Leads Im by 90˚
3. E1 and E2 are in phase and both opposing supply voltage V1, winding is purely inductive. So current has to lag voltage by 90˚.
4. The power input to the transformer

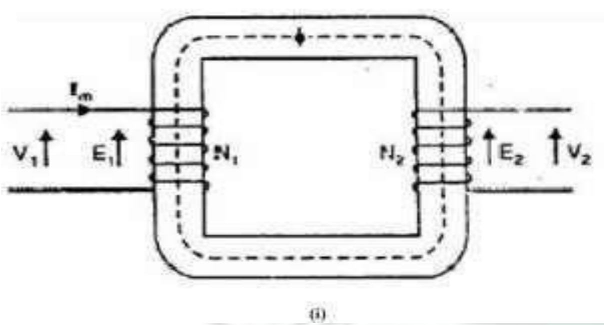
   $P = V_1.I_1. \cos(90˚)$..(cos90˚ = 0)

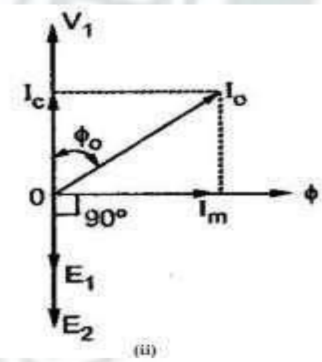
   P= 0 (ideal transformer)

***(i) Practical Transformer* on no load**

No load Transformer means a transformer which has no load connection at secondary winding only normal voltage is applied at the primary winding. Let V_1 is applied at the primary winding. After applying A.C voltage V_1, it is seen that small amount of current I_0 flows through the primary winding. In case of Ideal Transformer, no load primary current (I_0) will be equal to magnetizing current (I_m) of the transformer. We assumed there is no core losses and copper loss, So $I_0 = I_m$. But, in case of actual transformer, there is two losses, i.e i) Iron Losses in the core i.e hysteresis loss and eddy current loss , ii) and a very small amount copper loss in the primary winding.

Consider a practical transformer on no load i.e., secondary on open-circuit as shown in figure below.





No-load phasor diagram of a single phase transformer.

So, the primary current I_0 has two components:

1. I_C = Iron loss component which is same ph of applied voltage $V_1 = I_0 \cos\phi_0$
2. I_m = magnetizing component which is 90° behind $V_1 = I_0 \sin\phi_0$

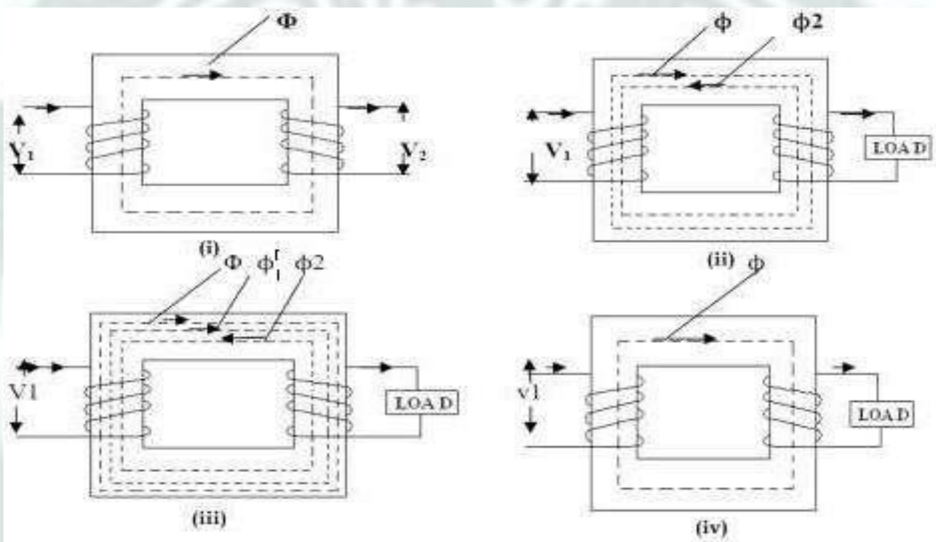
Hence, the primary current I_0 is vector summation of I_m & I_c, So, we can write that

$\mathbf{I_0} = \sqrt{\mathbf{I}_m^2 + \mathbf{I}_c^2}$

No-Load PF, CoseΦ_0 = I_c / I_0

and is not a 90⁰ behind V_1, but lags it by an angle $\varphi < 90^0$ Which is shown in figure 5. And no load input power, $\mathbf{W_0 = V_1 I_0 Cose\,\varphi_0}$. The magnitude of no load primary current is very small as compared to the full-load primary current. It is 1 percent of the full-load current. As I_0 is very small, the no load primary Cu loss $\mathbf{I_0^2 R_1}$ is negligible which means that no load primary input is practically equal to the iron loss in the transformer.

***ii) Practical Transformer* on Load**



An Ideal transformer at loaded condition

At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0. Hence, we can say that at no load, $V_1 = E_1$.

i) When transformer is loaded, the secondary current I_2 is flows through the secondary winding.

ii) Already I_m magnetizing current flow in the primary winding .

iii) The magnitude and phase of I_2 with respect to V_2 is determined by the characteristics of the load.

a) I_2 in phase with V_2 (resistive load)

b) I_2 lags with V_2 (Inductive load)

c) I_2 leads with V_2 (capacitive load)

iv) Flow of secondary current I_2 produce new Flux ϕ_2 figure above (ii)

v) Φ is main flux which is produced by the primary to maintain the transformer as constant magnetizingcomponent.

vi) Φ_2 opposes the main flux ϕ, the total flux in the core reduced. It is called demagnetizing Ampere- turns due to this E_1 reduced.

vii) To maintain the ϕ constant primary winding draws more current (I_1') from the supply (load component of primary) and produce ϕ_1' flux which is oppose ϕ_2 (but in same direction as ϕ), to maintainflux constant in the core shown in figure above (iii).

viii) The load component current I_1' always neutralizes the changes in the load.

ix) Whatever the load conditions, the net flux passing through the core is approximately the same as atno-load. An important deduction is that due to the constancy of core flux at all loads, the core loss is also practically the same under all load conditions figure above (iv).

$$\boxed{\Phi_2 = \phi_1' \quad N_2I_2 = N_1I_1' \quad I_1' = \frac{N2}{N1} X I_2 = KI_2}$$

Phasor Diagram

i) Take (ϕ) flux as reference for all load

ii) The no-load current I_0 lags by an angle Φ_0. $\mathbf{I_0} = \sqrt{\mathbf{I}m^2 + \mathbf{I}c^2}$

iii) The load component I_1', which is in anti-phase with I_2 and phase of I_2 is decided by the load.

iv) Primary current I_1 is vector sum of I_0 and I_1'i.e

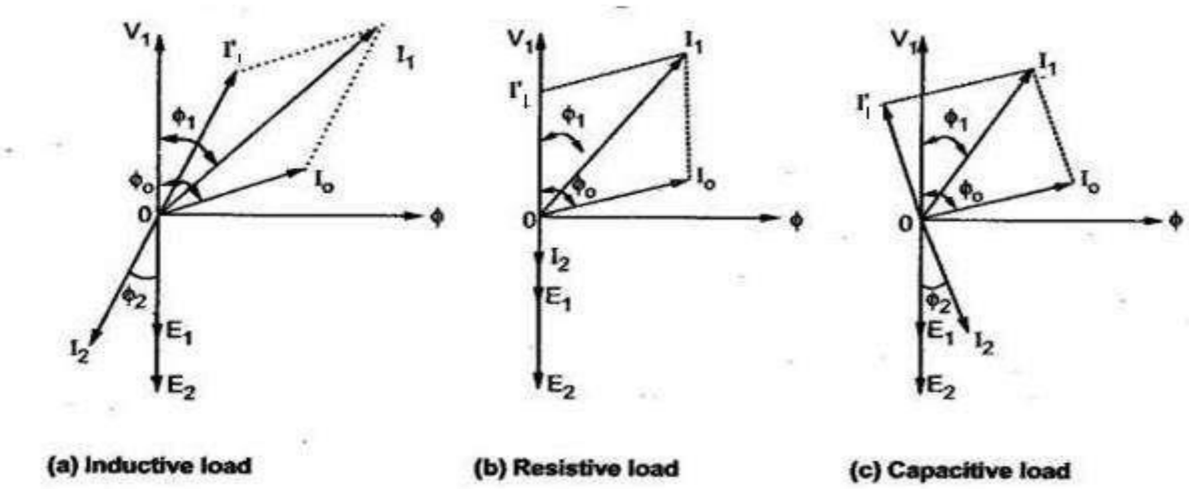
$$I_1 = \sqrt{I_0^2 + I_1'^2}$$

a) If load is Inductive, I_2 lags E_2 by ϕ_2, shown in phasor diagram fig 7 (a).

b) If load is resistive, I_2 in phase with E_2 shown in phasor diagram fig. 7 (b).

c) If load is capacitive load, I_2 leads E_2 by ϕ_2 shown in phasor diagram fig. 7 (c).

For easy understanding at this stage here we assumed E_2 is equal to V_2 neglecting various drops.



Phasor diagram for an ideal transformer on load

Now, N1I1’ = N2I2

Therefore,

$$I_1' = \left(\frac{N_2}{N_1}\right) I_2 = KI_2$$

5.8 LOSSES IN A TRANSFORMER

The power losses in a transformer are of two types, namely

1. Core or Iron losses
2. Copper losses

These losses appear in the form of heat and produce (i) an increase in Temperature and (ii) a drop in efficiency.

5.8.1 Core or Iron Losses (Pi)

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test.

Hysteresis loss = k_h f $Bm^{1.6}$ watts /m^3

K_h – hysteresis constant depend on material f - Frequency

B_m – maximum flux density

Eddy current loss = $K_e f^2 B_m^2 t^2$ watts /m^3

Ke – eddy current constant

t - Thickness of the core

Both hysteresis and eddy current losses depend upon

(i) Maximum flux density B_m in the core

(ii) Supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and Bm are constant. Hence, core or iron losses are practically the same at all loads.

Iron or Core losses, Pi = Hysteresis loss + Eddy current loss = Constant losses (Pi)

The hysteresis loss can be minimized by using steel of high silicon content. Whereas eddy current loss can be reduced by using core of thin laminations.

Copper losses (P_{cu})

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test. The copper loss depends on the magnitude of the current flowing through the windings.

Total copper loss = $I_1^2R_1 + I_2^2R_2 = I_1^2(R_1 + R_2') = I_2^2 (R_2 + R_1')$

$$\boxed{\text{Total loss = iron loss + copper loss = Pi + Pcu}}$$

5.9 EFFICIENCY OF A TRANSFORMER

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.

Power output = power input – Total lossesPower input = power output + Total losses

= power output + Pi + P_{cu}

$$\text{Efficiency} = \frac{power\ output}{power\ input}$$

$$\text{Efficiency} = \frac{power\ output}{power\ input + Pi + Pcu}$$

Power output = $V_2 I_2 \cos\phi$, $\cos\phi$ = load power factor

Transformer supplies full load of current I_2 and with terminal voltage V_2

Pcu = copper losses on full load = $I_2^2 R_{2e}$

$$\text{Efficiency} = \frac{\text{V2I2}\cos\phi}{\text{V2I2}\cos\phi + Pi + I2^2 R2e}$$

$V_2 I_2$ = VA rating of a transformer

$$\text{Efficiency} = \frac{(\text{VA rating})\text{X}\cos\phi}{(\text{VA rating})\text{X}\cos\phi + Pi + I2^2 R2e}$$

$$\boxed{\%\ \text{Efficiency} = \frac{(\text{VA rating})\text{X}\cos\phi}{(\text{VA rating})\text{X}\cos\phi + Pi + I2^2 R2e}\ \text{X}\ 100}$$

This is full load efficiency and I_2 = full load current.

