



GANDHI SCHOOL OF ENGINEERING

Brahmapur, Odisha

E-Notes

Basic Electrical Engineering

Complete E-Notes

B.Tech 1st Year · Semester 1 · Circuit Branch

Academic Year 2026-27



How to use these notes

These notes follow the official 2026-27 syllabus for Basic Electrical Engineering (2-0-0, 2 credits, 30 contact hours, 100 university marks + 50 internal). Each module has these parts:

- **Theory and derivations** in exam language, with circuit diagrams, phasor diagrams and characteristic curves.
- **Formula boxes** with every formula you must remember.
- **Solved numericals** worked step by step, with units.
- **Practice problems with solutions.**
- **MCQs** with answers and one-line reasons.

At the end: 2-mark questions with answers, a long-question bank, a model paper with solutions and a formula sheet. The Basic Electrical Engineering Laboratory has a separate lab manual.

Module	Topic	Hours
I	DC networks: Kirchhoff's laws, nodal and mesh analysis, star-delta conversion, superposition, Thevenin, Norton, maximum power transfer	6
II	Single-phase and three-phase AC circuits: average and rms values, R-L-C series and parallel circuits, resonance, star and delta connections, three-phase power measurement	6
III	Magnetic circuits and DC machines: laws of electromagnetic induction, magnetic circuits, DC machine construction, types, emf and torque equations, speed control of DC motors	6
IV	AC machines: single-phase transformer, three-phase induction motor, three-phase alternator	6
V	Introduction to power systems: generation, transmission and distribution, sources of electrical power	6

Reference books (from syllabus)

- G. Rizzoni, Principles and Applications of Electrical Engineering, TMH.
- I.J. Nagrath and D.P. Kothari, Basic Electrical Engineering, Tata McGraw Hill.
- S. Parker Smith, Problems in Electrical Engineering, Asia Publications.
- Edward Hughes, Electrical and Electronics Technology, Pearson.



Units and symbols used

Quantity	Symbol	Unit
Current	I, i	ampere (A)
Voltage, emf	V, E	volt (V)
Resistance, reactance, impedance	R, X, Z	ohm (Ω)
Conductance, admittance	G, Y	siemens (S)
Inductance	L	henry (H)
Capacitance	C	farad (F)
Active power	P	watt (W)
Reactive power	Q	volt-ampere reactive (VAR)
Apparent power	S	volt-ampere (VA)
Magnetic flux	Φ	weber (Wb)
Flux density	B	tesla (T) = Wb/m^2
Magnetising force	H	ampere-turn per metre (AT/m)
Reluctance	S	AT/Wb



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Basic Electrical Engineering

Subject: Basic Electrical Engineering · Semester 1 (Circuit branch) · 2-0-0 · 2 credits · 30 hours · University
100 + Internal 50

Module I: DC Networks

A network is any interconnection of circuit elements. This module develops systematic methods for finding currents and voltages in DC networks: Kirchhoff's laws, nodal and mesh analysis, source transformations and network theorems.

1.1 Basic definitions

- **Electric current:** rate of flow of charge, $I = dq/dt$ (ampere). Conventional current flows from + to – outside a source.
- **Potential difference (voltage):** work done per unit charge in moving it between two points, $V = W/Q$ (volt).
- **Ohm's law:** at constant temperature, the current through a conductor is proportional to the voltage across it: $V = IR$. Resistance $R = \rho l/A$ (ρ = resistivity).
- **Power and energy:** $P = VI = I^2R = V^2/R$ (watt); energy $W = Pt$ (joule or kWh; 1 kWh = 3.6×10^6 J = 1 "unit").
- **Active element:** supplies energy (voltage or current source). **Passive element:** receives energy (R, L, C).
- **Linear element:** V-I relation is a straight line (R, L, C). **Bilateral element:** same behaviour for current in either direction (resistor).
- **Node:** a point where two or more elements join. **Junction (principal node):** a node where three or more branches meet. **Branch:** a single path between two nodes. **Loop:** any closed path. **Mesh:** a loop that contains no other loop inside it.

Ideal and practical sources.

Ideal voltage source	Ideal current source
Maintains constant terminal voltage whatever the current	Maintains constant current whatever the voltage
Zero internal resistance	Infinite internal resistance
Practical: E in series with internal resistance r	Practical: I in parallel with internal resistance r

Source transformation. A practical voltage source (E in series with r) is equivalent, at its terminals, to a practical current source ($I = E/r$ in parallel with r), and vice versa. This is useful for simplifying circuits.

Series and parallel resistors.

Series: $R_{eq} = R_1 + R_2 + \dots$ (same current; voltage divides: $V_1 = V R_1/(R_1 + R_2)$)
 Parallel: $1/R_{eq} = 1/R_1 + 1/R_2 + \dots$; two resistors: $R_{eq} = R_1 R_2/(R_1 + R_2)$ (same voltage; current divides: $I_1 = I R_2/(R_1 + R_2)$)

1.2 Kirchhoff's laws

Kirchhoff's current law (KCL). The algebraic sum of currents meeting at a node is zero: $\sum I = 0$. Equivalently, total current entering = total current leaving. KCL expresses **conservation of charge** (charge cannot accumulate at a node).

Kirchhoff's voltage law (KVL). The algebraic sum of all voltages (emfs and voltage drops) around any closed loop is zero: $\sum V = 0$, or $\sum \text{emf} = \sum IR$. KVL expresses **conservation of energy** (a charge taken round a closed path returns to the same potential).

Sign convention for KVL. Going round a loop: a rise in potential (– to + through a source, or against the current through a resistor) is positive; a fall (+ to – through a source, or along the current through a resistor) is negative. Choose a direction for each unknown current; if the answer comes out negative, the actual current is opposite to the assumed direction.

1.3 Mesh (loop) current method

Idea. Assign a circulating current to each mesh (usually clockwise). The actual branch current in a branch shared by two meshes is the algebraic difference of the two mesh currents. Apply KVL to each mesh.

Steps.

- Identify the meshes (for a planar network, number of meshes = $b - n + 1$, where b = branches, n = nodes).
- Assign mesh currents $I_1, I_2, \dots$ clockwise.
- Write KVL for each mesh: (sum of resistances in the mesh) $\times$ (own mesh current) – (shared resistance) $\times$ (neighbouring mesh current) = (algebraic sum of source emfs in the mesh, taken positive if they drive current in the assumed direction).
- Solve the simultaneous equations (by elimination, Cramer's rule or Gauss elimination; see Mathematics-I, Module IV).

Matrix form (by inspection) for a network with only voltage sources:

$$[R_{11} \quad -R_{12}; -R_{21} \quad R_{22}] [I_1; I_2] = [E_1; E_2]$$

R_{11}, R_{22} = self-resistance of each mesh; $R_{12} = R_{21}$ = mutual (shared) resistance; E_1, E_2 = net emf in each mesh.

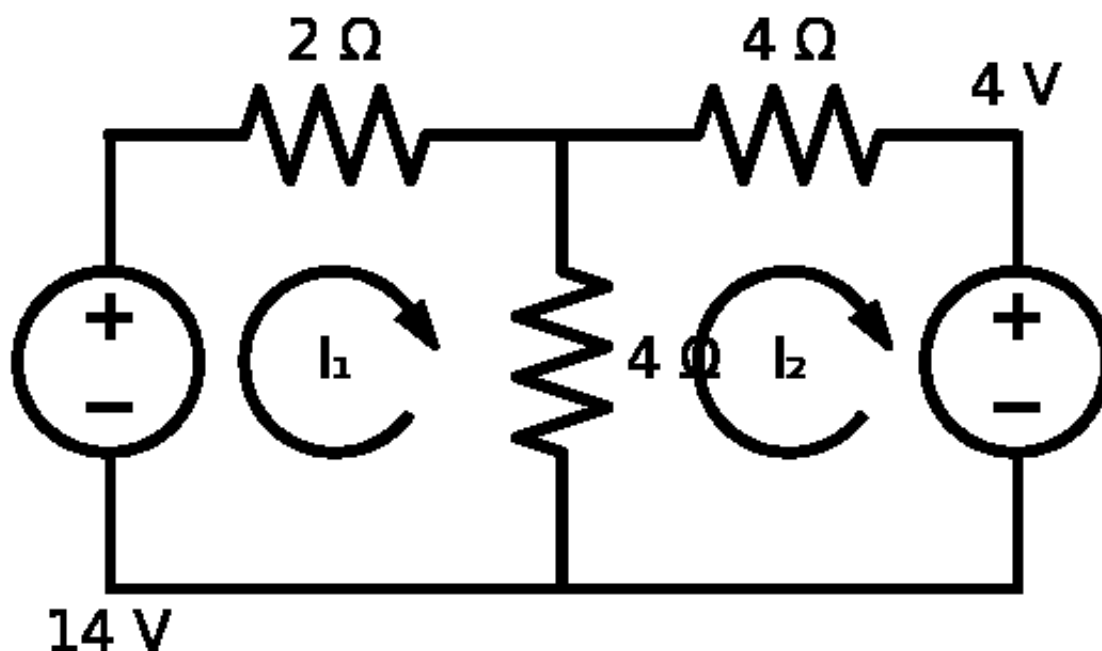


Figure 1.1: A two-mesh network with clockwise mesh currents I_1 and I_2 (solved in Problem 2 of Section 1.12).

Supermesh. If a current source lies in a branch common to two meshes, combine the two meshes into a supermesh: write KVL around the outer loop, and add the constraint equation $I_1 - I_2 = I_s$ given by the current source.

1.4 Node voltage (nodal) method

Idea. Choose one node as reference (ground, 0 V). Take the voltages of the other nodes as unknowns. Apply KCL at each non-reference node, writing each branch current in terms of node voltages ($I = (V_a - V_b)/R$).

Steps.

- Number of equations = $n - 1$ (n = number of principal nodes).
- At each node, write Σ (currents leaving through resistors) = Σ (source currents entering).
- Solve for node voltages, then find branch currents.

Matrix form (by inspection) with conductances $G = 1/R$:

$$[G_{11} \ -G_{12}; \ -G_{21} \ G_{22}][V_1; V_2] = [I_1; I_2]$$

G_{11} = sum of conductances connected to node 1, G_{12} = conductance between nodes 1 and 2, I_1 = net source current into node 1.

Supernode. If a voltage source connects two non-reference nodes, enclose it (and its nodes) in a supernode: apply KCL to the supernode as a whole and use the source to relate the two node voltages.

Which method to choose? Use the one with fewer equations: mesh analysis needs $b - n + 1$ equations; nodal analysis needs $n - 1$. Nodal analysis is preferred for circuits with many parallel branches and current sources; mesh analysis for circuits with many series elements and voltage sources.

1.5 Star-delta and delta-star transformation

Some networks (e.g. a bridge) contain resistors that are neither in series nor in parallel. They can be simplified by converting three resistors connected in delta (Δ , or mesh) to an equivalent star (Y), or vice versa. The two are equivalent if the resistance between any pair of terminals is the same.

Let the delta resistors be R_{AB} , R_{BC} , R_{CA} and the star resistors R_A , R_B , R_C (connected from the common point to A, B, C).

Delta $\rightarrow$ Star: $R_A = R_{AB}R_{CA}/(R_{AB} + R_{BC} + R_{CA})$, $R_B = R_{AB}R_{BC}/\Sigma$, $R_C = R_{BC}R_{CA}/\Sigma$
Star $\rightarrow$ Delta: $R_{AB} = R_A + R_B + R_A R_B/R_C$, $R_{BC} = R_B + R_C + R_B R_C/R_A$, $R_{CA} = R_C + R_A + R_C R_A/R_B$

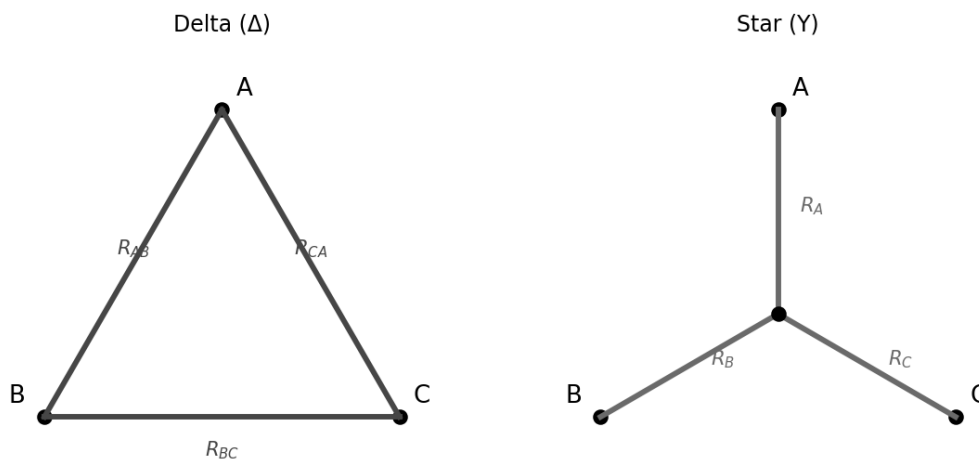


Figure 1.2: Delta and equivalent star connection of three resistors.