We can now find the full-load efficiency of the transformer at any p.f. without actually loading the transformer.

$$\text{Full load Efficiency} = \frac{(\text{Full load VA rating})\text{X}\cos\phi}{(\text{Full load VA rating})\text{X}\cos\phi + Pi + I2^2 R2e}$$

Also for any load equal to n x full-load,

Corresponding total losses = $P_i + n^2 P_{Cu}$

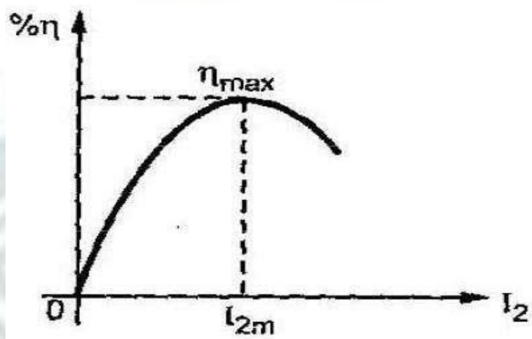
$$n = \text{fractional by which load is less than full load} = \frac{actual\ load}{full\ load}$$

$$n = \frac{half\ load}{fullload} = \frac{\left(\frac{1}{2}\right)}{1} = 0.5$$

$$\boxed{\text{Corresponding (n) \% Efficiency} = \frac{n(\text{VA rating})\text{X}\cos\phi}{n(\text{VA rating})\text{X}\cos\phi + Pi + n^2 Pcu}\ \text{X}\ 100}$$

Condition for Maximum Efficiency

Voltage and frequency supply to the transformer is constant the efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it loaded further the efficiency start decreases as shown in fig. 2.21.



The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as ηmax, now we find -

(a) condition for maximum efficiency

(b) load current at which ηmax occurs

(c) KVA supplied at maximum efficiency

Considering primary side,

Load output = $V_1 I_1 \cos\phi 1$

Copper loss = $I_1^2 R_1e$ or $I2^2 R2e$

Iron loss = hysteresis + eddy current loss = Pi

$$\text{Efficiency} = \frac{V1I1\cos\phi1 - \text{losses}}{V1I1\cos\phi1} = \frac{V1I1\cos\phi1 - I1^2R1e + Pi}{V1I1\cos\phi1}$$

$$= 1 - \frac{I1R1e}{V1I21\cos\phi1} = \frac{Pi}{V1I1\cos\phi1}$$

Differentiating both sides with respect to I2, we get

$$\frac{d\eta}{dI2} = 0 - \frac{R1e}{V1\cos\phi1} = \frac{Pi}{V1I1^2\cos\phi1}$$

For η to be maximum, $\frac{d\eta}{dI2} = 0$. Hence, the above equation becomes

$$\frac{R1e}{V1\cos\phi1} = \frac{Pi}{V1I1^2\cos\phi1} \text{ or } Pi = I_1^2R_{1e}$$

P_{cu} loss = P_i iron loss

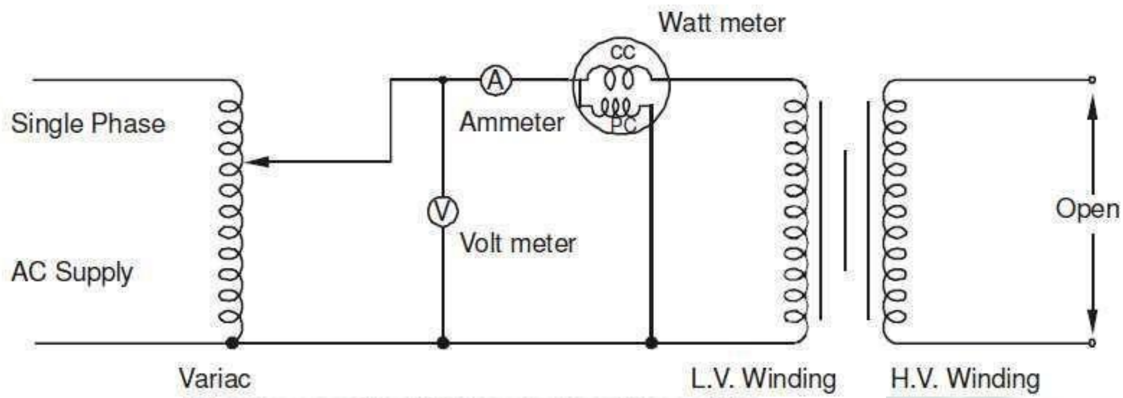
The output current which will make P_{cu} loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

5.10 TESTING OF TRANSFORMER

5.10.1 Open-Circuit or No-Load Test

This test is conducted to determine the iron losses (or core losses) and parameters R_0 and X_0 of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open circuited. The applied primary voltage V_1 is measured by the voltmeter, the no load current I_0 by ammeter and no-load input power W_0 by wattmeter as shown in Fig.2.24.a. As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current I_0 is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses. Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads.

This is the load current at η_{max} in terms of full load current



Iron losses, P_i = Wattmeter reading = W_0

No load current = Ammeter reading = I_0

Applied voltage = Voltmeter reading = V_1

Input power, $W_0 = V_1 I_0 \cos \phi_0$

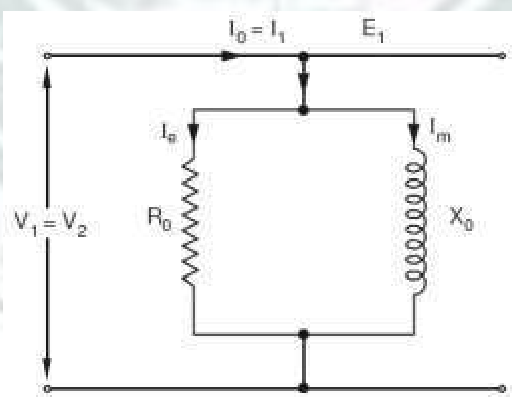
No - load p.f., $\cos\phi = \frac{W_o}{V_o I_o}$ = no load power factor

$I_m = I_o \sin \phi_o$ = magnetizing component

$I_c = I_o \cos \phi_o$ = Active component

$R_o = \frac{V_o}{I_c}\ \Omega, \quad X_o = \frac{V_o}{I_m}\ \Omega$

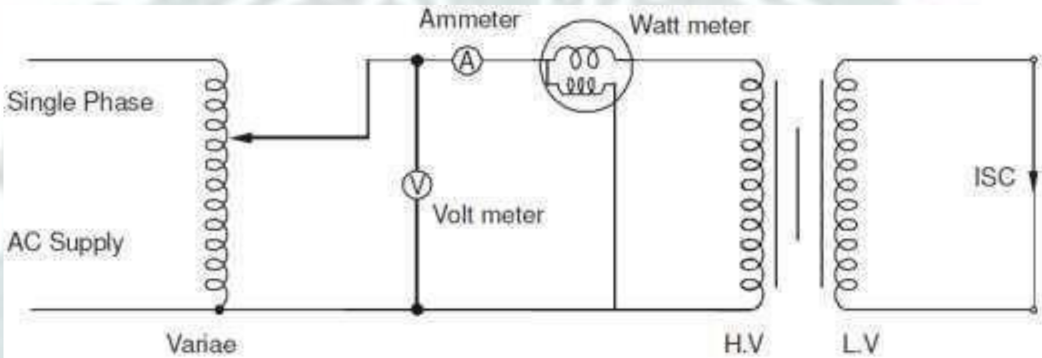
Under no load conditions the PF is very low (near to 0) in lagging region. By using the above data we can draw the equivalent parameter shown in Figure below.



Thus open-circuit test enables us to determine iron losses and parameters R_0 and X_0 of the transformer.

5.10.2 Short-Circuit or Impedance Test

This test is conducted to determine R1e (or R2e), X1e (or X2e) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in figure below. The low input voltageis gradually raised till at voltage VSC, full-load current I1 flows in the primary. Then I2 in the secondary also has full-load value since I1/I2 = N2/N1. Under such conditions, the copper loss in the windings is the same as that on full load. There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage VSC is very small. Hence, the wattmeter will practically register the full load copper losses in the transformer windings.



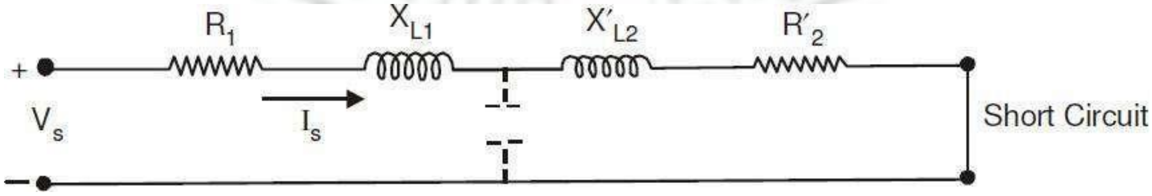
Full load Cu loss, PC = Wattmeter reading = W_{sc}
Applied voltage = Voltmeter reading = V_{SC}
F.L. primary current = Ammeter reading = I_1

$Pcu = I_1^2R_1 + I_1^2 R_2' = I_1^2 R_{1e}$, $R_{1e} = \dfrac{Pcu}{I1^2}$

Where R_{1e} is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z1e = \sqrt{Z1e^2 - R1e^2}$,

short – circuit P.F. $\cos\Phi = \dfrac{Pcu}{Vsc\, I1}$ Thus short-circuit lest gives full-load Cu loss, R1e and X1e.



From above figure we can calculate,

$$\text{equivalent resistance } R_{eq} = \frac{W_s}{I_s^2} = R_1 + R_2'$$

$$\text{and equivalent impedance } Z_{eq} = \frac{V_s}{I_s}$$

So we calculate equivalent reactance

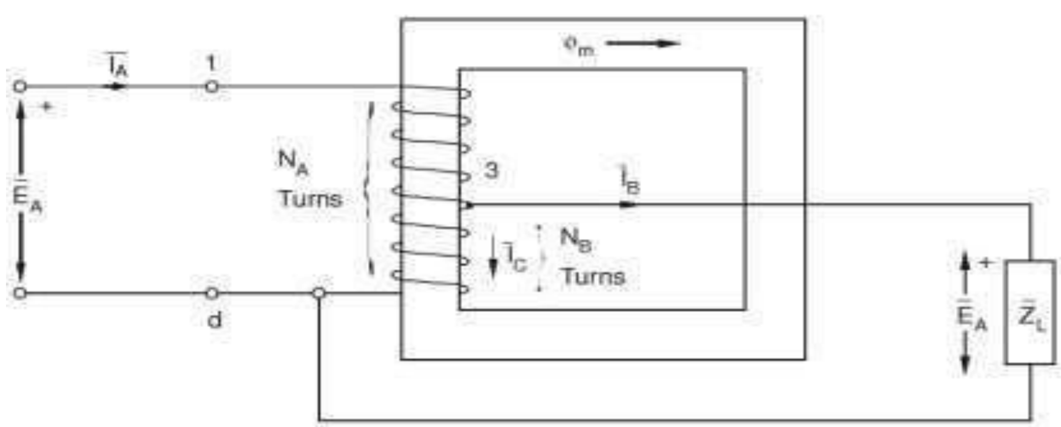
$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = X_{L1} + X_{L2}'$$

These R_{eq} and X_{eq} are equivalent resistance and reactance of both windings referred in HV side. These are know as equivalent circuit resistance and reactance.