Memory aid. Delta $\rightarrow$ star: each star arm = (product of the two delta resistors touching that terminal)/(sum of all three). Star $\rightarrow$ delta: each delta resistor = (sum of the two star arms it joins) + (their product)/(the third arm).

Balanced case. If all delta resistors equal R_Δ , then $R_Y = R_\Delta/3$; conversely $R_\Delta = 3R_Y$.

Derivation of delta $\rightarrow$ star. Resistance between A and B in delta: $R_{AB} \parallel (R_{BC} + R_{CA}) = R_{AB}(R_{BC} + R_{CA})/\Sigma$. In star: $R_A + R_B$. Writing the corresponding equations for BC and CA, adding two and subtracting the third gives the formulas above.

1.6 Superposition theorem

Statement. In a linear, bilateral network containing more than one independent source, the current through (or voltage across) any element is the algebraic sum of the currents (or voltages) produced by each source acting alone, with all other independent sources replaced by their internal resistances.

How to "remove" a source: replace an ideal voltage source by a **short circuit** and an ideal current source by an **open circuit** (keep any internal resistance).

Steps.

- Keep one source, remove the others; find the required current (with direction).
- Repeat for each source.
- Add the individual currents algebraically (taking directions into account).

Limitations. Applies only to linear networks. It cannot be used directly for power, because power is proportional to I^2 (non-linear): $(I_1 + I_2)^2 \neq I_1^2 + I_2^2$. Dependent sources

are never removed.

1.7 Thevenin's theorem

Statement. Any linear two-terminal network containing sources and resistances can be replaced, as seen from its terminals A-B, by an equivalent circuit consisting of a single voltage source V_{th} in series with a single resistance R_{th} .

- V_{th} (Thevenin voltage) = open-circuit voltage across A-B (load removed).
- R_{th} (Thevenin resistance) = resistance seen from A-B with all independent sources replaced by their internal resistances (voltage sources shorted, current sources opened).

$$\text{Load current } I_L = V_{th} / (R_{th} + R_L)$$

Steps.

- Remove the load resistor R_L and mark the terminals A, B.
- Find the open-circuit voltage $V_{AB} = V_{th}$ using any method (KVL, voltage divider, nodal).
- Replace sources by internal resistances and find R_{th} between A and B.
- Draw the Thevenin equivalent, reconnect R_L and find I_L .

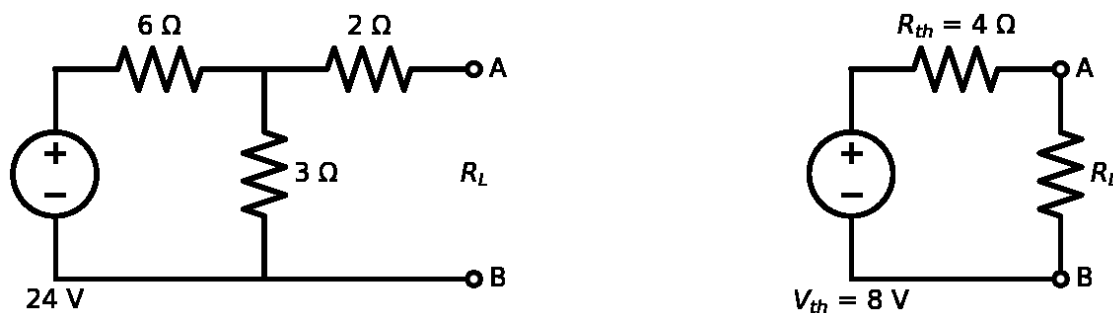


Figure 1.3: A network (left) and its Thevenin equivalent seen from terminals A-B (right).

Advantage. When the load changes (e.g. finding current for several values of R_L), the rest of the network need not be solved again.

1.8 Norton's theorem

Statement. Any linear two-terminal network can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistance R_N .

- I_N = short-circuit current through A-B (load replaced by a short).
- $R_N = R_{th}$ (found the same way).

$$\text{Load current } I_L = I_N R_N / (R_N + R_L)$$

Relation between Thevenin and Norton. They are source transformations of each other:

$$V_{th} = I_N R_N, R_{th} = R_N = V_{th} / I_N = V_{oc} / I_{sc}$$

1.9 Maximum power transfer theorem

Statement. A load resistance R_L receives maximum power from a DC network when R_L equals the Thevenin (internal) resistance of the network as seen from the load terminals:

$$R_L = R_{th}$$

Proof. $P_L = I_L^2 R_L = V_{th}^2 R_L / (R_{th} + R_L)^2$. Differentiate with respect to R_L and set to zero: $(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L) = 0 \Rightarrow R_{th} + R_L = 2R_L \Rightarrow R_L = R_{th}$. The second derivative is negative, so this is a maximum.

$$P_{max} = V_{th}^2 / (4R_{th}) \text{ Efficiency at maximum power transfer} = 50\%$$

Remarks. Half of the power is wasted in R_{th} , so maximum power transfer is used in communication and electronic circuits (e.g. matching an amplifier to a loudspeaker or an antenna), where the signal power is small. Power systems are not operated at maximum power transfer because efficiency would be only 50%. For AC circuits the condition becomes $Z_L = Z_{th}^*$ (complex conjugate).

1.10 Application summary: which method to use?

Situation	Best method
Current in one particular branch, especially for several load values	Thevenin or Norton
Several sources, current in one element	Superposition (or Thevenin)
All branch currents, few meshes, voltage sources	Mesh analysis
All node voltages, many parallel branches, current sources	Nodal analysis
Bridge or lattice network that is not series-parallel	Star-delta transformation
Value of load for maximum power	Maximum power transfer theorem

1.11 Common mistakes to avoid in Module I

- Forgetting to take the shared resistance with a negative sign in mesh equations.
- Shorting a current source (it must be opened) or opening a voltage source (it must be shorted) when applying superposition or finding R_{th} .
- Applying superposition to power.
- Finding R_{th} with the load still connected.
- Mixing up delta→star (divide by the sum) and star→delta (divide by the opposite arm).

1.12 Worked method: solving a network by all four methods

To see how the methods compare, consider the network of Figure 1.3 (24 V source, 6 Ω series, 3 Ω shunt, 2 Ω to terminal A) with a 4 Ω load across A-B.

(a) Series-parallel reduction. Load branch = 2 + 4 = 6 Ω, in parallel with 3 Ω: $6 \times 3 / 9 = 2$ Ω. Total = 6 + 2 = 8 Ω. Source current = $24 / 8 = 3$ A. Voltage across the 3 Ω = $3 \times 2 = 6$ V. Load current = $6 / 6 = 1$ A.

(b) Mesh analysis. Mesh 1 (source, 6 Ω, 3 Ω): $9I_1 - 3I_2 = 24$. Mesh 2 (3 Ω, 2 Ω, 4 Ω): $-3I_1 + 9I_2 = 0 \Rightarrow I_1 = 3I_2$. Then $27I_2 - 3I_2 = 24 \Rightarrow I_2 = 1$ A (load current), $I_1 = 3$ A.

(c) Nodal analysis. Let V be the voltage at the top of the 3 Ω resistor. $(V - 24) / 6 + V / 3 + V / 6 = 0 \Rightarrow V(1/6 + 1/3 + 1/6) = 4 \Rightarrow V = 6$ V. Load current = $6 / 6 = 1$ A.

(d) Thevenin. $V_{th} = 8$ V, $R_{th} = 4$ Ω $\Rightarrow I_L = 8 / (4 + 4) = 1$ A.

All four methods give the same answer, which is a good check. Thevenin's method is fastest if the load changes; nodal analysis needs only one equation here.

1.13 Derivation of star-delta formulas (complete)

Resistance between A and B must be equal in both networks (with C left open):

- Star: $R_A + R_B$. Delta: $R_{AB} \parallel (R_{BC} + R_{CA}) = R_{AB}(R_{BC} + R_{CA})/\Sigma$, where $\Sigma = R_{AB} + R_{BC} + R_{CA}$.

So (1) $R_A + R_B = R_{AB}(R_{BC} + R_{CA})/\Sigma$. Similarly (2) $R_B + R_C = R_{BC}(R_{CA} + R_{AB})/\Sigma$ and (3) $R_C + R_A = R_{CA}(R_{AB} + R_{BC})/\Sigma$.

(1) - (2) + (3): $2R_A = [R_{AB}R_{BC} + R_{AB}R_{CA} - R_{BC}R_{CA} - R_{BC}R_{AB} + R_{CA}R_{AB} + R_{CA}R_{BC}]/\Sigma = 2R_{AB}R_{CA}/\Sigma$. Hence $R_A = R_{AB}R_{CA}/\Sigma$, and similarly for R_B and R_C .

Star→delta. From the delta→star results: $R_A R_B + R_B R_C + R_C R_A = R_{AB} R_{BC} R_{CA}/\Sigma$. Dividing by $R_C = R_{BC} R_{CA}/\Sigma$ gives $R_{AB} = (R_A R_B + R_B R_C + R_C R_A)/R_C = R_A + R_B + R_A R_B/R_C$.

1.14 Proof of Thevenin's theorem (outline)

Connect a current source I (equal to the load current) in place of the load. By superposition, the terminal voltage V is the sum of (i) the voltage due to all internal sources with I removed (open circuit), which is $V_{oc} = V_{th}$, and (ii) the voltage due to I alone with the internal sources removed, which is $-IR_{th}$. So $V = V_{th} - IR_{th}$, which is exactly the terminal equation of a source V_{th} in series with R_{th} . This proof also shows why the network must be linear.

1.15 Reciprocity and other network theorems (for awareness)

- **Reciprocity theorem:** in a linear bilateral network with a single source, the ratio of the source voltage in one branch to the current in another branch is unchanged if the positions of the source and the ammeter are interchanged.
- **Millman's theorem:** several voltage sources E_k with series resistances R_k connected in parallel can be replaced by a single source $E = (\Sigma E_k/R_k)/(\Sigma 1/R_k)$ with $R = 1/(\Sigma 1/R_k)$.
- **Tellegen's theorem:** in any network, the sum of powers delivered to all branches is zero ($\Sigma v_k i_k = 0$), an expression of energy conservation.
- **Compensation theorem:** any resistance R carrying current I can be replaced by a voltage source IR ; used to find the effect of a small change in a resistance.

1.16 Temperature effect on resistance and practical notes

- $R_t = R_0(1 + \alpha_0 t)$, where α_0 is the temperature coefficient of resistance at 0 °C (copper $\alpha_0 \approx 0.00427/^\circ\text{C}$). Metals have positive α ; carbon, semiconductors and insulators have negative α ; alloys such as manganin and constantan have nearly zero α (used for standard resistors).
- Colour code of resistors: black 0, brown 1, red 2, orange 3, yellow 4, green 5, blue 6, violet 7, grey 8, white 9; gold $\pm 5\%$, silver $\pm 10\%$ tolerance. Example: yellow-violet-red-gold = $47 \times 10^2 \Omega = 4.7 \text{ k}\Omega \pm 5\%$.
- Measuring instruments: an ammeter is connected in series and should have very low resistance; a voltmeter is connected in parallel and should have very high resistance. (See BEE Lab, Experiment 1.)

1.17 Solved numericals: Module I

Problem 1. Three resistors of $2\ \Omega$, $3\ \Omega$ and $6\ \Omega$ are connected in parallel, and the combination is in series with a $4\ \Omega$ resistor across a $12\ \text{V}$ supply. Find the total current and the current in the $6\ \Omega$ resistor.