5.11 AUTO-TRANSFORMERS

The transformers we have considered so far are two-winding transformers in which the electricalcircuit connected to the primary is electrically isolated from that connected to the secondary. An auto-transformer does not provide such isolation, but has economy of cost combined with increased efficiency. Figure below illustrates the auto-transformer which consists of a coil of N_A turns between terminals 1 and 2, with a third terminal 3 provided after N_B turns. If we neglect coil resistances and leakage fluxes, the flux linkages of the coil between 1 and 2 equals N_A ɸm while the portion of coil between 3 and 2 has a flux linkage N_B ɸm. If the induced voltages are designated as E_A and E_B, just as in a two winding transformer,

$$\frac{\bar{E}_A}{\bar{E}_B} = \frac{N_A}{N_B}$$



Neglecting the magnetizing ampere-turns needed by the core for producing flux, as in an ideal transformer, the current *IA* flows through only (*NA* – *NB*) turns. If the load current is *IB*, as shown by

Kirchhoff's current law, the current IC flowing from terminal 3 to terminal 2 is (IA - IB). This current flows through NB turns. So, the requirement of a net value of zero ampere-turns across the core demands that consequently, as far as voltage, current converting properties are concerned, the autotransformer of

$$(N_A - N_B)\,\bar{I}_A + (\bar{I}_A - \bar{I}_B)\,N_B = 0$$

or $$N_A\,\bar{I}_A - N_B\,\bar{I}_B = 0$$

Hence, just as in a two-winding transformer,

$$\frac{\bar{I}_A}{\bar{I}_B} = \frac{N_B}{N_A}$$

Figure above behaves just like a two-winding transformer. However, in the autotransformer we don't needtwo separate coils, each designed to carry full load values of current.

5.12 PARALLEL OPERATION OF TRANSFORMERS

It is economical to install numbers of smaller rated transformers in parallel than installing a bigger rated electrical power transformers. This has mainly the following advantages,

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfil the total demand. In this way we can run the system with maximum efficiency.

To maximize electrical power system availability: If numbers of transformers run in parallel, we can shut down any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

To maximize power system reliability: if any one of the transformers run in parallel, is tripped due to fault of other parallel transformers is the system will share the load, hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfil the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

Conditions for Parallel Operation of Transformers

When two or more transformers run in parallel, they must satisfy the following conditions for satisfactory performance. These are the conditions for parallel operation of transformers.

- ***Same voltage ratio of transformer.***
- ***Same percentage impedance.***
- ***Same polarity.***
- ***Same phase sequence.***
- ***Same Voltage Ratio***

Same voltage ratio of transformer.

If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages. Now say the secondary of these transformers are connected to same bus, there will be a circulating current between secondaries and therefore between primaries also. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra I2R loss.

Same Percentage Impedance

The current shared by two transformers running in parallel should be proportional to their MVA ratings. Again, current carried by these transformers are inversely proportional to their internal impedance. From these two statements it can be said that, impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words, percentage impedance or per unit values of impedance should be identical for all the transformers that run in parallel.

Same Polarity

Polarity of all transformers that run in parallel, should be the same otherwise huge circulating current that flows in the transformer but no load will be fed from these transformers. Polarity of transformer means the instantaneous direction of induced emf in secondary. If the instantaneous directions of induced secondary emf in two transformers are opposite to each other when same input power is fed to both of the transformers, the transformers are said to be in opposite polarity. If the instantaneous directions of induced secondary e.m.f in two transformers are same when same input power is fed to the both of the transformers, the transformers are said to be in same polarity.

Same Phase Sequence

The phase sequence or the order in which the phases reach their maximum positive voltage, must be identical for two parallel transformers. Otherwise, during the cycle, each pair of phases will be short circuited. The above said conditions must be strictly followed for parallel operation of transformers but totally identical percentage impedance of two different transformers is difficult to achieve practically, that is why the transformers run in parallel may not have exactly same percentage impedance but the values would be as nearer as possible.

5.13 WHY TRANSFORMER RATING IN kVA?

An important factor in the design and operation of electrical machines is the relation between thelife of the insulation and operating temperature of the machine. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. We know that copper loss in a transformer depends on current and iron loss depends on voltage. Therefore, the total loss in a transformer depends on the volt-ampere product only and not on the phase angle between voltage and current i.e., it is independent of load power factor. For this reason, the rating of a transformer is in kVAand not kW.

CHAPTER - 6

THREE PHASE INDUCTION MOTORS

6.1 INTRODUCTION

An electrical motor is an electromechanical device which converts electrical energy into mechanical energy. In the case of three phase AC (Alternating Current) operation, the most widely used motor is a **3 phase induction motor**, as this type of motor does not require an additional starting device. These types of motors are known as self-starting induction motors.

6.2 TYPES AND CONSTRUCTION OF THREE PHASE INDUCTION MOTOR

A 3 phase induction motor consists of two major parts:

- A stator
- A rotor

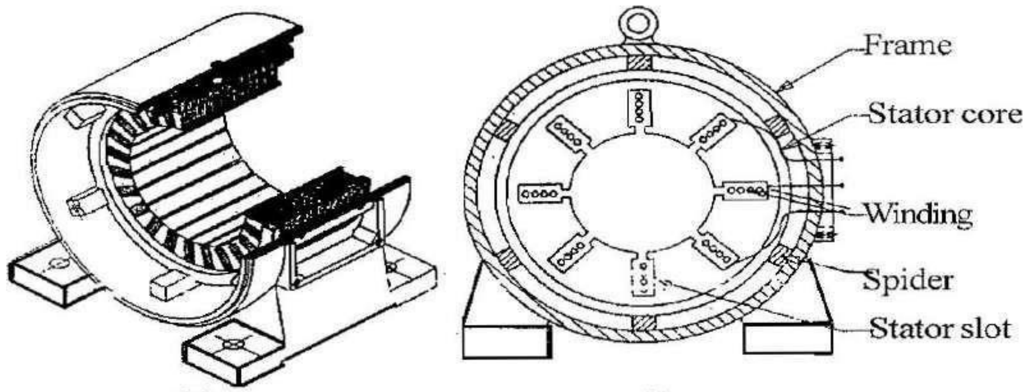
According to rotor construction three phase induction motors are constructed into two majortypes:

1. Squirrel cage Induction Motors
2. Slip ring Induction Motors

6.2.1 Stator Construction

The **stator** of three phase induction motor is made up of numbers of slots to construct a 3 phase winding circuit which we connect with 3 phase AC source. We arrange the three- phase winding in such a manner in the slots that they produce rotating magnetic field when we switch on the three-phase AC supply source which is of constant magnitude but which revolves at synchronous speed (N_S = 120f/P). This revolving magnetic field induces an EMF in the rotor by mutual induction. It is wound with a definite number of poles as per the requirement of the speed. Greater the number of pole lesser the speed and vice versa.

The stator or the stationary part consists of three phase winding held in place in the slots of alaminated steel core which is enclosed and supported by a cast iron or a steel frame as shown in below.



(a) (b)

6.2.2 Rotor Construction

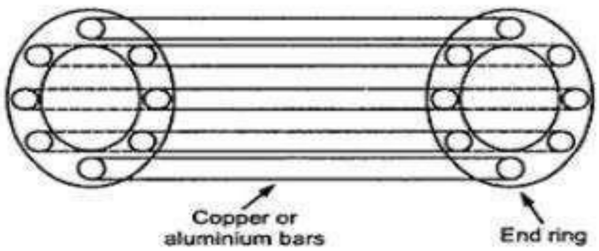
6.2.2.1 Squirrel Cage Rotor Construction

The rotor of the squirrel cage motor shown in Fig as given below contains no windings. Instead it is a cylindrical core constructed of steel laminations with conductor bars mounted parallel to the shaft and embedded near the surface of the rotor core.

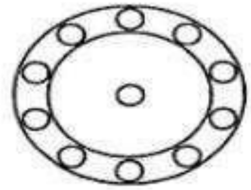
These conductor bars are short circuited by an end rings at both end of the rotor core. In large machines, these conductor bars and the end rings are made up of copper with the bars brazed or welded to the end rings .In small machines the conductor bars and end rings are sometimes madeof aluminium with the bars and rings cast in as part of the rotor core. Actually the entire construction (bars and end-rings) resembles a squirrel cage, from which the name is derived.

The rotor or rotating part is not connected electrically to the power supply but has voltage inducedin it by transformer action from the stator. For this reason, the stator is sometimes called the primary and the rotor is referred to as the secondary of the motor since the motor operates on the principle of induction and as the construction of the rotor with the bars and end rings resembles a squirrel cage, the squirrel cage induction motor is used.

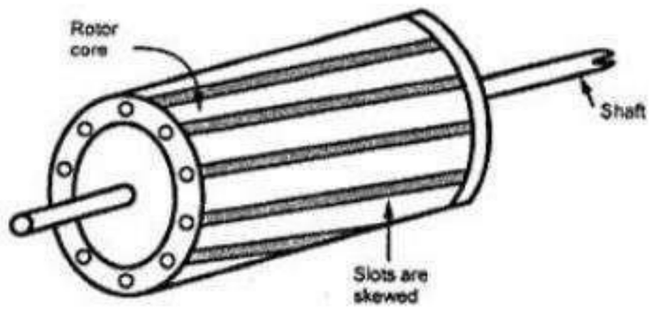
The rotor bars are not insulated from the rotor core because they are made of metals having less resistance than the core. The induced current will flow mainly in them. Also the rotor bars are usually not quite parallel to the rotor shaft but are mounted in a slightly skewed position. This feature tends to produce a more uniform rotor field and torque. Also it helps to reduce some of theinternal magnetic noise when the motor is running.



Cage type structure of rotor



Symbolic representation



Squirrel Cage Rotor

(a) **End Shields**

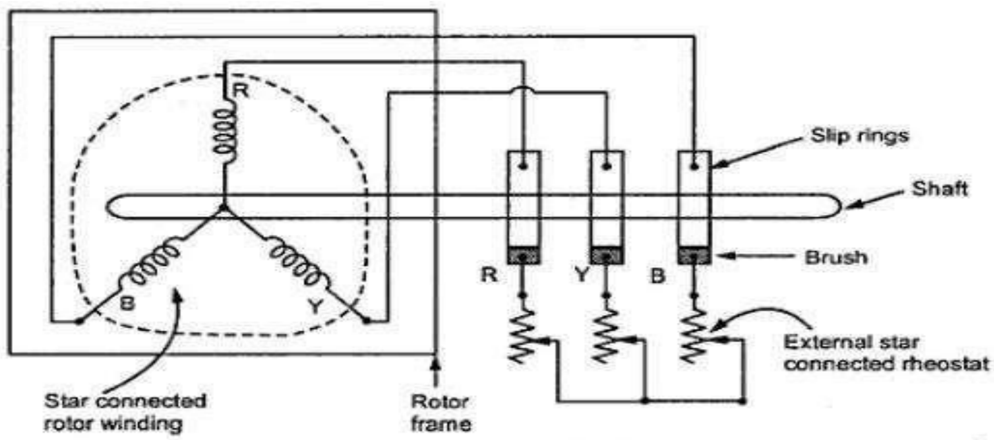
The function of the two end shields is to support the rotor shaft. They are fitted with bearings and attached to the stator frame with the help of studs or bolts attention.

6.2.2.2 Slip Ring or Phase wound Rotor Construction

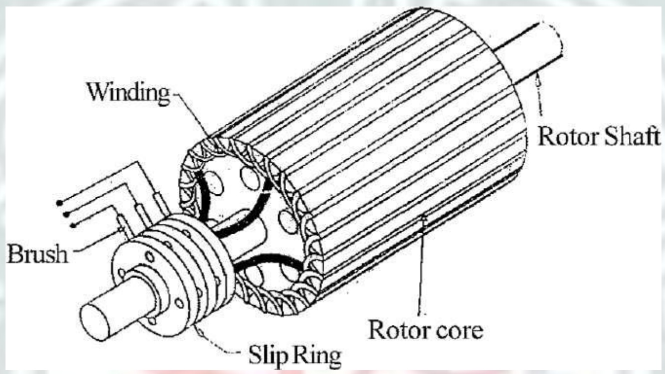
The rotor of the slip ring induction motor is also cylindrical or constructed of lamination.

Squirrel cage motors have a rotor with short circuited bars whereas slip ring motors have woundrotors having "three windings" each connected in star.

The winding is made of copper wire. The terminals of the rotor windings of the slip ring motorsare brought out through slip rings which are in contact with stationary brushes as shown in Fig:



Slip ring Rotor



6.2.2.3 Comparison of Squirrel Cage and Slip Ring Motor

Sl.No.	*Property*	*Squirrel cage motor*	*Slip ring motor*
1.	**Rotor Construction**	*Bars are used in rotor. Squirrel cage motor is very simple, rugged andlong lasting. No slip rings and brushes*	*Winding wire is to beused. Wound rotor requiredattention. Slip ring and brushes are needed also need frequent maintenance.*
2.	**Starting**	*Can be started by D.O.L., star-delta, autotransformer starters*	*Rotor resistance starteris required.*
3.	**Starting torque**	*Low*	*Very high*
4.	**Starting Current**	*High*	*Low*

5.	**Speed variation**	*Not easy, but could be varied in large steps by pole changing or through smaller incremental steps through thyristors or byfrequency variation.*	*Easy to vary speed. Speed change is possibleby inserting rotor resistance using thyristors or by using frequency variation injecting emf in the rotor circuit cascading.*
6.	**Maintenance**	*Almost maintenance Zero*	*Requires frequent maintenance*
7.	**Cost**	*Low*	*High*

6.3 PRINCIPLE OF OPERATION

Induction motor works on the principle of electromagnetic induction. When three phase supply is given to the stator winding, a rotating magnetic field of constant magnetic field is produced.
The speed of rotating magnetic field (R.M.F.) is synchronous speed, NS r.p.m.

- $N_S = \frac{120f}{P}$ = speed of rotating magnetic field
- f = supply frequency

This rotating field produces an effect of rotating poles around a rotor. Let direction of this magnetic field is clockwise as shown.

Now at this instant rotor is stationary and stator flux R.M.F. is rotating. So its obvious that there exists a relative motion between the R.M.F. and rotor conductors. Now the R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors. Whenever a conductor cuts the flux, emf. gets induced in it. So e.m.f. gets induced in the rotor conductors called rotor induced emf. this is electro – magnetic induction. As rotor forms closed circuit, induced emf. circulates current through rotor called rotor current. Any current carrying conductor produces its own flux. So rotor produces its flux called rotor flux. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown.

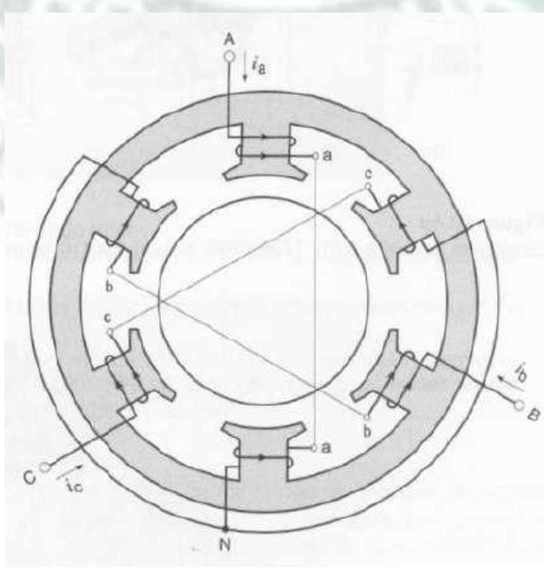
This direction can be easily determined using right hand thumb rule. Now there are two fluxes, one R.M.F. and another rotor flux. Both the fluxes interact with each. On left of rotor conductor, two fluxes are in same direction hence added up to get high flux area. On right side of rotor conductor, two fluxes are in opposite direction hence they cancel each other to produce low flux area. So rotor conductor experiences a force from left to right, due to interaction of the two fluxes. As all rotor conductor experiences a force, overall rotor experiences a torque and starts rotating. So interaction of the two fluxes is very essential for a motoring action.

6.3.1 Rotating Magnetic Field and Induced Voltages

Consider a simple stator having 6 salient poles, each of which carries a coil having 5 turns Coils that are diametrically opposite are connected in series by means of three jumpers that respectively connect terminals a-a, b-b, and c-c. This creates three identical sets of windings AN, BN, CN, which are mechanically spaced at 120 degrees to each other. The two coils in each winding produce magneto motive forces that act in the same direction.

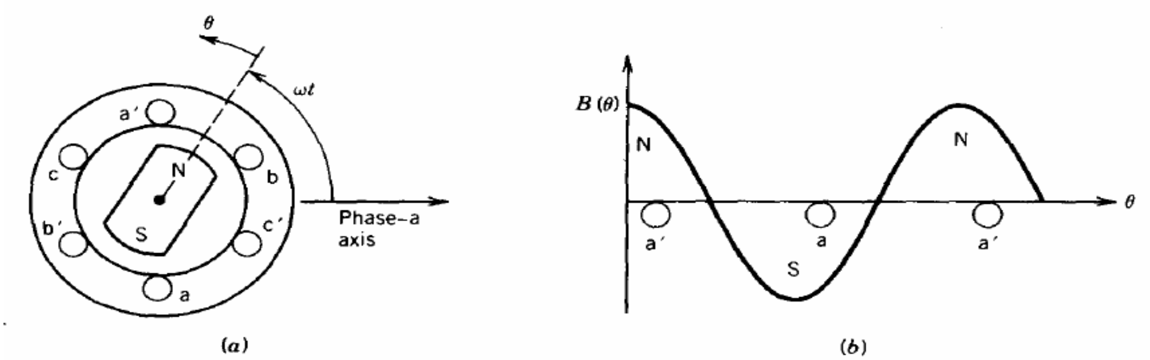
The three sets of windings are connected in wye, thus forming a common neutral N. Owing to the perfectly symmetrical arrangement, the line to neutral impedances are identical. In other words, as regards terminals A, B, C, the windings constitute a balanced 3-phase system.

For a two-pole machine, rotating in the air gap, the magnetic field (i.e., flux density) being sinusoidally distributed with the peak along the centre of the magnetic poles. The result is illustrated in figure below. The rotating field will induce voltages in the phase coils aa', bb', and cc'. Expressions for the induced voltages can be obtained by using Faraday laws of induction.



Elementary stator having terminals A, B, Cconnected to a 3-phase source (not shown).

Currents flowing from line to neutral are considered to be positive.



Air gap flux density distribution.

The flux density distribution in the air gap can be expressed as:

$$B(\theta) = B_{\max} \cos \theta$$

The air gap flux per pole, ϕ_P, is:

$$\phi_p = \int_{-\pi/2}^{\pi/2} B(\theta) lr d\theta = 2B_{\max} lr$$

Where,

l is the axial length of the stator.

r is the radius of the stator at the air gap.

Let us consider that the phase coils are full-pitch coils of N turns (the coil sides of each phase are 180 electrical degrees apart as shown in figure above). It is obvious that as the rotating field moves (or the magnetic poles rotate) the flux linkage of a coil will vary. The flux linkage for coil aa' will be maximum.

($= N\phi_P$ at $\omega t = 0^o$) (Fig.3.5a) and zero at $\omega t = 90^o$. The flux linkage $\lambda_a(\omega t)$ will vary as the cosine of the angle ωt.

Hence,

$$\lambda_a(\omega t) = N\phi_p \cos \omega t$$

Therefore, the voltage induced in phase coil **aa'** is obtained from *Faraday law* as:

$$e_a = -\frac{d\lambda_a(\omega t)}{dt} = \omega N\phi_p \sin \omega t = E_{\max} \sin \omega t$$

The voltages induced in the other phase coils are also sinusoidal, but phase-shifted from each other by 120 electrical degrees. Thus,

$$e_b = E_{\max} \sin(\omega t - 120)$$

$$e_c = E_{\max} \sin(\omega t + 120).$$

the *rms* value of the induced voltage is:

$$E_{rms} = \frac{\omega N\phi_p}{\sqrt{2}} = \frac{2\pi f}{\sqrt{2}} N\phi_p = 4.44 fN\phi_p$$

Where f is the frequency in hertz. Above equation has the same form as that for the inducedvoltage in transformers. However, $Ø_P$ represents the flux per pole of the machine.

The above equation also shows the rms voltage per phase. The N is the total number of series turnsper phase with the turns forming a concentrated full-pitch winding. In an actual AC machine eachphase winding is distributed in a number of slots for better use of the iron and copper and to improve the waveform. For such a distributed winding, the EMF induced in various coils placed in different slots are not in time phase, and therefore the phasor sum of the EMF is less than their numerical sum when they are connected in series for the phase winding. A reduction factor K_W, called the winding factor, must therefore be applied. For most three-phase machine windings K_W is about 0.85 to 0.95.

Therefore, for a distributed phase winding, the rms voltage per phase is

$Erms = 4.44fN_{ph}\varphi_P K_W$

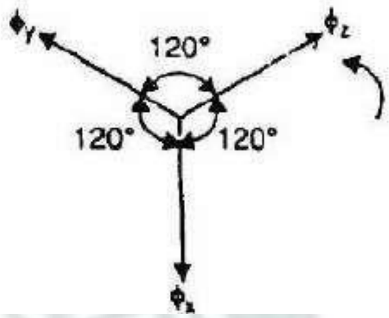
Where N_{ph} is the number of turns in series per phase.

6.3.2 Alternate Analysis for Rotating Magnetic Field

When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced.This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 m where m is the maximum flux dueto any phase.