Parallel: $1/R_p = 1/2 + 1/3 + 1/6 = 1 \Rightarrow R_p = 1\ \Omega$. Total $R = 1 + 4 = 5\ \Omega$.

$I = 12/5 = 2.4\ \text{A}$. Voltage across the parallel group $= 2.4 \times 1 = 2.4\ \text{V}$.

Current in $6\ \Omega = 2.4/6 = 0.4\ \text{A}$.

Answer: $I = 2.4\ \text{A}$; $I_{6\Omega} = 0.4\ \text{A}$.

Problem 2. In a two-mesh circuit, mesh 1 contains a $14\ \text{V}$ source and a $2\ \Omega$ resistor; mesh 2 contains a $4\ \text{V}$ source (opposing the clockwise direction) and a $4\ \Omega$ resistor; the two meshes share a $4\ \Omega$ resistor. Find the mesh currents and the current in the shared resistor.

Mesh 1 (clockwise I_1): $(2 + 4)I_1 - 4I_2 = 14 \Rightarrow 6I_1 - 4I_2 = 14$.

Mesh 2 (clockwise I_2): $-4I_1 + (4 + 4)I_2 = -4 \Rightarrow -4I_1 + 8I_2 = -4$.

From the second, $I_1 = 2I_2 + 1$. Substituting: $6(2I_2 + 1) - 4I_2 = 14 \Rightarrow 8I_2 = 8 \Rightarrow I_2 = 1\ \text{A}$, $I_1 = 3\ \text{A}$.

Current in the shared $4\ \Omega$ resistor $= I_1 - I_2 = 2\ \text{A}$ (in the direction of I_1).

Answer: $I_1 = 3\ \text{A}$, $I_2 = 1\ \text{A}$, shared branch $2\ \text{A}$.

Problem 3. A three-mesh ladder network has $5\ \Omega$ resistors in each mesh-only branch and $5\ \Omega$ resistors in the two shared branches. Sources: $20\ \text{V}$ in mesh 1 and $10\ \text{V}$ in mesh 2 (both aiding clockwise current). Find the mesh currents.

Mesh 1: $10I_1 - 5I_2 = 20$. Mesh 2: $-5I_1 + 15I_2 - 5I_3 = 10$. Mesh 3: $-5I_2 + 10I_3 = 0$.

From mesh 3: $I_2 = 2I_3$. From mesh 1: $I_1 = 2 + I_2/2 = 2 + I_3$.

Mesh 2: $-5(2 + I_3) + 30I_3 - 5I_3 = 10 \Rightarrow 20I_3 = 20 \Rightarrow I_3 = 1\ \text{A}$, $I_2 = 2\ \text{A}$, $I_1 = 3\ \text{A}$.

Answer: $I_1 = 3\ \text{A}$, $I_2 = 2\ \text{A}$, $I_3 = 1\ \text{A}$. (Check with Gauss elimination, Mathematics-I Module IV.)

Problem 4. Using nodal analysis, find V_1 and V_2 : node 1 is connected to a $20\ \text{V}$ source through $4\ \Omega$, to ground through $8\ \Omega$ and to node 2 through $2\ \Omega$; node 2 is connected to ground through $1\ \Omega$ and receives $2\ \text{A}$ from a current source.

KCL at node 1: $(V_1 - 20)/4 + V_1/8 + (V_1 - V_2)/2 = 0 \Rightarrow$ multiply by 8: $2V_1 - 40 + V_1 + 4V_1 - 4V_2 = 0 \Rightarrow 7V_1 - 4V_2 = 40$.

KCL at node 2: $(V_2 - V_1)/2 + V_2/1 = 2 \Rightarrow -V_1 + 3V_2 = 4$.

From the second, $V_1 = 3V_2 - 4$. Then $7(3V_2 - 4) - 4V_2 = 40 \Rightarrow 17V_2 = 68 \Rightarrow V_2 = 4\ \text{V}$, $V_1 = 8\ \text{V}$.

Answer: $V_1 = 8\ \text{V}$, $V_2 = 4\ \text{V}$. (Current from source $= (20 - 8)/4 = 3\ \text{A}$.)

Problem 5. Convert a delta network with $R_{AB} = 10\ \Omega$, $R_{BC} = 20\ \Omega$, $R_{CA} = 30\ \Omega$ into an equivalent star.

$\Sigma = 60\ \Omega$. $R_A = R_{AB}R_{CA}/\Sigma = 300/60 = 5\ \Omega$.

$R_B = R_{AB}R_{BC}/\Sigma = 200/60 = 3.33\ \Omega$. $R_C = R_{BC}R_{CA}/\Sigma = 600/60 = 10\ \Omega$.



Answer: $R_A = 5 \Omega$, $R_B = 3.33 \Omega$, $R_C = 10 \Omega$.

Problem 6. Convert a star network with $R_A = 2 \Omega$, $R_B = 3 \Omega$, $R_C = 6 \Omega$ into an equivalent delta.

$$R_{AB} = 2 + 3 + (2 \times 3)/6 = 6 \Omega. R_{BC} = 3 + 6 + 18/2 = 18 \Omega. R_{CA} = 6 + 2 + 12/3 = 12 \Omega.$$

Answer: $R_{AB} = 6 \Omega$, $R_{BC} = 18 \Omega$, $R_{CA} = 12 \Omega$.

Problem 7. Find the equivalent resistance between A and B of a bridge in which $AC = 10 \Omega$, $AD = 20 \Omega$, $CD = 30 \Omega$ (the bridge arm), $CB = 15 \Omega$ and $DB = 5 \Omega$.

The bridge is not balanced ($10/20 \neq 15/5$), so convert the delta ACD ($AC = 10$, $AD = 20$, $CD = 30$) to star with centre O. $\Sigma = 60$.

$$R_{AO} = 10 \times 20/60 = 3.33 \Omega; R_{CO} = 10 \times 30/60 = 5 \Omega; R_{DO} = 20 \times 30/60 = 10 \Omega.$$

Now O-C-B = $5 + 15 = 20 \Omega$ and O-D-B = $10 + 5 = 15 \Omega$ are in parallel: $20 \times 15/35 = 8.57 \Omega$.

Answer: $R_{AB} = 3.33 + 8.57 = 11.9 \Omega$.

Problem 8. Using superposition, find the current in the 2Ω resistor of a circuit in which a 12 V source with 3Ω and a 6 V source with 6Ω feed a common node, and the 2Ω resistor connects that node to the common return.

12 V alone (6 V shorted): the 6Ω and 2Ω are in parallel = 1.5Ω ; total = 4.5Ω ; $I = 12/4.5 = 2.667 \text{ A}$. Current in $2 \Omega = 2.667 \times 6/8 = 2.0 \text{ A}$.

6 V alone (12 V shorted): $3 \parallel 2 = 1.2 \Omega$; total = 7.2Ω ; $I = 6/7.2 = 0.833 \text{ A}$. Current in $2 \Omega = 0.833 \times 3/5 = 0.5 \text{ A}$.

Both currents flow in the same direction through the 2Ω resistor.

Answer: $I = 2.0 + 0.5 = 2.5 \text{ A}$. (Check by nodal: $V = (12/3 + 6/6)/(1/3 + 1/6 + 1/2) = 5 \text{ V}$; $I = 5/2 = 2.5 \text{ A}$ □.)

Problem 9. A 24 V source with a series 6Ω resistor feeds a 3Ω shunt resistor; a 2Ω resistor connects the top of the 3Ω resistor to terminal A (B is the common return). Find the Thevenin equivalent and the load current for $R_L = 2, 4$ and 8Ω .

V_{th} (open-circuit): no current in 2Ω , so $V_{th} =$ voltage across $3 \Omega = 24 \times 3/(6 + 3) = 8 \text{ V}$.

R_{th} : short the source: $(6 \parallel 3) + 2 = 2 + 2 = 4 \Omega$.

$I_L = 8/(4 + R_L)$: $R_L = 2 \Omega$: 1.33 A ; $R_L = 4 \Omega$: 1.0 A ; $R_L = 8 \Omega$: 0.67 A .

Answer: $V_{th} = 8 \text{ V}$, $R_{th} = 4 \Omega$; $I_L = 1.33 \text{ A}$, 1.0 A , 0.67 A .

Problem 10. For the network of the previous problem, find the Norton equivalent.

$I_N = V_{th}/R_{th} = 8/4 = 2 \text{ A}$ (or directly: short A-B; $2 \parallel 3 = 1.2 \Omega$, total 7.2Ω , $I = 3.33 \text{ A}$; current into short = $3.33 \times 3/5 = 2 \text{ A}$ □).

Answer: $I_N = 2 \text{ A}$ in parallel with $R_N = 4 \Omega$.

Problem 11. For the same network, find the value of R_L for maximum power transfer and the maximum power.

$$R_L = R_{th} = 4 \Omega. P_{max} = V_{th}^2 / (4R_{th}) = 64/16 = 4 \text{ W}.$$

Answer: $R_L = 4 \Omega$; $P_{max} = 4 \text{ W}$ (efficiency 50%).

Problem 12. A battery of emf 12 V and internal resistance 0.5Ω supplies a load. Find the load resistance for maximum power and the maximum power.

$$R_L = r = 0.5 \Omega. P_{max} = E^2 / (4r) = 144/2 = 72 \text{ W}.$$

Answer: 0.5Ω ; 72 W .

Problem 13. A current source of 10 A with an internal resistance of 5Ω is connected to a 20Ω load. Convert it to an equivalent voltage source and find the load current.

$$E = IR = 10 \times 5 = 50 \text{ V in series with } 5 \Omega.$$

$$I_L = 50 / (5 + 20) = 2 \text{ A. (Check by current division: } 10 \times 5/25 = 2 \text{ A } \square.)$$

Answer: 50 V in series with 5Ω ; $I_L = 2 \text{ A}$.

Problem 14. In a Wheatstone bridge, $P = 10 \Omega$, $Q = 20 \Omega$, $R = 15 \Omega$, $S = 30 \Omega$, and the supply is 6 V. What is the current through the galvanometer?

$$P/Q = 10/20 = 0.5 \text{ and } R/S = 15/30 = 0.5. \text{ The bridge is balanced.}$$

Answer: 0 A (no current through the galvanometer).

Problem 15. Two batteries, 10 V (internal resistance 1Ω) and 8 V (internal resistance 2Ω), are connected in parallel (like terminals together) across a 5Ω load. Find the load current using Millman's method (nodal).

$$V = (10/1 + 8/2) / (1/1 + 1/2 + 1/5) = 14/1.7 = 8.235 \text{ V}.$$

$$I_L = 8.235/5 = 1.647 \text{ A. Battery currents: } (10 - 8.235)/1 = 1.765 \text{ A (discharging); } (8 - 8.235)/2 = -0.118 \text{ A (the 8 V battery is being charged).}$$

Answer: $I_L \approx 1.65 \text{ A}$.

1.18 More solved numericals: Module I

Problem 1. A copper coil has a resistance of 20Ω at 20°C . Find its resistance at 70°C ($\alpha_{20} = 0.00393/^\circ\text{C}$).

$$R_{70} = R_{20}[1 + \alpha_{20}(70 - 20)] = 20(1 + 0.1965).$$

Answer: $R_{70} \approx 23.93 \Omega$.

Problem 2. Find the current through each resistor in a circuit where a 30 V source feeds 5Ω in series with a parallel combination of 10Ω and $(5 \Omega + 5 \Omega)$.

$$10 \parallel 10 = 5 \Omega. \text{ Total} = 10 \Omega. I = 3 \text{ A}.$$

The current divides equally between the $10\ \Omega$ branch and the $5 + 5\ \Omega$ branch: 1.5 A each.

Answer: 3 A in the $5\ \Omega$ series resistor; 1.5 A in each parallel branch.

Problem 3. Using nodal analysis, find the voltage V of a node fed by a 10 A current source, with a $5\ \Omega$ resistor to ground and a $10\ \Omega$ resistor to the positive terminal of a 20 V source (whose negative terminal is grounded).

Current leaving through $5\ \Omega$: $V/5$. Current leaving through the $10\ \Omega$ -source branch: $(V - 20)/10$ (the branch connects the node to a point at +20 V).

KCL: $V/5 + (V - 20)/10 = 10 \Rightarrow 2V + V - 20 = 100 \Rightarrow V = 40\text{ V}$.

Answer: $V = 40\text{ V}$.