To see how rotating field is produced, consider a 2-pole, 3-phase winding as shown in figure below.The three phases X, Y and Z are energized from a 3-phase source and currents in these phases areindicated as Ix, Iy and Iz. Referring to Fig. as given below, the fluxes produced by these currents are given by:

$$\phi_x = \phi_m \sin \omega t$$
$$\phi_y = \phi_m \sin(\omega t - 120°)$$
$$\phi_z = \phi_m \sin(\omega t - 240°)$$



Here ϕ_m is the maximum flux due to any phase. Above figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 ϕ_m.At instant 1, the current in phase X is zero and currents in phases Y andZ are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of theresultant flux is constant and is equal to 1.5 ϕ_m as proved under:

At instant 1, $\omega t = 0°$. Therefore, the three fluxes are given by;

$$\phi_x = 0; \qquad \phi_y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m;$$
$$\phi_z = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

So,

Resultant flux, $\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5\ \phi_m$

At instant 2 [Figure below], the current is maximum (negative) in ϕ_y phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5 ϕ_m as proved under:

At instant 2, $\omega t = 30°$. Therefore, the three fluxes are given by;

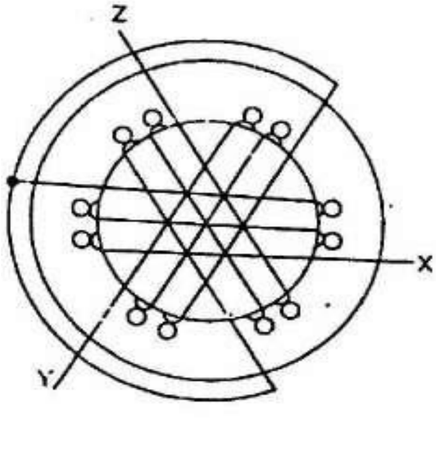
$$\phi_x = \phi_m \sin 30° = \frac{\phi_m}{2}$$
$$\phi_y = \phi_m \sin(-90°) = -\phi_m$$
$$\phi_z = \phi_m \sin(-210°) = \frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

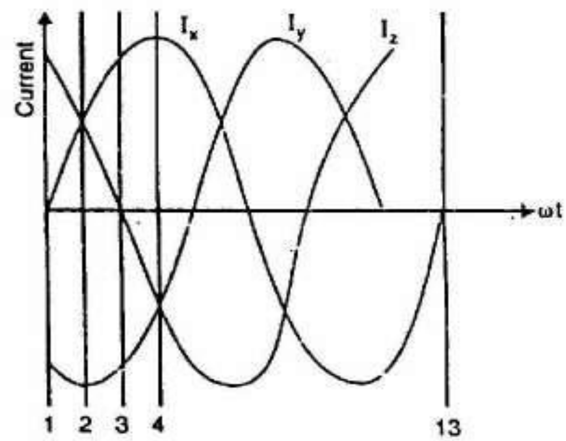
Phasor sum of ϕ_x and ϕ_z, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

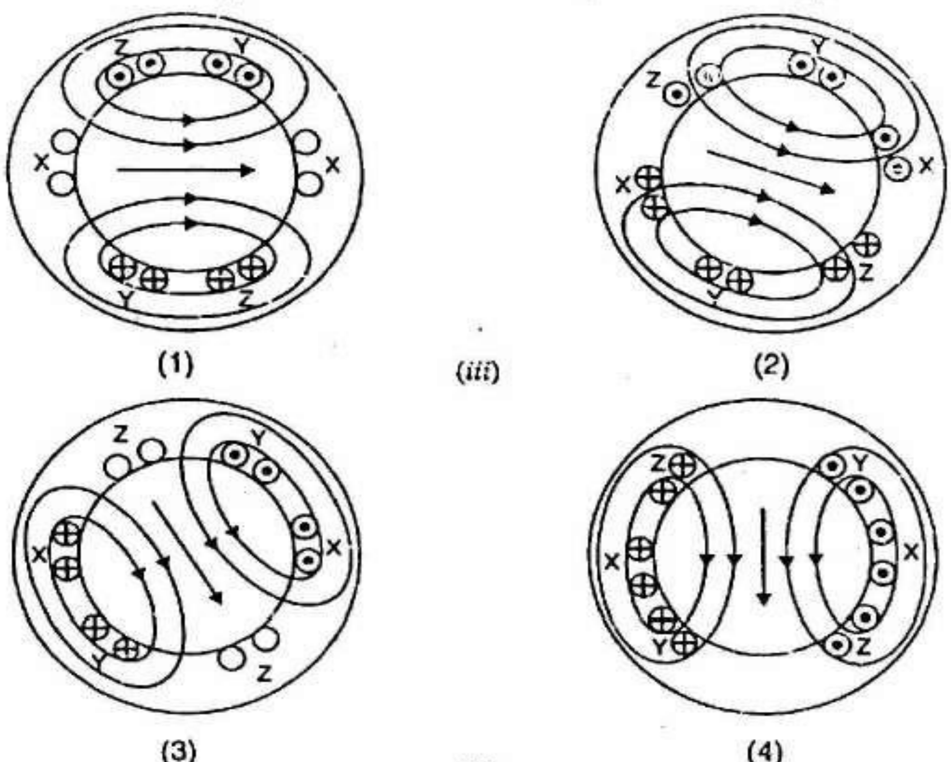
Note that resultant flux is displaced 30° clockwise from position 1.



(i)

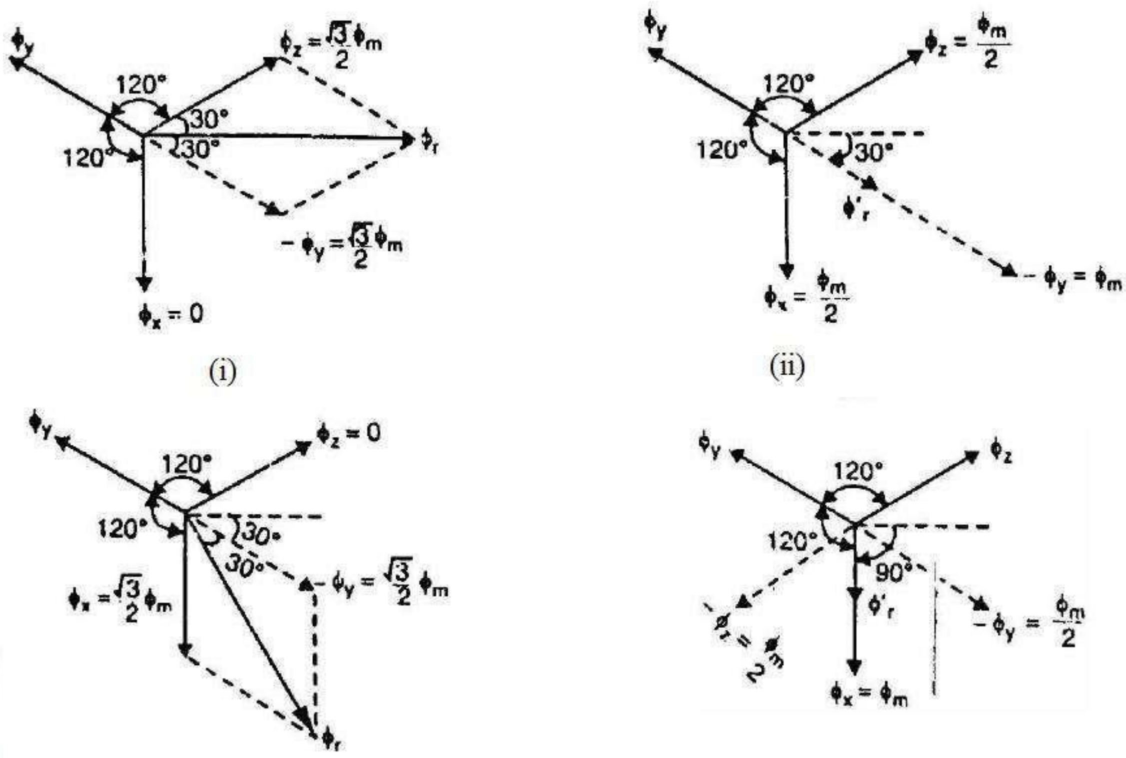


(ii)



(iii)

At instant 3, current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc 0.866 × max. value). The magnitude of resultant flux is 1.5 ϕ_m as proved under:



(i) (ii) (iii) (iv)

At instant 3, $\omega t = 60°$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 60° = \frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_y = \phi_m \sin(-60°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-180°) = 0$$

The resultant flux ϕ_r is the phasor sum of ϕ_x and $-\phi_y$ $(\because\ \phi_z = 0)$.

$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 1.5\ \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

At instant 4 the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are 0.5 × max. value). This establishes a resultant flux downward as shown under:

At instant 4, $\omega t = 90°$. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90° = \phi_m$$

$$\phi_y = \phi_m \sin(-30°) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150°) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

Phasor sum of $-\phi_z$ and $-\phi_y$, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and ϕ_x, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\,\phi_m$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1.

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5 ϕ_m, where ϕ_m is the maximum flux due to any phase).

6.3.3 Speed of Rotating Magnetic Field

The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns). The time instant 4 represents the completion of one-quarter cycle of alternating current Ix from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 or one complete cycle of current Ix from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

$$\therefore \quad \text{Cycles of current} = \frac{P}{2} \times \text{revolutions of field}$$

or $$\text{Cycles of current per second} = \frac{P}{2} \times \text{revolutions of field per second}$$

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

or $$N_s = \frac{120\,f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

6.3.4 Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, therotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

$$\% \text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity $N_s - N$ is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

6.3.5 Rotor Current Frequency

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula;

$$\text{Frequency} = \frac{NP}{120}$$

where N = Relative speed between magnetic field and the winding
P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is $N_s - N$. Consequently, the rotor current frequency f' is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{sN_sP}{120} \qquad \left(\because s = \frac{N_s - N}{N_s}\right)$$

$$= sf \qquad \left(\because f = \frac{N_sP}{120}\right)$$

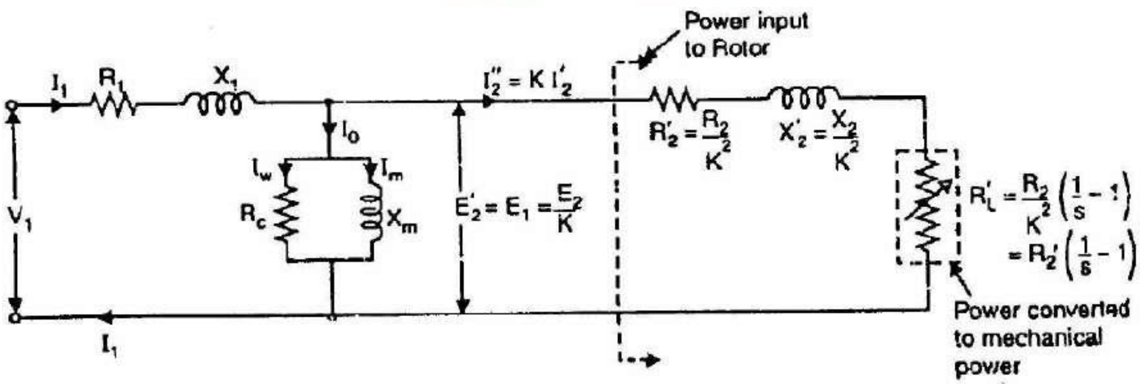
i.e., Rotor current frequency = Fractional slip x Supply frequency

(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency (f' = sf = 1× f = f).

(ii) As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

6.4 POWER AND TORQUE RELATIONS OF THREE PHASE INDUCTION MOTOR

The transformer equivalent circuit of an induction motor is quite helpful in analyzing the various power relations in the motor. Fig. shows below the equivalent circuit per phase of an induction motor where all values have been referred to primary (i.e., stator).



(i) Total electrical load $= R'_2\left(\frac{1}{s}-1\right) + R'_2 = \frac{R'_2}{s}$

Power input to stator $= 3V_1 I_1 \cos\phi_1$

There will be stator core loss and stator Cu loss. The remaining power will be the power transferred across the air-gap i.e., input to the rotor.

(ii) Rotor input $= \frac{3(I''_2)^2 R'_2}{s}$

Rotor Cu loss $= 3(I''_2)^2 R'_2$

Total mechanical power developed by the rotor is

$$P_m = \text{Rotor input} - \text{Rotor Cu loss}$$

$$= \frac{3(I''_2)^2 R'_2}{s} 3(I''_2)^2 R'_2 = 3(I''_2)^2 R'_2 \left(\frac{1}{s} - 1\right)$$

(iii) If T_g is the gross torque developed by the rotor, then,

$$P_m = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1}{s}-1\right) = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1-s}{s}\right) = \frac{2\pi N\, T_g}{60}$$

or $$3(I''_2)^2 R'_2\left(\frac{1-s}{s}\right) = \frac{2\pi N_s(1-s)\, T_g}{60} \qquad [\because N = N_s(1-s)]$$

$$\therefore \quad T_g = \frac{3(I''_2)^2 R'_2/s}{2\pi N_s/60} \quad \text{N - m}$$

or $$T_g = 9.55\frac{3(I''_2)^2 R'_2/s}{N_s} \quad \text{N - m}$$

Note that shaft torque T_{sh} will be less than T_g by the torque required to meet windage and frictional losses.

6.4.1 Induction Motor Torque

The mechanical power P available from any electric motor can be expressed as:

$$P = \frac{2\pi N T}{60} \quad \text{watts}$$

where N = speed of the motor in r.p.m.
T = torque developed in N-m

$$\therefore \quad T = \frac{60}{2\pi}\frac{P}{N} = 9.55\frac{P}{N} \text{ N - m}$$

If the gross output of the rotor of an induction motor is P_m and its speed is N r.p.m., then gross torque T developed is given by:

$$T_g = 9.55\frac{P_m}{N} \text{ N - m}$$

Similarly, $$T_{sh} = 9.55\frac{P_{out}}{N} \text{ N - m}$$

Note. Since windage and friction loss is small, $T_g = T_{sh}$,. This assumption hardly leads to any significant error.

6.4.2 Rotor Output

If T_g newton-metre is the gross torque developed and N r.p.m. is the speed of the rotor, then,

$$\text{Gross rotor output} = \frac{2\pi N T_g}{60} \quad \text{watts}$$

If there were no copper losses in the rotor, the output would equal rotor input and the rotor wouldrun at synchronous speed Ns.

$$\therefore \quad \text{Rotor input} = \frac{2\pi N_s T_g}{60} \quad \text{watts}$$

$$\therefore \quad \text{Rotor Cu loss} = \text{Rotor input} - \text{Rotor output}$$

$$= \frac{2\pi T_g}{60}(N_s - N)$$

(i) $$\frac{\text{Rotor Cu loss}}{\text{Rotor input}} = \frac{N_s - N}{N_s} = s$$

$$\therefore \quad \text{Rotor Cu loss} = s \times \text{Rotor input}$$

(ii) $$\text{Gross rotor output, } P_m = \text{Rotor input} - \text{Rotor Cu loss}$$

$$= \text{Rotor input} - s \times \text{Rotor input}$$

$$\therefore \quad P_m = \text{Rotor input } (1 - s)$$

(iii) $$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s = \frac{N}{N_s}$$

(iv) $$\frac{\text{Rotor Cu loss}}{\text{Gross rotor output}} = \frac{s}{1 - s}$$

It is clear that if the input power to rotor is "Pr" then "s.Pr" is lost as rotor Cu loss and the remaining(1 - s) Pr is converted into mechanical power. Consequently, induction motor operating at high slip has poor efficiency.

Note.

$$\frac{\text{Gross rotor output}}{\text{Rotor input}} = 1 - s$$

If the stator losses as well as friction and windage losses arc neglected, then,

$$\text{Gross rotor output} = \text{Useful output}$$

$$\text{Rotor input} = \text{Stator input}$$

$$\therefore \quad \frac{\text{Useful output}}{\text{Stator output}} = 1 - s = \text{Efficiency}$$

Hence the approximate efficiency of an induction motor is 1 − s. Thus if the slip of an induction motor is 0.125, then its approximate efficiency is = 1 − 0.125 = 0.875 or 87.5%.

6.4.3 Torque Equations

The gross torque Tg developed by an induction motor is given by;

$$T_g = \frac{\text{Rotor input}}{2\pi N_s} \qquad \text{... } N_s \text{ is r.p.s.}$$

$$= \frac{60 \times \text{Rotor input}}{2\pi N_s} \qquad \text{... } N_s \text{ is r.p.s.}$$

Now $\quad \text{Rotor input} = \dfrac{\text{Rotor Cu loss}}{s} = \dfrac{3(I'_2)^2 R_2}{s} \qquad$ (i)

As shown in Sec. 8.16, under running conditions,

$$I'_2 = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} = \frac{s K E_1}{\sqrt{R_2^2 + (s X_2)^2}}$$

where $\quad K = \text{Transformation ratio} = \dfrac{\text{Rotor turns/phase}}{\text{Stator turns/phase}}$

$$\therefore \quad \text{Rotor input} = 3 \times \frac{s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{1}{s} = \frac{3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

(Putting me value of I'_2 in eq.(i))

Also $\quad \text{Rotor input} = 3 \times \dfrac{s^2 K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \times \dfrac{1}{s} = \dfrac{3 s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2}$

(Putting me value of I'_2 in eq.(i))

$$\therefore \quad T_g = \frac{\text{Rotor input}}{2\pi N_s} = \frac{3}{2\pi N_s} \times \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2} \qquad \text{...in terms of } E_2$$

$$= \frac{3}{2\pi N_s} \times \frac{s K^2 E_1^2 R_2}{R_2^2 + (s X_2)^2} \qquad \text{...in terms of } E_1$$

Note that in the above expressions of T_g, the values E_1, E_2, R_2 and X_2 represent the phase values.

6.4.4 Rotor Torque

The torque T developed by the rotor is directly proportional to:

(i) rotor current
(ii) rotor e.m.f.
(iii) power factor of the rotor circuit

$$\therefore \quad T \propto E_2 I_2 \cos\phi_2$$

or $$T = K E_2 I_2 \cos\phi_2$$

where I_2 = rotor current at standstill
E_2 = rotor e.m.f. at standstill
$\cos\phi_2$ = rotor p.f. at standstill

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions.

6.4.5 Starting Torque (Ts)

Let,

E_2 = rotor e.m.f. per phase at standstill

X_2 = rotor reactance per phase at standstillR_2 = rotor resistance per phase

Rotor impedance/phase, $Z_2 = \sqrt{R_2^2 + X_2^2}$ ...at standstill

Rotor current/phase, $I_2 = \dfrac{E_2}{Z_2} = \dfrac{E_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

Rotor p.f., $\cos\phi_2 = \dfrac{R_2}{Z_2} = \dfrac{R_2}{\sqrt{R_2^2 + X_2^2}}$ ...at standstill

$\therefore$ Starting torque, $T_s = K E_2 I_2 \cos\phi_2$

$$= K E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$= \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Generally, the stator supply voltage V is constant so that flux per pole ϕ set up by the stator is also fixed. This in turn means that e.m.f. E_2 induced in the rotor will be constant.

$$\therefore \qquad T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} = \frac{K_1 R_2}{Z_2^2}$$

where K_1 is another constant.

It is clear that the magnitude of starting torque would depend upon the relative values of R_2 and X_2 i.e., rotor resistance/phase and standstill rotor reactance/phase.

It can be shown that $K = 3/2 \pi N_s$.

$$\therefore \qquad T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

Note that here N_s is in r.p.s.

6.4.6 Condition for Maximum Starting Torque

It can be proved that starting torque will be maximum when rotor resistance/phase is equal tostandstill rotor reactance/phase.

Now $$T_s = \frac{K_1 R_2}{R_2^2 + X_2^2} \qquad \text{(i)}$$

Differentiating eq. (i) w.r.t. R_2 and equating the result to zero, we get,

$$\frac{dT_s}{dR_2} = K_1 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{\left(R_2^2 + X_2^2\right)^2} \right] = 0$$

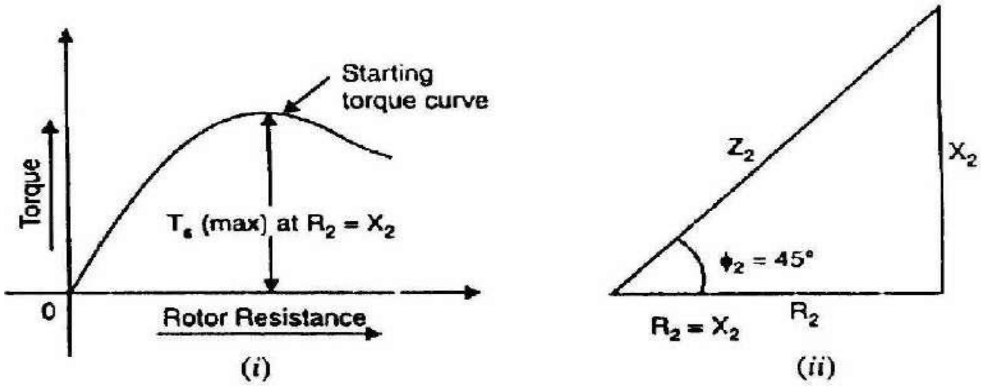
or $$R_2^2 + X_2^2 = 2R_2^2$$

or $$R_2 = X_2$$

Hence starting torque will be maximum when:

Rotor resistance/phase = Standstill rotor reactance/phase

Under the condition of maximum starting torque, $\phi_2 = 45°$ and rotor power factor is 0.707 lagging.



Above figure shows the variation of starting torque with rotor resistance. As the rotor resistance is increased from a relatively low value, the starting torque increases until it becomes maximum when $R_2 = X_2$. If the rotor resistance is increased beyond this optimum value, the starting torque will decrease.

6.4.7 Effect of Change of Supply Voltage

$$T_s = \frac{K E_2^2 R_2}{R_2^2 + X_2^2}$$

Since $E_2 \propto$ Supply voltage V

$$\therefore \quad T_s = \frac{K_2 V^2 R_2}{R_2^2 + X_2^2}$$

where K_2 is another constant.

$$\therefore \quad T_s \propto V^2$$

Therefore, the starting torque is very sensitive to changes in the value of supply voltage. For example, a drop of 10% in supply voltage will decrease the starting torque by about 20%. This could mean the motor failing to start if it cannot produce a torque greater than the load torque plusfriction torque.