Problem 4. Find the equivalent resistance between A and B of a network of five $10\ \Omega$ resistors arranged as a balanced Wheatstone bridge (four arms and a middle link).

All arms equal $\Rightarrow$ bridge balanced $\Rightarrow$ no current in the middle link, which can be removed.

Two paths of $20\ \Omega$ each in parallel.

Answer: $R_{AB} = 10\ \Omega$.

Problem 5. A network has three branches in parallel between nodes A and B: 10 V with $2\ \Omega$, 20 V with $4\ \Omega$, and a $4\ \Omega$ load. Using Millman's theorem, find the load current.

$E = (10/2 + 20/4)/(1/2 + 1/4) = 10/0.75 = 13.33\text{ V}$; $R = 1/0.75 = 1.333\ \Omega$.

$I_L = 13.33/(1.333 + 4) = 2.5\text{ A}$.

Answer: $I_L = 2.5\text{ A}$.

Problem 6. A Thevenin circuit has $V_{th} = 20\text{ V}$ and $R_{th} = 5\ \Omega$. Plot (tabulate) the power in the load for $R_L = 1, 2.5, 5, 10, 20\ \Omega$ and comment.

$P = 400R_L/(5 + R_L)^2$.

$R_L = 1: 11.1\text{ W}; 2.5: 17.8\text{ W}; 5: 20\text{ W}; 10: 17.8\text{ W}; 20: 12.8\text{ W}$.

Answer: Power is maximum (20 W) at $R_L = R_{th} = 5\ \Omega$ and falls on either side.

Problem 7. A 12 V battery has internal resistance $0.2\ \Omega$. Find its terminal voltage when supplying 10 A, and the power lost inside.

$V = E - Ir = 12 - 2 = 10\text{ V}$. Internal loss $= I^2r = 20\text{ W}$.

Answer: $V = 10\text{ V}$; loss = 20 W.

Problem 8. Find the Norton equivalent of a 2 A current source in parallel with $10\ \Omega$, followed by a $5\ \Omega$ series resistor to terminal A.

Convert: $2\text{ A} \parallel 10\ \Omega \rightarrow 20\text{ V}$ in series with $10\ \Omega$. Add the $5\ \Omega$: 20 V in series with $15\ \Omega$ (Thevenin).

$I_N = 20/15 = 1.333\text{ A}$; $R_N = 15\ \Omega$.

Answer: 1.33 A in parallel with 15 Ω .

Problem 9. Determine the current in a 6 Ω resistor connected between the midpoints of two branches: branch 1 is 4 Ω + 8 Ω and branch 2 is 8 Ω + 4 Ω , both across 24 V (unbalanced bridge). Use Thevenin's theorem.

Open-circuit voltages at the midpoints (measured from the negative rail): branch 1: $24 \times 8/12 = 16$ V; branch 2: $24 \times 4/12 = 8$ V. $V_{th} = 8$ V.

R_{th} (source shorted) = $(4 \parallel 8) + (8 \parallel 4) = 2.667 + 2.667 = 5.333 \Omega$.

$I = 8/(5.333 + 6) = 0.706$ A.

Answer: $I \approx 0.71$ A.

Problem 10. A 220 V, 60 W lamp and a 220 V, 100 W lamp are connected in series across 220 V. Which glows brighter and what power does each take?

$R_{60} = 220^2/60 = 806.7 \Omega$; $R_{100} = 484 \Omega$. $I = 220/1290.7 = 0.1705$ A.

$P_{60} = 0.1705^2 \times 806.7 = 23.4$ W; $P_{100} = 0.1705^2 \times 484 = 14.1$ W.

Answer: The 60 W lamp glows brighter (23.4 W vs 14.1 W), because in series the larger resistance takes more power.

1.19 DC transients in R-L and R-C circuits

The course outcome CO1 includes transients. When a switch is closed or opened in a circuit containing an inductor or a capacitor, the currents and voltages do not change instantly; they pass through a **transient** period before reaching the new **steady state**.

Key facts.

- The current through an inductor cannot change instantaneously ($v = L di/dt$ would be infinite). At the instant of switching an inductor behaves like an **open circuit** (if initially uncharged) and in DC steady state like a **short circuit**.
- The voltage across a capacitor cannot change instantaneously ($i = C dv/dt$ would be infinite). At the instant of switching an uncharged capacitor behaves like a **short circuit**, and in DC steady state like an **open circuit**.

(A) R-L circuit: growth of current. A DC voltage V is applied to R in series with L at $t = 0$. KVL: $V = Ri + L di/dt$. Solving with $i(0) = 0$:

$$i(t) = (V/R)(1 - e^{-t/\tau}), \tau = L/R \text{ (time constant)}$$

$$v_L(t) = V e^{-t/\tau}, v_R(t) = V(1 - e^{-t/\tau})$$

- At $t = \tau$, the current reaches 63.2% of its final value V/R ; after about 5τ it is practically at steady state (99.3%).
- The time constant is also the time the current would take to reach its final value if it continued rising at its initial rate V/L .

(B) R-L circuit: decay of current. If the source is removed and the coil is shorted through R at $t = 0$ with initial current I_0 : $i(t) = I_0 e^{-t/\tau}$. The stored energy $(1/2)LI_0^2$ is dissipated in R . Opening an inductive circuit suddenly produces a very high voltage ($L di/dt$), which causes sparking at switch contacts; a discharge resistor or freewheeling diode is connected across field windings for protection.

(C) R-C circuit: charging. A DC voltage V is applied to R in series with an uncharged C at $t = 0$. KVL: $V = Ri + q/C$ with $i = dq/dt$:

$$v_C(t) = V(1 - e^{-t/\tau}), i(t) = (V/R)e^{-t/\tau}, \tau = RC$$

- At $t = \tau$, the capacitor voltage is 63.2% of V ; after 5τ it is practically fully charged.
- The initial current is V/R (capacitor acts as a short circuit), decaying to zero.

(D) R-C circuit: discharging. A capacitor charged to V_0 discharges through R : $v_C(t) = V_0 e^{-t/\tau}$, $i(t) = -(V_0/R)e^{-t/\tau}$. After one time constant the voltage falls to 36.8% of V_0 .

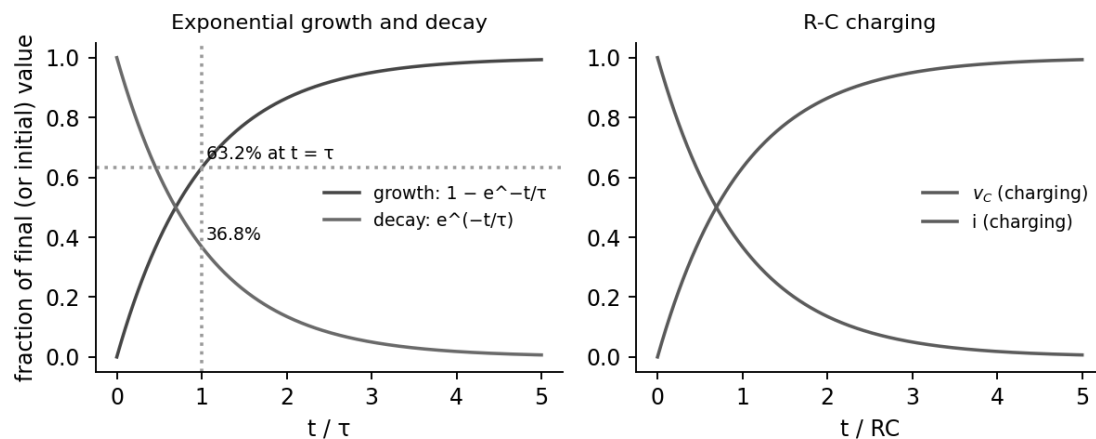


Figure 1.4: Exponential growth and decay in R-L and R-C circuits. After one time constant a quantity completes 63.2% of its change.

Quantity	R-L circuit	R-C circuit
Time constant	$\tau = L/R$	$\tau = RC$
Growth	$i = (V/R)(1 - e^{-t/\tau})$	$v_C = V(1 - e^{-t/\tau})$
Decay	$i = I_0 e^{-t/\tau}$	$v_C = V_0 e^{-t/\tau}$
At $t = 0+$ (initially uncharged)	Open circuit	Short circuit
In DC steady state	Short circuit	Open circuit
Energy stored	$(1/2)LI^2$	$(1/2)CV^2$

Applications. Time-delay circuits, camera flash (capacitor charging), smoothing filters in power supplies, relay timing, RC timing in oscillators (555 timer), and the inrush current and switching surges studied in power systems. (Compare with damped oscillations in Physics, Module I: an RLC circuit gives oscillatory or non-oscillatory transients.)

1.20 Solved problems on transients

Problem 1. A coil of $R = 10 \Omega$ and $L = 2 \text{ H}$ is connected to a 100 V DC supply. Find the time constant, the final current, the current after 0.2 s , and the time for the current to reach 5 A .

$$\tau = L/R = 0.2 \text{ s}. I_{\text{final}} = 100/10 = 10 \text{ A}.$$

$$\text{At } t = 0.2 \text{ s } (= \tau): i = 10(1 - e^{-1}) = 6.32 \text{ A}.$$

$$5 = 10(1 - e^{-t/0.2}) \Rightarrow e^{-t/0.2} = 0.5 \Rightarrow t = 0.2 \ln 2 = 0.139 \text{ s}.$$

Answer: $\tau = 0.2 \text{ s}$; 10 A ; 6.32 A ; 0.139 s .

Problem 2. A 10 μF capacitor is charged through a 100 k Ω resistor from a 200 V supply. Find the time constant, the initial current, the capacitor voltage after 1 s and the energy stored when fully charged.

$$\tau = RC = 10^5 \times 10^{-5} = 1 \text{ s. } I_0 = 200/10^5 = 2 \text{ mA.}$$

$$v_C(1 \text{ s}) = 200(1 - e^{-1}) = 126.4 \text{ V.}$$

$$W = (1/2)CV^2 = 0.5 \times 10^{-5} \times 40000 = 0.2 \text{ J.}$$

Answer: $\tau = 1 \text{ s}$; $I_0 = 2 \text{ mA}$; 126.4 V; 0.2 J.

Problem 3. A 50 μF capacitor charged to 400 V is discharged through a 20 k Ω resistor. Find the voltage after 2 s and the time for the voltage to fall to 100 V.

$$\tau = 20000 \times 50 \times 10^{-6} = 1 \text{ s. } v(2) = 400e^{-2} = 54.1 \text{ V.}$$

$$100 = 400e^{-t} \Rightarrow t = \ln 4 = 1.386 \text{ s.}$$

Answer: 54.1 V; 1.39 s.

Problem 4. The field winding of a DC machine has $R = 50 \Omega$ and $L = 10 \text{ H}$ and carries 4 A. When the supply is switched off, the winding is discharged through a 150 Ω resistor. Find the initial voltage across the winding and the time for the current to fall to 1 A.

At switch-off, the current (4 A) continues through $R + 150 = 200 \Omega$ total: the voltage across the discharge resistor is $4 \times 150 = 600 \text{ V}$ (which is why the resistor must be chosen carefully).

$$\tau = L/R_{\text{total}} = 10/200 = 0.05 \text{ s. } 1 = 4e^{-t/0.05} \Rightarrow t = 0.05 \ln 4 = 0.069 \text{ s.}$$

Answer: 600 V across the discharge resistor; $t \approx 69 \text{ ms}$.

Problem 5. In an R-C circuit, what fraction of the final voltage is reached after 2τ , 3τ and 5τ ?

$$1 - e^{-2} = 0.865; 1 - e^{-3} = 0.950; 1 - e^{-5} = 0.993.$$

Answer: 86.5%, 95.0%, 99.3%.

1.21 Practice problems with solutions: Module I

Q1. Find the equivalent resistance of three 12 Ω resistors connected in delta, as seen between two terminals.

$$\text{Between two terminals: } 12 \parallel (12 + 12) = 12 \times 24/36.$$

Answer: 8 Ω .

Q2. A 100 W, 220 V lamp is connected across 110 V. Find the power consumed (assume constant resistance).

$$R = V^2/P = 48400/100 = 484 \Omega. P' = 110^2/484 = 25 \text{ W.}$$

Answer: 25 W.