6.5 METHODS OF STARTING THREE PHASE INDUCTION MOTORS

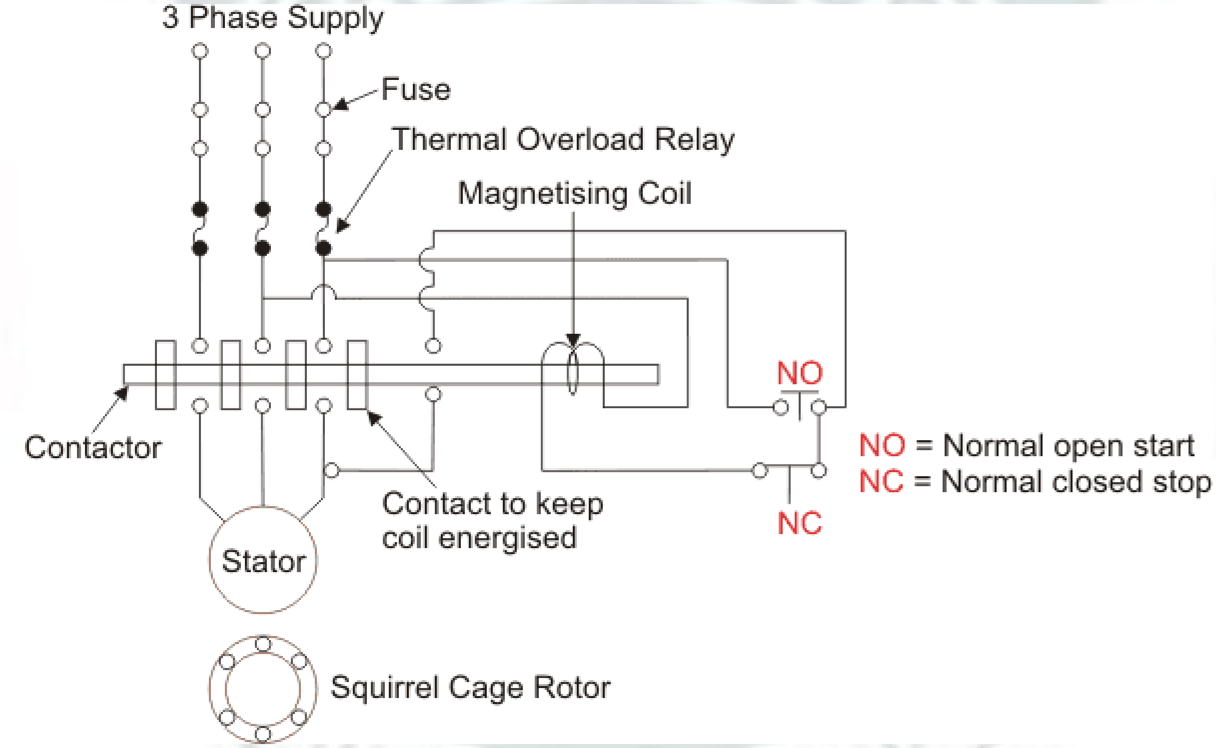
The method to be employed in starting a given induction motor depends upon the size of the motorand the type of the motor. The common methods used to start induction motors are:

(i) Direct-on-line starting

(ii) Star-delta starting

6.5.1 Direct-On-Line Starting

A **DOL starter** (or **Direct On Line starter** or **across the line starter**) is a method of starting of a 3 phase induction motor. In DOL Starter an induction motor is connected directly across its 3-phase supply, and the DOL starter applies the full line voltage to the motor terminals. Despite this direct connection, no harm is done to the motor. A DOL motor starter contains protection devices, and in some cases, condition monitoring. A wiring diagram of a DOL starter is shown below:



Since the DOL starter connects the motor directly to the main supply line, the motor draws a very high inrush current compared to the full load current of the motor (up to 5-8 times higher). The value of this large current decreases as the motor reaches its rated speed. A direct on line starter can only be used if the high inrush current of the motor does not cause an excessive voltage drop in the supply circuit. If a high voltage drop needs to be avoided, a star delta starter should be used instead. Direct on line starters are commonly used to start small motors, especially 3 phase squirrel cage induction motors.

$$I_a = \frac{(V - E)}{R_a}$$

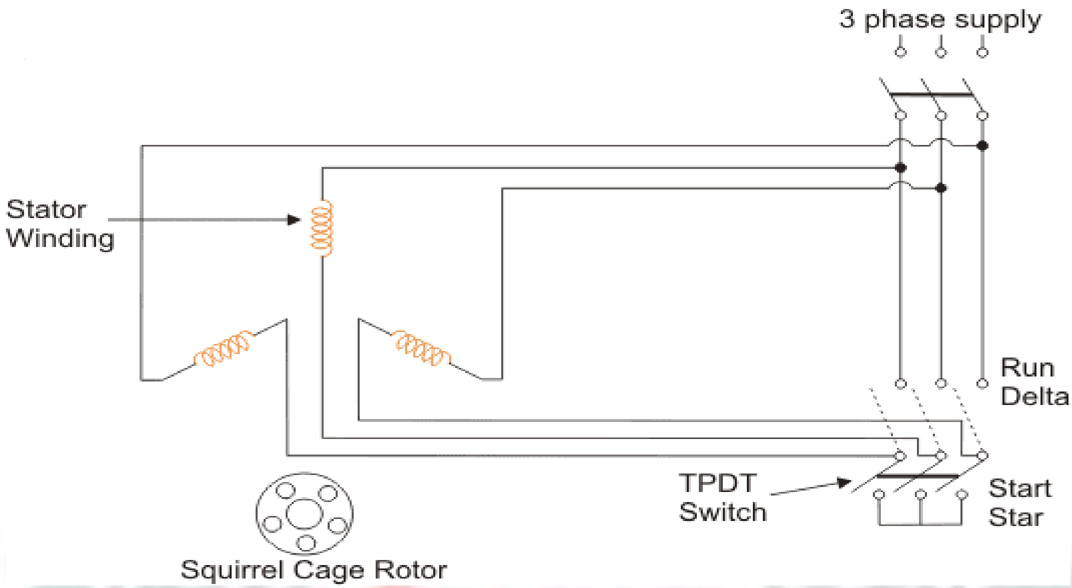
As we know, the equation for armature current in the motor. The value of back emf (E) depends upon speed (N), i.e. E is directly proportional to N.

At starting, the value of E is zero. So starting current is very high. In a small rating motor, the rotor has more considerable axial length and small diameter. So it gets accelerated fastly. Hence, speed increases and thus the value of armature current decreases rapidly. Therefore, small rating motors smoothly run when it is connected directly to a 3-phase supply. If we connect a large motor directly across 3-phase line, it would not run smoothly and will be damaged, because it does not get accelerated as fast as a smaller motor since it has short axial length and larger diameter more massive rotor. However, for large rated motors, we can use an oil immersed DOL starter.

6.5.1.1 Working of DOL Starter

A direct online starter consists of two buttons, a GREEN button for starting and a RED for stopping purpose ofthe motor. The **DOL starter** comprises of an MCCB or circuit breaker, contactor and an overload relay for protection. These two buttons, i.e. Green and Red or start and stop buttons control the contacts. To start the motor, we close the contact by pushing Green Button, and the full line voltage appears to the motor. A contactor can be of 3 poles or 4-poles. Below given contactor is of 4-pole type. It contains three NO (normally open) contacts that connect the motor to supply lines, and the fourth contact is “hold on contact” (auxiliary contact) which energizes the contactor coil after the start button is released. If any fault occurs, the auxiliary coil gets de-energized, and hence the starter disconnects the motor from supply mains.

6.5.2 Star-Delta Starting



A star delta starter will start a motor with a star connected stator winding. When motor reaches about 80% ofits full load speed, it will begin to run in a delta connected stator winding. **A s**tar delta starter is a type of reduced voltage starter. We use it to reduce the starting current of the motor without using any external device or apparatus. This is a big advantage of a star delta starter, as it typically has around 1/3 of the inrush current compared to a DOL starter. The starter mainly consists of a TPDP switch which stands for Tripple Pole Double Throw switch. This switch changes stator winding from star to delta. During starting condition stator winding is connected in the form of a star.

CHAPTER - 7

SINGLE PHASE INDUCTION MOTORS

7.1 INTRODUCTION

Single phase Induction motors perform a great variety of useful services at home, office, farm, factory and in business establishments. Single phase motors are generally manufactured in fractional HP ratings below 1 HP for economic reasons. Hence, those motors are generally referred to as fractional horsepower motors with a rating of less than 1 HP. Most single phase motors fall into this category. Single phase Induction motors are also manufactured in the range of 1.5, 2, 3 and up to 10 HP as a special requirement.

a) Stator **(b) Squirrel cage rotor**

1.1.1 Capacitor-Start, Induction-Run Motor

A drive which requires a large starting torque may be fitted with a capacitor-start, induction- run motor as it has excellence starting torque as compared to the resistance-start, induction-runmotor.

CONSTRUCTION AND WORKING

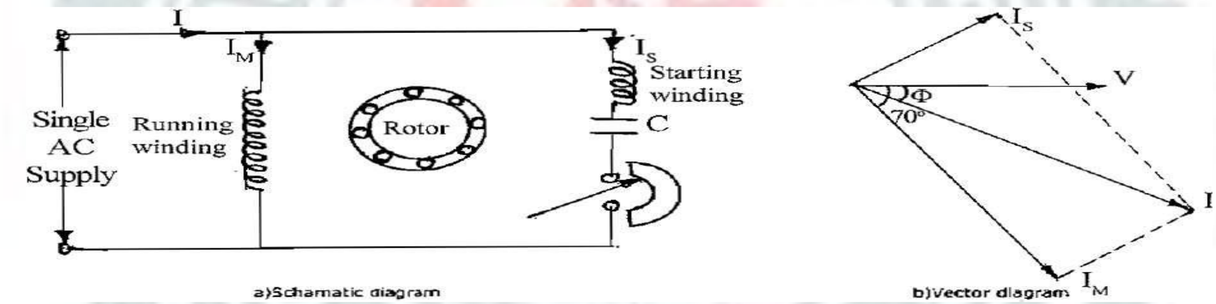
Figure below (a) shows the schematic diagram of a capacitor-start, induction-run motor. As shown, the main winding is directly connected across the main supply whereas the starting winding is connected across the main supply through a capacitor and centrifugal switch.

Both these windings are placed in a stator slot at 90 degree electrical apart, and a squirrel cage type rotor is used.

As shown in figure below (b), at the time of starting the current in the main winding lags the supply voltages by 90 degrees, depending upon its inductance and resistance. On the other hand, the current in the starting winding due to its capacitor will lead the applied voltage, by say 20 degrees.

Hence, the phase difference between the main and starting winding becomes near to 90 degrees. This in turn makes the line current to be more or less in phase with its applied voltage, making the power factor to be high, thereby creating an excellent starting torque.

However, after attaining 75% of the rated speed, the centrifugal switch operates opening the starting winding and the motor then operates as an induction motor, with only the main winding connected to the supply.



As shown in above figure, the displacement of current in the main and starting winding is about 80/90 degrees, and the power factor angle between the applied voltage and line current is very small. This results in producing a high power factor and an excellent starting torque, several times higher than the normal running torque.

APPLICATIONS

Due to the excellent starting torque and easy direction-reversal characteristics,

- Used in belted fans,
- Used in blowers dryers,
- Used in washing machines,Used in pumps and compressors

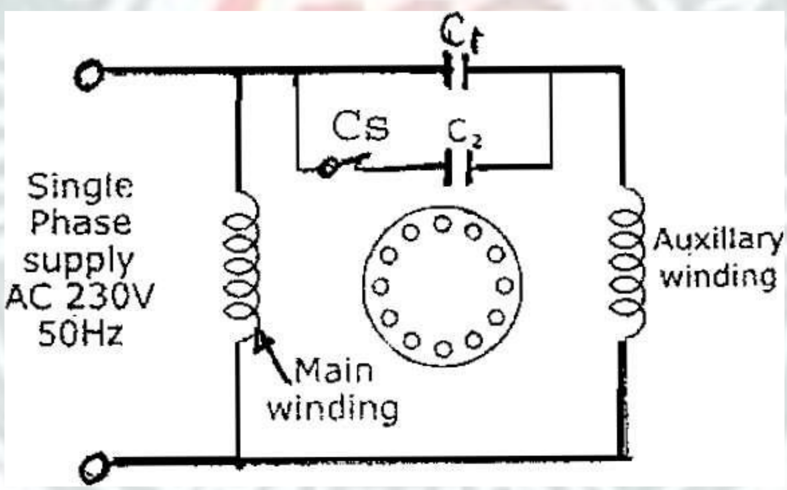
1.1.2 Capacitor-Start, Capacitor-Run Motors

As discussed earlier, one capacitor-start, induction-run motors have excellent starting torque, say about 300% of the full load torque and their power factor during starting in high.

However, their running torque is not good, and their power factor, while running is low. Theyalso have lesser efficiency and cannot take overloads.

CONSTRUCTION AND WORKING

The aforementioned problems are eliminated by the use ofa two valve capacitor motor in whichone large capacitor of electrolytic (short duty) type is used for starting whereas a smaller capacitor of oil filled (continuous duty) type is used for running, by connecting them with the starting winding as shown in figure below. A general view of such a two valve capacitor motor is shown in figure below.



This motor also works in the same way as a capacitor-start, induction-run motor, with exception, that the capacitor C1 is always in the circuit, altering the running performance to a great extent.

The starting capacitor which is of short duty rating will be disconnected from the starting winding with the help of a centrifugal switch, when the starting speed attains about 75% of the rated speed.

ADVANTAGES

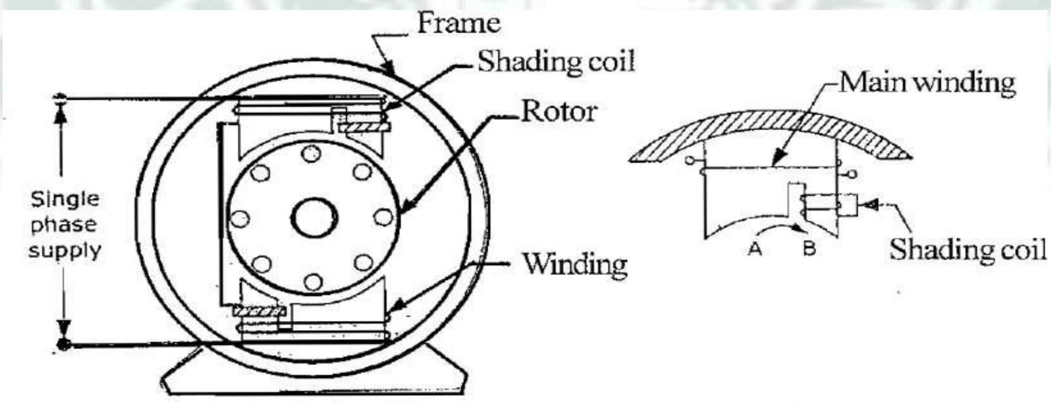
- The starting torque is 300% of the full load torque
- The starting current is low, say 2 to 3 times of the running current.
- Starting and running power factor are good.
- Highly efficient running.
- Extremely noiseless operation.
- Can be loaded upto 125% of the full load capacity.

APPLICATIONS

- Used for compressors, refrigerators, air-conditioners, etc.
- Higher starting torque.
- High efficiency, higher power factor and overloading.
- Costlier than the capacitor-start — Induction run motors of the same capacity.

7.1.3 Shaped Pole Starting

The motor consists of a yoke to which salient poles are fitted as shown in Fig: 4(a) and it has asquirrel cage type rotor.



(a) (b)

A shaded pole made of laminated sheets has a slot cut across the lamination at about one third the distance from the edge of the pole. Around the smaller portion of the pole, a short-circuited copper ring is placed which is called the shading coil, and this part of the pole is known as the shaded part of the pole.The remaining part of the pole is called the unshaded part which is clearly shown in above figure.

Around the poles, exciting coils are placed to which an AC supply is connected. When AC supply is effected to the exciting coil, the magnetic axis shifts from the unshaded part of the pole to the shaded part as will be explained in details in the next paragraph. This shifting of axis is equivalent to the physical movement of the pole.

This magnetic axis, which is moving, cuts the rotor conductors and hence, a rotational torque is developed in the rotor. By this torque the rotor starts rotating in the direction of the shifting of the magnetic axis that is from the unshaded part to the shaded part.

7.2 SINGLE PHASE SERIES MOTOR

The single-phase series motor is a commutator-type motor. If the polarity of the line terminals of a dc series motor is reversed, the motor will continue to run in the same direction. Thus, it might be expected that a dc series motor would operate on alternating current also. The direction of the torque developed in a dc series motor is determined by both filed polarity and the direction of current through the armature [T a ϕI_a].

7.2.1 Operation

Let a dc series motor be connected across a single-phase ac supply. Since the same current flows through the field winding and the armature, it follows that ac reversals from positive to negative, or from negative to positive, will simultaneously affect both the field flux polarity and the current direction through the armature. This means that the direction of the developed torque will remain positive, and rotation will continue in the same direction. Thus, a series motor can run both on dc and ac.

However, a series motor which is specifically designed for dc operation suffers from the following drawbacks when it is used on single-phase ac supply:

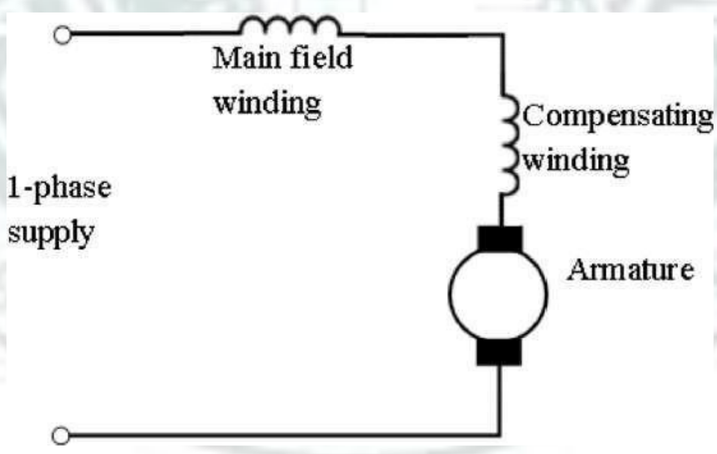
1. Its efficiency is low due to hysteresis and eddy-current losses.
2. The power factor is low due to the large reactance of the field and the armature winding.
3. The sparking at the brushes is excessive.

In order to overcome these difficulties, the following modifications are made in a D.C. series motor that is to operate satisfactorily on alternating current:

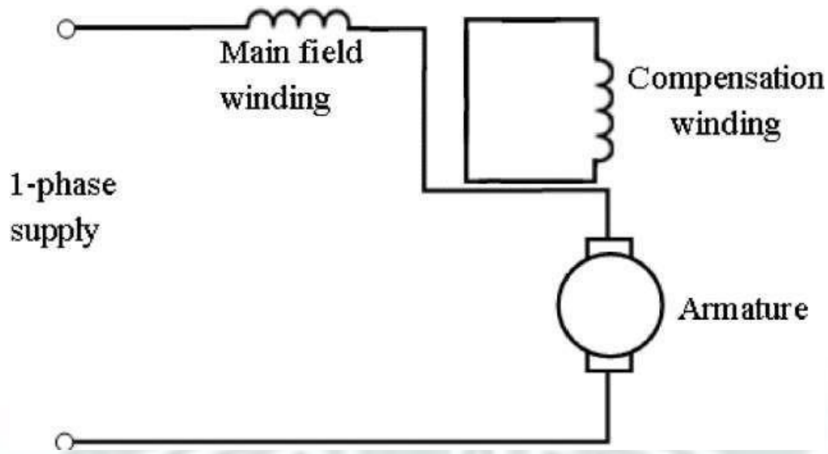
1. The field core is constructed of a material having low hysteresis loss. It is laminated to reduceeddy-current loss.
2. The field winding is provided with small number of turns. The field-pole areas is increased so thatthe flux density is reduced. This reduces the iron loss and the reactive voltage drop.
3. The number of armature conductors is increased in order to get the required torque with the lowflux.
4. In order to reduce the effect of armature reaction, thereby improving commutation and reducingarmature reactance, a compensating winding is used.

The compensating winding is put in the stator slots. The axis of the compensating winding is 90 (electrical) with the main field axis. It may be connected in series with both the armature and field as shown in figure (a). In such a case the motor is conductively compensated.

The compensating winding may be short circuited on itself, in which case the motor is said to beinductively compensated shown in figure (b).

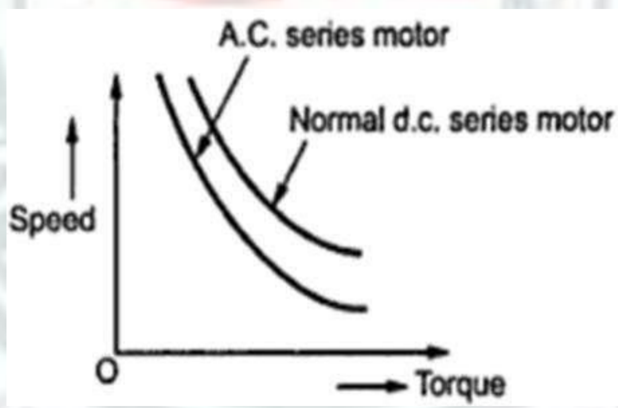


(a)



(b)

The characteristics of single-phase series motor are very much similar to those of D.C. series motors, but the series motor develops less torque when operating from an a.c. supply than when working from an equivalent D.C. supply (figure below) The direction of rotation can be changed by interchanging connections to the field with respect to the armature as in D.C. series motor.



Speed control of universal motors is best obtained by solid-state devices. Since the speed of these is not limited by the supply frequency and may be as high as 20,000 r.p.m. (greater than the maximum synchronous speed of 3000 r.p.m. at 50 Hz), they are most suitable for applications requiring high speeds.

CHAPER-8

ALTERNATOR

The **working principle of an alternator** is very simple. It is just like the basic principle of DC generator. It also depends upon Faraday's law of electromagnetic induction which says the current is induced in the conductor inside a magnetic field when there is a relative motion between that conductor and the magnetic field.

8.1 APPLICATION

Electric generator: Most power generation stations use synchronous machines as their generators. Connection of these generators to the utility grid requires synchronization conditions to be met.

Automotive alternators: Alternators are used in modern automobiles to charge the battery and to power the electrical system when its engine is running.

Diesel electric locomotive alternators: The traction alternator usually incorporates integral silicon diode rectifiers to provide the traction motors with up to 1200 volts DC (DC traction, which is used directly) or thecommon inverter bus (AC traction, which is first inverted from dc to three-phase ac). The first diesel electric locomotives, and many of those still in service, use DC generators as, before silicon power electronics, it was easier to control the speed of DC traction motors. Most of these had two generators: one to generate the excitation current for a larger main generator.

Marine alternators: Marine alternators used in yachts are similar to automotive alternators, with appropriate adaptations to the salt-water environment. Marine alternators are designed to be explosion proof so that brush sparking will not ignite explosive gas mixtures in an engine room environment. They may be 12 or 24 volt depending on the type of system installed.

Radio alternators: High frequency alternators of the variable-reluctance type were applied commercially to radio transmissionin the low-frequency radio bands.