Q3. Two resistors of 4 Ω and 12 Ω are in parallel and carry a total current of 8 A. Find the current in each.



$$I_4 = 8 \times 12/16 = 6 \text{ A}; I_{12} = 8 \times 4/16 = 2 \text{ A}.$$

Answer: 6 A and 2 A.

Q4. In a single-loop circuit, a 20 V source and an opposing 5 V source are connected in series with 2 Ω and 3 Ω resistors. Find the current.

$$\text{KVL: } 20 - 5 = I(2 + 3) \Rightarrow I = 3 \text{ A}.$$

Answer: 3 A (in the direction driven by the 20 V source).

Q5. Find R_{th} and V_{th} across a 10 Ω resistor that is connected across the 6 Ω resistor of a series circuit (4 Ω + 6 Ω) fed by 20 V.

$$V_{th} = 20 \times 6/10 = 12 \text{ V}. R_{th} = 4 \parallel 6 = 2.4 \Omega.$$

$$I_{10\Omega} = 12/(2.4 + 10) = 0.968 \text{ A}.$$

Answer: $V_{th} = 12 \text{ V}$; $R_{th} = 2.4 \Omega$; $I = 0.97 \text{ A}$.

Q6. A network has $V_{oc} = 30 \text{ V}$ and $I_{sc} = 6 \text{ A}$ at its terminals. Find the Thevenin and Norton equivalents and the maximum power it can deliver.

$$R_{th} = 30/6 = 5 \Omega. P_{max} = 30^2/(4 \times 5) = 45 \text{ W}.$$

Answer: 30 V with 5 Ω ; 6 A with 5 Ω ; $P_{max} = 45 \text{ W}$.

Q7. Three equal star resistors of 5 Ω are converted to delta. Find each delta resistance.

$$R_{\Delta} = 3R_Y.$$

Answer: 15 Ω each.

Q8. A 2 A current source and a 10 V source (in series with 5 Ω) both feed a 10 Ω resistor connected across the current source. Using superposition, find the current in the 10 Ω resistor.

$$10 \text{ V alone (current source opened): } I = 10/(5 + 10) = 0.667 \text{ A}.$$

$$2 \text{ A alone (voltage source shorted): current divides between 10 Ω and 5 Ω : } I = 2 \times 5/15 = 0.667 \text{ A}.$$

Answer: If both act in the same direction through the 10 Ω resistor: $I = 1.33 \text{ A}$.

Q9. A load of 3 Ω is connected to a source with internal resistance 1 Ω and emf 8 V. Find the efficiency of power transfer.

$$I = 8/4 = 2 \text{ A}. P_L = 4 \times 3 = 12 \text{ W}; P_{total} = 8 \times 2 = 16 \text{ W}.$$

Answer: $\eta = 75\%$.

1.22 MCQs: Module I

1. KCL is based on the conservation of

(a) flux (b) energy (c) momentum (d) charge

□ (d) — Charge cannot accumulate at a node.



- 2.** KVL is based on the conservation of
(a) mass (b) charge (c) energy (d) current
☐ (c) — Work around a closed loop is zero.
- 3.** An ideal voltage source has internal resistance
(a) equal to load (b) $1\ \Omega$ (c) zero (d) infinite
☐ (c) — Constant terminal voltage.
- 4.** An ideal current source has internal resistance
(a) $1\ \Omega$ (b) zero (c) equal to load (d) infinite
☐ (d) — Constant current.
- 5.** When removing sources in superposition, a voltage source is replaced by
(a) a resistor of $1\ \Omega$ (b) an open circuit (c) a current source (d) a short circuit
☐ (d) — Keep internal resistance only.
- 6.** When removing sources in superposition, a current source is replaced by
(a) a voltage source (b) nothing (c) an open circuit (d) a short circuit
☐ (c) — Infinite internal resistance.
- 7.** Superposition theorem cannot be applied directly to
(a) current (b) power (c) voltage (d) linear circuits
☐ (b) — $P \propto I^2$ is non-linear.
- 8.** Thevenin's equivalent circuit consists of
(a) a current source in parallel with a resistance (b) a voltage source in series with a resistance (c) two voltage sources (d) only a resistance
☐ (b) — V_{th} and R_{th} .
- 9.** Norton's equivalent circuit consists of
(a) a voltage source in series with R (b) a short circuit (c) a current source in parallel with R (d) a current source in series with R
☐ (c) — I_N and R_N .
- 10.** V_{th} is the
(a) source voltage (b) open-circuit voltage (c) short-circuit current (d) load voltage
☐ (b) — Load removed.
- 11.** I_N is the
(a) open-circuit voltage (b) short-circuit current (c) source current (d) load current
☐ (b) — Terminals shorted.
- 12.** R_{th} equals
(a) $V_{oc} \times I_{sc}$ (b) I_{sc}/V_{oc} (c) V_{oc}/I_{sc} (d) $V_{oc} + I_{sc}$
☐ (c) — Relation between the two equivalents.
- 13.** Maximum power is transferred when
(a) $R_L = 2R_{th}$ (b) $R_L = R_{th}$ (c) $R_L = 0$ (d) $R_L = \infty$
☐ (b) — Maximum power transfer theorem.
- 14.** Efficiency at maximum power transfer is
(a) 25% (b) 75% (c) 50% (d) 100%
☐ (c) — Half the power is lost in R_{th} .
- 15.** The maximum power delivered to the load is
(a) $V_{th}^2/2R_{th}$ (b) $4V_{th}^2/R_{th}$ (c) $V_{th}^2/4R_{th}$ (d) V_{th}^2/R_{th}
☐ (c) — With $R_L = R_{th}$.
- 16.** Three equal resistors R in delta are equivalent to star resistors of
(a) $R/3$ (b) $R/2$ (c) R (d) $3R$
☐ (a) — $R_Y = R_\Delta/3$.



- 17.** In star-to-delta, R_{AB} equals
(a) $R_A + R_B$ (b) $R_A + R_B + R_A R_B / R_C$ (c) $R_A R_B / R_C$ (d) $R_A R_B / (R_A + R_B + R_C)$
□ (b) — Standard formula.
- 18.** In delta-to-star, R_A equals
(a) $R_{AB} R_{BC} / R_{CA}$ (b) $R_{AB} R_{CA} / (R_{AB} + R_{BC} + R_{CA})$ (c) $R_{AB} + R_{CA}$ (d) $R_{BC} / 3$
□ (b) — Product of adjacent over sum.
- 19.** The number of independent mesh equations for a network with b branches and n nodes is
(a) $b - n$ (b) $b + n$ (c) $n - 1$ (d) $b - n + 1$
□ (d) — Planar network.
- 20.** The number of nodal equations needed for a network with n principal nodes is
(a) $n - 1$ (b) $2n$ (c) n (d) $n + 1$
□ (a) — One node is reference.
- 21.** A mesh is a loop that
(a) has only one element (b) contains other loops inside (c) contains no other loop inside (d) is always open
□ (c) — Definition.
- 22.** Two resistors of $6\ \Omega$ and $3\ \Omega$ in parallel give
(a) $4.5\ \Omega$ (b) $18\ \Omega$ (c) $9\ \Omega$ (d) $2\ \Omega$
□ (d) — $18/9$.
- 23.** In a series circuit, the voltage across a resistor is proportional to
(a) the total current squared (b) the source frequency (c) its conductance (d) its resistance
□ (d) — Voltage divider.
- 24.** In a parallel circuit, the current through a branch is inversely proportional to
(a) frequency (b) voltage (c) its resistance (d) power
□ (c) — Current divider.
- 25.** A 10 V source with $2\ \Omega$ internal resistance is equivalent to a current source of
(a) 10 A with $2\ \Omega$ (b) 5 A in parallel with $2\ \Omega$ (c) 20 A in parallel with $2\ \Omega$ (d) 5 A in series with $2\ \Omega$
□ (b) — $I = E/r$.
- 26.** A supermesh is used when
(a) a current source is shared by two meshes (b) a voltage source is shared by two meshes (c) all resistors are equal (d) there are no sources
□ (a) — Combine the meshes.
- 27.** A supernode is used when
(a) all nodes are grounded (b) a current source connects two nodes (c) there is a short circuit (d) a voltage source connects two non-reference nodes
□ (d) — Combine the nodes.
- 28.** Superposition theorem applies to
(a) non-linear circuits (b) linear bilateral circuits (c) only AC circuits (d) only DC circuits with one source
□ (b) — Linearity required.
- 29.** A Wheatstone bridge is balanced when
(a) $PQ = RS$ always (b) $P/Q = R/S$ (c) $P + Q = R + S$ (d) $P = S$
□ (b) — No galvanometer current.



- 30.** If a network has $V_{th} = 12\text{ V}$ and $R_{th} = 3\ \Omega$, the current in a $3\ \Omega$ load is
(a) 1 A (b) 4 A (c) 6 A (d) 2 A
□ (d) — $12/6$.
- 31.** 1 kWh equals
(a) $3.6 \times 10^3\text{ J}$ (b) 36 J (c) $3.6 \times 10^6\text{ J}$ (d) 1000 J
□ (c) — $1000\text{ W} \times 3600\text{ s}$.
- 32.** The resistance of a wire of length l and area A is
(a) $l/(\rho A)$ (b) $\rho A/l$ (c) $\rho l A$ (d) $\rho l/A$
□ (d) — $R = \rho l/A$.
- 33.** If the length of a wire is doubled by stretching (volume constant), its resistance becomes
(a) R (b) $4R$ (c) $R/2$ (d) $2R$
□ (b) — $R \propto l^2$.
- 34.** Maximum power transfer is commonly used in
(a) power transmission (b) generators (c) household wiring (d) communication circuits
□ (d) — Efficiency is not critical there.
- 35.** A linear element is one whose
(a) voltage is constant (b) resistance changes with current (c) power is constant (d) V-I graph is a straight line
□ (d) — Definition.
- 36.** A diode is an example of
(a) a passive linear element (b) a linear bilateral element (c) a non-linear unilateral element (d) an ideal source
□ (c) — Conducts in one direction only.
- 37.** The current in the shared branch of two meshes carrying I_1 and I_2 (both clockwise) is
(a) zero (b) $I_1 I_2$ (c) $I_1 + I_2$ (d) $I_1 - I_2$
□ (d) — Algebraic difference.
- 38.** Norton resistance is found in the same way as
(a) internal resistance of the load (b) the short-circuit current (c) Thevenin resistance (d) load resistance
□ (c) — $R_N = R_{th}$.
- 39.** A network delivers 10 A into a short circuit and 50 V on open circuit. Its internal resistance is
(a) $0.2\ \Omega$ (b) $5\ \Omega$ (c) $500\ \Omega$ (d) $60\ \Omega$
□ (b) — $50/10$.
- 40.** The power dissipated in a $5\ \Omega$ resistor carrying 2 A is
(a) 2.5 W (b) 10 W (c) 20 W (d) 50 W
□ (c) — $I^2 R$.

Module II: Single-Phase and Three-Phase AC Circuits

Almost all electrical power is generated, transmitted and used as alternating current (AC), because AC voltages can be stepped up and down easily with transformers and AC machines are simple and robust. This module analyses single-phase circuits with R, L and C, and balanced three-phase systems.

2.1 Generation of a sinusoidal emf

A coil of N turns and area A rotating with angular velocity ω in a uniform magnetic field B has flux linkage $N\Phi = NBA \cos \omega t$. By Faraday's law the induced emf is

$$e = -d(NBA \cos \omega t)/dt = NBA\omega \sin \omega t = E_m \sin \omega t, E_m = NBA\omega$$

Terms. Cycle; time period T ; frequency $f = 1/T$ (50 Hz in India); angular frequency $\omega = 2\pi f$ (314 rad/s at 50 Hz); amplitude (peak value) E_m ; phase and phase difference. A quantity $v = V_m \sin(\omega t + \phi)$ **leads** one with phase 0 by ϕ if $\phi > 0$, and **lags** it if $\phi < 0$.

2.2 Average and effective (rms) values

Average value. The average of a symmetrical alternating wave over a full cycle is zero, so the average is taken over a half cycle:

$$I_{av} = (1/\pi) \int_0^\pi I_m \sin \theta d\theta = 2I_m/\pi = 0.637 I_m$$

RMS (effective) value. The rms value of an AC is the steady DC that would produce the same heat in the same resistance in the same time.

$$I_{rms} = \sqrt{[(1/2\pi) \int_0^{2\pi} I_m^2 \sin^2 \theta d\theta]} = I_m/\sqrt{2} = 0.707 I_m$$

Form factor = rms/average = 1.11 (sine wave); Peak (crest) factor = peak/rms = 1.414 (sine wave)

- AC meters read rms values. The 230 V domestic supply is an rms value; its peak is $230\sqrt{2} = 325$ V.
- For a half-wave rectified sine: $I_{av} = I_m/\pi$, $I_{rms} = I_m/2$. For a full-wave rectified sine: $I_{av} = 2I_m/\pi$, $I_{rms} = I_m/\sqrt{2}$. For a square wave: $I_{av} = I_{rms} = I_m$, form factor 1.

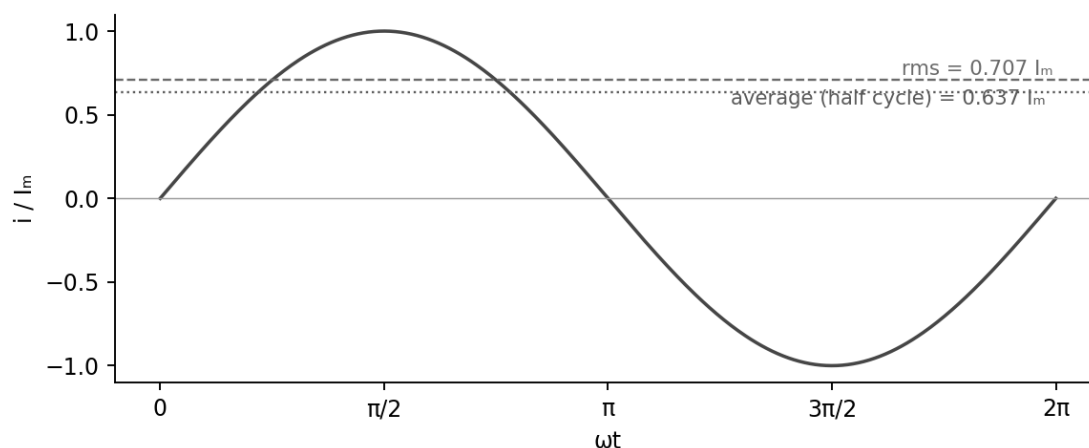


Figure 2.1: A sinusoidal current with its rms and half-cycle average values.

Phasor representation. A sinusoid $V_m \sin(\omega t + \phi)$ is represented by a rotating vector (phasor) of length V_m (or V_{rms}) at angle ϕ . Phasors of the same frequency can be added

like vectors. In complex (rectangular/polar) form: $V = V \angle \phi = V(\cos \phi + j \sin \phi)$, $j = \sqrt{-1}$. Multiplying by j rotates a phasor by $+90^\circ$.

2.3 Single elements in AC

Element	Opposition	Phase of current relative to voltage	Average power
Resistance R	R	In phase	$P = VI = I^2R$
Inductance L	$X_L = \omega L = 2\pi fL$	Lags by 90°	0 (energy stored and returned)
Capacitance C	$X_C = 1/(\omega C) = 1/(2\pi fC)$	Leads by 90°	0

Pure inductor. $v = L di/dt$. If $i = I_m \sin \omega t$, $v = \omega L I_m \cos \omega t = V_m \sin(\omega t + 90^\circ)$. Current lags voltage by 90° . $X_L \propto f$: an inductor blocks high frequencies and passes DC ($X_L = 0$ at $f = 0$).

Pure capacitor. $i = C dv/dt$. If $v = V_m \sin \omega t$, $i = \omega C V_m \cos \omega t$. Current leads voltage by 90° . $X_C \propto 1/f$: a capacitor blocks DC and passes high frequencies.

Memory aid: **CIVIL** — in a Capacitor, I leads V; V leads I in an inductor (L).

2.4 Series R-L, R-C and R-L-C circuits

Series R-L. $V = V_R + V_L$ (phasor sum). $V_R = IR$ in phase with I ; $V_L = IX_L$ leading I by 90° .

$$Z = \sqrt{R^2 + X_L^2}, I = V/Z, \phi = \tan^{-1}(X_L/R) \text{ (current lags)}, \cos \phi = R/Z$$

In complex form $Z = R + jX_L$.

Series R-C. $Z = \sqrt{R^2 + X_C^2}$, current leads by $\phi = \tan^{-1}(X_C/R)$. $Z = R - jX_C$.

Series R-L-C.

$$Z = R + j(X_L - X_C), |Z| = \sqrt{R^2 + (X_L - X_C)^2}, \phi = \tan^{-1}[(X_L - X_C)/R]$$

- $X_L > X_C$: circuit is inductive, current lags.
- $X_L < X_C$: circuit is capacitive, current leads.
- $X_L = X_C$: circuit is resistive, current in phase (resonance).

Series RLC phasor diagram ($X_L > X_C$)

Impedance triangle ($\times I^2 \rightarrow$ power triangle P, Q, S)

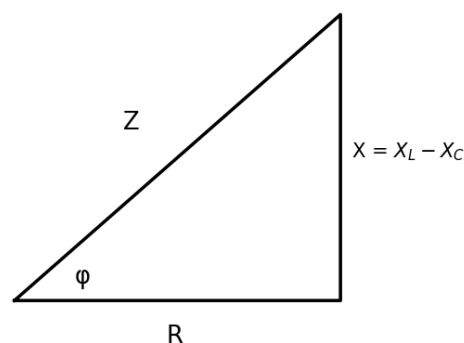
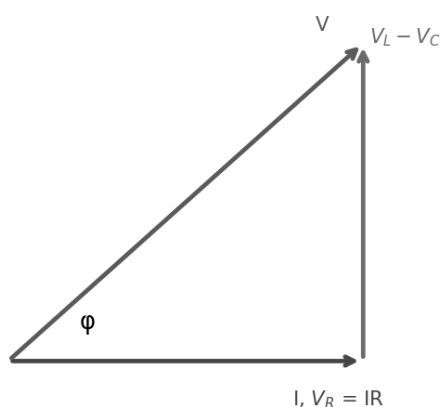


Figure 2.2: Phasor diagram of a series RLC circuit and the impedance triangle.

Impedance triangle. Sides R , $X = X_L - X_C$ and Z , with angle ϕ between R and Z . The **power triangle** is the impedance triangle multiplied by I^2 : sides P , Q , S .

2.5 Power in AC circuits

If $v = V_m \sin \omega t$ and $i = I_m \sin(\omega t - \phi)$, instantaneous power $p = vi = VI \cos \phi - VI \cos(2\omega t - \phi)$. The second term averages to zero.

Active (real) power $P = VI \cos \phi = I^2 R$ (W)

Reactive power $Q = VI \sin \phi = I^2 X$ (VAR)

Apparent power $S = VI = I^2 Z = \sqrt{P^2 + Q^2}$ (VA)

Power factor $= \cos \phi = P/S = R/Z$

- The power factor is **lagging** for inductive loads (motors, fans, transformers) and **leading** for capacitive loads.
- A low power factor means a larger current for the same power, which increases $I^2 R$ losses and needs larger cables and equipment. Power factor is improved by connecting capacitors in parallel with inductive loads.
- In complex form, $S = VI^* = P + jQ$.

2.6 Parallel AC circuits

In parallel, the voltage is common and branch currents add as phasors. Methods:

- **Phasor (vector) method:** find each branch current and its phase angle, resolve into components ($I \cos \phi$ and $I \sin \phi$), and add.
- **Admittance method:** $Y = 1/Z = G - jB$ (siemens). For parallel branches, $Y_{\text{total}} = Y_1 + Y_2 + \dots$, and $I = VY$.
- **Complex (symbolic) method:** $Z_{\text{eq}} = Z_1 Z_2 / (Z_1 + Z_2)$ using complex arithmetic.

For a branch $Z = R + jX$: $G = R/(R^2 + X^2)$, $B = X/(R^2 + X^2)$.

2.7 Series resonance

In a series R-L-C circuit, as the frequency is varied, X_L increases and X_C decreases. At the **resonant frequency** f_0 they are equal:

$$X_L = X_C = \omega_0 L = 1/(\omega_0 C) \Rightarrow f_0 = 1/(2\pi\sqrt{LC})$$

At series resonance:

- Impedance is minimum and purely resistive: $Z = R$.
- Current is maximum: $I_0 = V/R$, and in phase with the voltage (unity power factor).
- $V_L = V_C = I_0 X_L$; they are equal and opposite, so they cancel. Each can be much larger than the supply voltage: **voltage magnification**.

$$Q\text{-factor} = V_L/V = \omega_0 L/R = 1/(\omega_0 CR) = (1/R)\sqrt{L/C}$$

- Below f_0 the circuit is capacitive; above f_0 it is inductive.

Bandwidth. The half-power frequencies f_1 and f_2 are those at which the current is $I_0/\sqrt{2}$ (power is half the maximum).

$$\text{Bandwidth } BW = f_2 - f_1 = R/(2\pi L); Q = f_0/BW; f_0 = \sqrt{f_1 f_2}$$

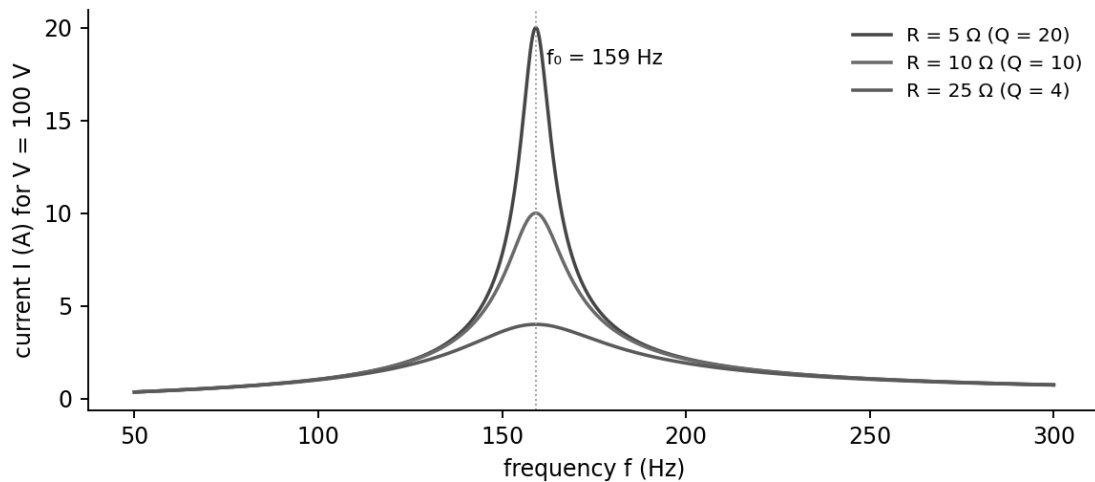


Figure 2.3: Current versus frequency for a series RLC circuit ($L = 0.1 \text{ H}$, $C = 10 \mu\text{F}$, $V = 100 \text{ V}$). Smaller R gives a sharper peak.

The series resonant circuit is called an **acceptor circuit** because it accepts (passes a large current at) the resonant frequency. Uses: radio tuning, filters. (Compare with Physics, Module I: forced electrical oscillator.)

2.8 Parallel resonance

A practical parallel circuit consists of a coil (R, L) in parallel with a capacitor C . Resonance occurs when the supply current is in phase with the voltage, i.e. the reactive (quadrature) components of the branch currents cancel:

$$f_0 = (1/2\pi)\sqrt{1/(LC) - R^2/L^2} (\approx 1/(2\pi\sqrt{LC})) \text{ when } R \text{ is small}$$

Dynamic impedance at resonance: $Z_D = L/(CR)$ (maximum, purely resistive)

- Supply current is **minimum** (V/Z_D) and in phase with the voltage.
- The currents in L and C can be much larger than the supply current: **current magnification** $Q = \omega_0 L/R$.
- Called a **rejector circuit** because it rejects (draws minimum current at) the resonant frequency. Used in the tank circuits of oscillators and in filters.

Series resonance	Parallel resonance
Z minimum = R	Z maximum = L/CR
Current maximum	Current minimum
Voltage magnification	Current magnification
Acceptor circuit	Rejector circuit
$f_0 = 1/(2\pi\sqrt{LC})$	$f_0 = (1/2\pi)\sqrt{1/LC - R^2/L^2}$

2.9 Three-phase systems: advantages

A three-phase supply has three emfs of equal magnitude and frequency, displaced by 120° from each other. It is generated by three identical windings placed 120° apart on the stator of an alternator.

$$e_R = E_m \sin \omega t, e_Y = E_m \sin(\omega t - 120^\circ), e_B = E_m \sin(\omega t - 240^\circ); e_R + e_Y + e_B = 0$$

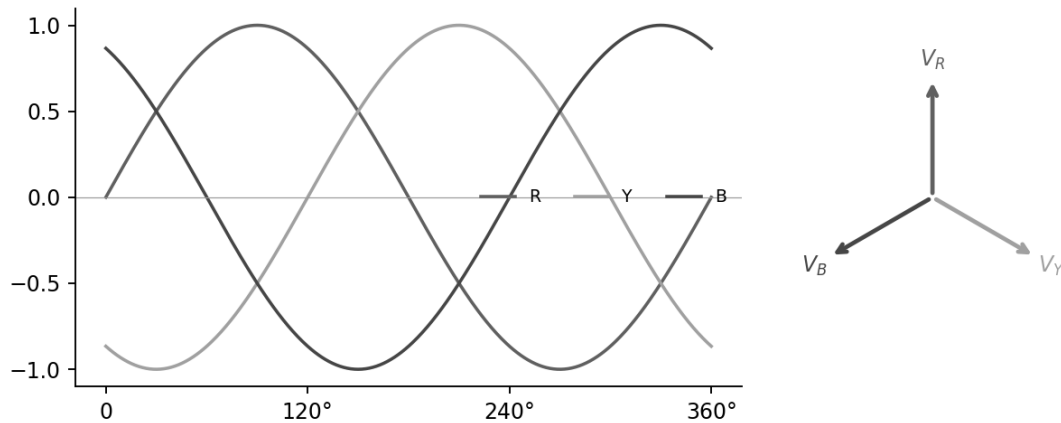


Figure 2.4: Balanced three-phase voltages 120° apart and their phasors.

Phase sequence: the order in which the phases reach their maximum (R-Y-B, positive sequence). Reversing the sequence reverses the direction of rotation of a three-phase motor.

Advantages over single phase:

- For the same power and voltage, three-phase transmission needs less conductor material (about 75% of single phase).
- Three-phase motors are self-starting, simpler and cheaper; single-phase motors are not self-starting.
- Power delivered by a balanced three-phase system is constant (not pulsating), giving smooth torque.
- A three-phase machine gives more output than a single-phase machine of the same size.
- Three-phase systems can supply single-phase loads as well.

2.10 Star (Y) connection

The three similar ends (the "stars") of the phase windings are joined to a common neutral point N; the other three ends are taken to the lines R, Y, B. A fourth (neutral) wire may be run from N.

- **Line current = phase current:** $I_L = I_{ph}$.
- **Line voltage = $\sqrt{3} \times$ phase voltage**, and line voltages lead the corresponding phase voltages by 30°: $V_L = \sqrt{3} V_{ph}$.

Proof. $V_{RY} = V_{RN} - V_{YN}$ (phasor difference). The two phasors are equal in magnitude and 120° apart, so the magnitude of the difference is $2V_{ph} \cos 30^\circ = \sqrt{3} V_{ph}$.

Example: Indian low-voltage supply 400 V (line) / 230 V (phase), 3-phase 4-wire.

2.11 Delta (Δ) connection

The end of each winding is joined to the start of the next, forming a closed mesh; lines are taken from the three junctions. There is no neutral.

- **Line voltage = phase voltage:** $V_L = V_{ph}$.
- **Line current = $\sqrt{3} \times$ phase current:** $I_L = \sqrt{3} I_{ph}$, and line currents lag the corresponding phase currents by 30°.

Proof. $I_R = I_{RY} - I_{BR}$ (KCL at the junction), phasor difference of two equal currents 120° apart $\Rightarrow$ magnitude $\sqrt{3} I_{ph}$.

Quantity	Star (Y)	Delta (Δ)
Voltage	$V_L = \sqrt{3} V_{ph}$	$V_L = V_{ph}$
Current	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$
Neutral	Available (3-phase 4-wire)	Not available
Used for	Distribution, generators, long lines	Motors, transformer windings

2.12 Power in balanced three-phase circuits

In each phase $P_{ph} = V_{ph} I_{ph} \cos \phi$, where ϕ is the angle between phase voltage and phase current (the load angle). Total power $P = 3 V_{ph} I_{ph} \cos \phi$. In terms of line quantities (for both star and delta):

$$P = \sqrt{3} V_L I_L \cos \phi \text{ (W)}; Q = \sqrt{3} V_L I_L \sin \phi \text{ (VAR)}; S = \sqrt{3} V_L I_L \text{ (VA)}$$

Solving a balanced circuit. Work with one phase: find V_{ph} , then $I_{ph} = V_{ph}/Z_{ph}$, then convert to line values with the relations above.

Balanced star-delta equivalence. A balanced delta load of Z_Δ per phase is equivalent to a star load of $Z_Y = Z_\Delta/3$ per phase. A load connected in delta takes 3 times the power it takes when connected in star across the same supply.

2.13 Measurement of three-phase power: two-wattmeter method

In a three-phase, three-wire system (balanced or unbalanced, star or delta), total power can be measured with two wattmeters: the current coils are in two lines (say R and B) and the pressure coils are connected between each of these lines and the third line (Y).

$$\text{Total power } P = W_1 + W_2$$

For a balanced load with power factor angle ϕ :

$$W_1 = V_L I_L \cos(30^\circ - \phi), W_2 = V_L I_L \cos(30^\circ + \phi)$$

$$\tan \phi = \sqrt{3} (W_1 - W_2)/(W_1 + W_2); \text{ reactive power } Q = \sqrt{3}(W_1 - W_2)$$

Effect of power factor on the readings.

Power factor ($\cos \phi$)	ϕ	Readings
1 (resistive)	0°	$W_1 = W_2$
0.866	30°	$W_2 = W_1/2$
0.5	60°	$W_2 = 0$; all power read by W_1
Below 0.5	$> 60^\circ$	W_2 is negative (reverse its pressure-coil connection and subtract the reading)
0 (pure reactance)	90°	$W_1 = -W_2$; total = 0

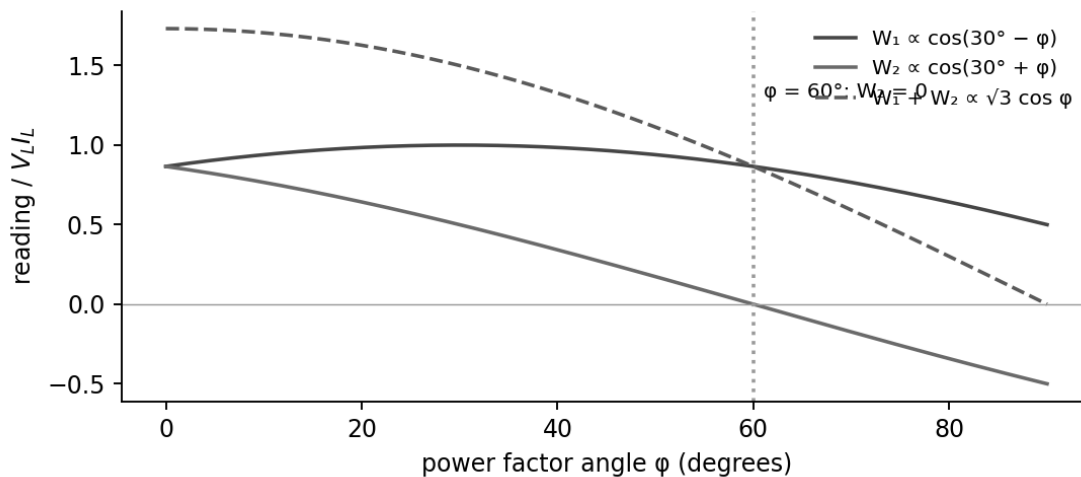


Figure 2.5: Variation of the two wattmeter readings with the load power factor angle.

One-wattmeter method (balanced load with accessible neutral): $P = 3 \times$ wattmeter reading.

2.14 Common mistakes to avoid in Module II

- Adding voltages in AC circuits arithmetically instead of as phasors.
- Using peak values in power formulas. $P = V_{\text{rms}} I_{\text{rms}} \cos \phi$.
- Using the line voltage as the phase voltage for a star-connected load.
- Taking ϕ in $P = \sqrt{3} V_L I_L \cos \phi$ as the angle between line voltage and line current. It is the angle between phase voltage and phase current.
- Forgetting that a negative wattmeter reading must be subtracted.

2.15 Derivations for long answers

(A) RMS value of a sinusoid. $i = I_m \sin \theta$. Mean of i^2 over a cycle $= (1/2\pi) \int_0^{2\pi} I_m^2 \sin^2 \theta d\theta = (I_m^2/2\pi) \int_0^{2\pi} (1 - \cos 2\theta)/2 d\theta = (I_m^2/2\pi)(\pi) = I_m^2/2$. Hence $I_{\text{rms}} = I_m/\sqrt{2}$.

(B) Average value. Over a half cycle: $(1/\pi) \int_0^{\pi} I_m \sin \theta d\theta = (I_m/\pi)[- \cos \theta]_0^{\pi} = 2I_m/\pi$.

(C) Power in an AC circuit. $p = vi = V_m \sin \omega t \times I_m \sin(\omega t - \phi) = (V_m I_m/2)[\cos \phi - \cos(2\omega t - \phi)]$. The average of $\cos(2\omega t - \phi)$ over a cycle is zero, so $P = (V_m I_m/2) \cos \phi = V_{\text{rms}} I_{\text{rms}} \cos \phi$. The power oscillates at twice the supply frequency. For a pure inductor or capacitor ($\phi = \pm 90^\circ$), $P = 0$: energy flows in during one quarter-cycle and back out during the next.

(D) Current in a series R-L circuit. KVL: $Ri + L di/dt = V_m \sin \omega t$. The steady-state solution is $i = (V_m/Z) \sin(\omega t - \phi)$ with $Z = \sqrt{R^2 + \omega^2 L^2}$, $\tan \phi = \omega L/R$. (Check by substitution.) At switch-on there is also a transient term $Ae^{-Rt/L}$, which dies out with time constant $\tau = L/R$.

(E) Half-power frequencies of a series RLC circuit. At the half-power points $|Z| = \sqrt{2} R$, so $\omega L - 1/(\omega C) = \pm R$. Solving the two quadratics: $\omega_2 - \omega_1 = R/L$, and $\omega_1 \omega_2 = 1/(LC) = \omega_0^2$. Hence BW (Hz) $= R/(2\pi L)$ and $f_0 = \sqrt{f_1 f_2}$.

(F) Line and phase relations in star (phasor proof). Take $V_{\text{RN}} = V \angle 0^\circ$, $V_{\text{YN}} = V \angle -120^\circ$. $V_{\text{RY}} = V_{\text{RN}} - V_{\text{YN}} = V[1 - (-0.5 - j0.866)] = V(1.5 + j0.866) = \sqrt{3}V \angle 30^\circ$. So $V_L = \sqrt{3}V_{\text{ph}}$, leading by 30° .

(G) Two-wattmeter method proof (balanced load). Drawing the phasor diagram for a lagging balanced load shows that one wattmeter sees its line current and line voltage separated by $(30^\circ - \phi)$ and the other by $(30^\circ + \phi)$. $W_1 + W_2 = V_L I_L [\cos(30^\circ - \phi) + \cos(30^\circ + \phi)] = V_L I_L \times 2 \cos 30^\circ \cos \phi = \sqrt{3} V_L I_L \cos \phi = \text{total power}$. $W_1 - W_2 = V_L I_L \times 2 \sin 30^\circ \sin \phi = V_L I_L \sin \phi$. Dividing gives $\tan \phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$.

2.16 Complex (j) notation: a quick guide

- Rectangular form: $Z = a + jb$. Polar form: $Z = r \angle \theta$ with $r = \sqrt{a^2 + b^2}$, $\theta = \tan^{-1}(b/a)$. Conversion back: $a = r \cos \theta$, $b = r \sin \theta$.
- Add and subtract in rectangular form; multiply and divide in polar form:
 $(r_1 \angle \theta_1)(r_2 \angle \theta_2) = r_1 r_2 \angle (\theta_1 + \theta_2)$; $(r_1 \angle \theta_1)/(r_2 \angle \theta_2) = (r_1/r_2) \angle (\theta_1 - \theta_2)$.
- Impedances: resistor R ; inductor $jX_L = j\omega L$; capacitor $-jX_C = 1/(j\omega C)$.
- All DC methods (series-parallel, mesh, nodal, Thevenin, superposition) apply to AC circuits when resistances are replaced by complex impedances and quantities by phasors.

Example. $Z_1 = 3 + j4$ and $Z_2 = 6 - j8$ in parallel: $Z = Z_1 Z_2 / (Z_1 + Z_2) = (5 \angle 53.13^\circ)(10 \angle -53.13^\circ) / (9 - j4) = 50 \angle 0^\circ / (9.849 \angle -23.96^\circ) = 5.077 \angle 23.96^\circ = 4.64 + j2.06 \Omega$.

2.17 Power factor improvement

Causes of low power factor: induction motors (especially at light load), transformers on no load, arc furnaces, fluorescent-lamp chokes.

Disadvantages: larger current for the same power $\Rightarrow$ larger conductors, higher $I^2 R$ losses, larger voltage drop, poor regulation, and larger kVA rating of generators and transformers. Supply companies charge a penalty for low pf.

Methods: static capacitors (banks) in parallel with the load (most common); synchronous condensers (over-excited synchronous motors running without load); phase advancers for induction motors.

Capacitor kVAR required $Q_C = P(\tan \phi_1 - \tan \phi_2)$; $C = Q_C / (2\pi f V^2)$

2.18 Solved numericals: Module II

Problem 1. An alternating current is given by $i = 14.14 \sin(314t)$ A. Find the rms value, average value, frequency and the time period.

$$I_m = 14.14 \text{ A} \Rightarrow I_{\text{rms}} = 14.14/\sqrt{2} = 10 \text{ A}; I_{\text{av}} = 0.637 \times 14.14 = 9.0 \text{ A}.$$

$$\omega = 314 \Rightarrow f = 314/2\pi = 50 \text{ Hz}; T = 0.02 \text{ s}.$$

Answer: $I_{\text{rms}} = 10 \text{ A}$, $I_{\text{av}} = 9 \text{ A}$, $f = 50 \text{ Hz}$, $T = 20 \text{ ms}$.

Problem 2. Find the rms and average values of a half-wave rectified sine current of peak 10 A.

$$I_{\text{av}} = I_m / \pi = 3.18 \text{ A}. I_{\text{rms}} = I_m / 2 = 5 \text{ A} \text{ (the wave exists for only half the time).}$$

$$\text{Form factor} = 5/3.18 = 1.57.$$

Answer: $I_{\text{rms}} = 5 \text{ A}$; $I_{\text{av}} = 3.18 \text{ A}$; form factor 1.57.

Problem 3. A coil of resistance 10 Ω and inductance 0.1 H is connected to a 230 V, 50 Hz supply. Find the reactance, impedance, current, power factor and power.

$$X_L = 2\pi \times 50 \times 0.1 = 31.42 \, \Omega. \quad Z = \sqrt{(10^2 + 31.42^2)} = 32.97 \, \Omega.$$

$$I = 230/32.97 = 6.98 \, \text{A}. \quad \cos \phi = R/Z = 10/32.97 = 0.303 \text{ (lagging)}, \quad \phi = 72.3^\circ.$$

$$P = I^2 R = 6.98^2 \times 10 = 487 \, \text{W (check: } VI \cos \phi = 230 \times 6.98 \times 0.303 = 487 \, \text{W)}.$$

Answer: $X_L = 31.4 \, \Omega$, $Z = 33.0 \, \Omega$, $I = 6.98 \, \text{A}$, $\text{pf} = 0.303 \text{ lag}$, $P \approx 487 \, \text{W}$.

Problem 4. A 100 μF capacitor is connected across 230 V, 50 Hz. Find the capacitive reactance and current. What happens if the frequency is doubled?

$$X_C = 1/(2\pi \times 50 \times 10^{-4}) = 31.83 \, \Omega. \quad I = 230/31.83 = 7.23 \, \text{A (leading by } 90^\circ).$$

At 100 Hz, X_C halves to 15.9 Ω and the current doubles to 14.45 A.

Answer: $X_C = 31.8 \, \Omega$; $I = 7.23 \, \text{A}$; at 100 Hz, $I = 14.45 \, \text{A}$.

Problem 5. A series R-L-C circuit with $R = 10 \, \Omega$, $L = 0.2 \, \text{H}$ and $C = 50 \, \mu\text{F}$ is connected to 230 V, 50 Hz. Find the impedance, current, power factor, power and the voltage across each element.

$$X_L = 2\pi \times 50 \times 0.2 = 62.83 \, \Omega; \quad X_C = 1/(2\pi \times 50 \times 50 \times 10^{-6}) = 63.66 \, \Omega.$$

$$X = X_L - X_C = -0.83 \, \Omega \text{ (slightly capacitive)}. \quad Z = \sqrt{(10^2 + 0.69)} = 10.03 \, \Omega.$$

$$I = 230/10.03 = 22.9 \, \text{A}. \quad \cos \phi = 10/10.03 = 0.997 \text{ (leading)}.$$

$$P = I^2 R = 22.9^2 \times 10 = 5.25 \, \text{kW}. \quad V_R = 229 \, \text{V}, \quad V_L = 22.9 \times 62.83 = 1440 \, \text{V}, \quad V_C = 22.9 \times 63.66 = 1459 \, \text{V}.$$

Answer: $Z \approx 10.03 \, \Omega$; $I \approx 22.9 \, \text{A}$; $\text{pf} \approx 0.997 \text{ leading}$; $P \approx 5.25 \, \text{kW}$. The circuit is almost at resonance ($f_0 = 50.3 \, \text{Hz}$), so V_L and V_C are much larger than the supply voltage.

Problem 6. A series R-L-C circuit has $R = 10 \, \Omega$, $L = 0.1 \, \text{H}$ and $C = 10 \, \mu\text{F}$, supplied at 100 V. Find the resonant frequency, current at resonance, Q-factor, voltage across the capacitor at resonance and bandwidth.

$$f_0 = 1/(2\pi\sqrt{0.1 \times 10^{-5}}) = 1/(2\pi \times 10^{-3}) = 159.2 \, \text{Hz}.$$

$$I_0 = V/R = 10 \, \text{A}. \quad Q = (1/R)\sqrt{L/C} = (1/10)\sqrt{10^4} = 10.$$

$$V_C = QV = 1000 \, \text{V}. \quad \text{BW} = R/(2\pi L) = 10/(0.628) = 15.9 \, \text{Hz}.$$

Answer: $f_0 = 159.2 \, \text{Hz}$; $I_0 = 10 \, \text{A}$; $Q = 10$; $V_C = 1000 \, \text{V}$; $\text{BW} = 15.9 \, \text{Hz}$.

Problem 7. Two impedances $Z_1 = 6 + j8 \, \Omega$ and $Z_2 = 8 - j6 \, \Omega$ are connected in parallel across 100 V. Find the branch currents, total current, power factor and total power.

$$I_1 = 100/(6 + j8) = 100/(10 \angle 53.13^\circ) = 10 \angle -53.13^\circ \, \text{A} = 6 - j8 \, \text{A}.$$

$$I_2 = 100/(8 - j6) = 10 \angle 36.87^\circ \, \text{A} = 8 + j6 \, \text{A}.$$

$$I = I_1 + I_2 = 14 - j2 \, \text{A} \Rightarrow |I| = 14.14 \, \text{A}, \quad \phi = -8.13^\circ \text{ (lagging)}.$$

$$\text{pf} = \cos 8.13^\circ = 0.99 \text{ lag}. \quad P = VI \cos \phi = 100 \times 14.14 \times 0.99 = 1400 \, \text{W (check: } 10^2 \times 6 + 10^2 \times 8 = 1400 \, \text{W } \square).$$

Answer: $I_1 = 10 \text{ A}$, $I_2 = 10 \text{ A}$, $I = 14.14 \text{ A}$, $\text{pf} = 0.99 \text{ lag}$, $P = 1400 \text{ W}$.

Problem 8. A single-phase motor takes 5 kW at 0.7 pf lagging from a 230 V, 50 Hz supply. Find the capacitance required in parallel to raise the pf to 0.95 lagging.

$$Q_1 = P \tan \phi_1 = 5000 \times \tan(\cos^{-1} 0.7) = 5000 \times 1.0202 = 5101 \text{ VAR.}$$

$$Q_2 = 5000 \times \tan(\cos^{-1} 0.95) = 5000 \times 0.3287 = 1643 \text{ VAR.}$$

$$Q_C = Q_1 - Q_2 = 3458 \text{ VAR} = V^2 \omega C \Rightarrow C = 3458 / (230^2 \times 314.16) = 208 \mu\text{F.}$$

Answer: $C \approx 208 \mu\text{F}$.

Problem 9. A balanced star-connected load with $Z_{\text{ph}} = 6 + j8 \Omega$ is connected to a 400 V, 3-phase supply. Find the line current, power factor and total power.

$$V_{\text{ph}} = 400/\sqrt{3} = 230.9 \text{ V. } |Z_{\text{ph}}| = 10 \Omega. I_{\text{ph}} = I_L = 23.09 \text{ A.}$$

$$\cos \phi = 6/10 = 0.6 \text{ lagging.}$$

$$P = \sqrt{3} \times 400 \times 23.09 \times 0.6 = 9600 \text{ W (check: } 3I^2R = 3 \times 533.3 \times 6 = 9600 \text{ W } \square).$$

Answer: $I_L = 23.1 \text{ A}$; $\text{pf} = 0.6 \text{ lag}$; $P = 9.6 \text{ kW}$.

Problem 10. The same three impedances are connected in delta across 400 V. Find the phase current, line current and power. Compare with star.

$$V_{\text{ph}} = 400 \text{ V. } I_{\text{ph}} = 400/10 = 40 \text{ A. } I_L = \sqrt{3} \times 40 = 69.3 \text{ A.}$$

$$P = \sqrt{3} \times 400 \times 69.3 \times 0.6 = 28.8 \text{ kW.}$$

Answer: $I_{\text{ph}} = 40 \text{ A}$, $I_L = 69.3 \text{ A}$, $P = 28.8 \text{ kW} = 3 \times \text{star power}$.

Problem 11. A 3-phase, 400 V induction motor delivers 10 kW output at 85% efficiency and 0.8 pf. Find the line current.

$$\text{Input} = 10/0.85 = 11.76 \text{ kW. } I_L = 11760 / (\sqrt{3} \times 400 \times 0.8) = 21.2 \text{ A.}$$

Answer: $I_L \approx 21.2 \text{ A}$.

Problem 12. Two wattmeters measuring power in a balanced 3-phase load read 5 kW and 2 kW. Find the total power and power factor.

$$P = 5 + 2 = 7 \text{ kW. } \tan \phi = \sqrt{3}(5 - 2)/(5 + 2) = 0.742 \Rightarrow \phi = 36.6^\circ.$$

Answer: $P = 7 \text{ kW}$; $\text{pf} = \cos 36.6^\circ = 0.80$.

Problem 13. In the two-wattmeter method, the readings are 3 kW and -1 kW (after reversing the pressure coil). Find the total power and pf.

$$P = 3 + (-1) = 2 \text{ kW. } \tan \phi = \sqrt{3}(3 - (-1))/(3 + (-1)) = \sqrt{3} \times 4/2 = 3.464 \Rightarrow \phi = 73.9^\circ.$$

Answer: $P = 2 \text{ kW}$; $\text{pf} = \cos 73.9^\circ = 0.277$ (below 0.5, as expected when one reading is negative).

Problem 14. A coil with $R = 10 \Omega$ and $L = 0.1 \text{ H}$ is in parallel with a $10 \mu\text{F}$ capacitor. Find the resonant frequency and the dynamic impedance.

$$f_0 = (1/2\pi)\sqrt{1/(LC) - R^2/L^2} = (1/2\pi)\sqrt{10^6 - 10^4} = (1/2\pi) \times 994.99 = 158.4 \text{ Hz.}$$
$$Z_D = L/(CR) = 0.1/(10^{-5} \times 10) = 1000 \Omega.$$

Answer: $f_0 \approx 158.4 \text{ Hz}$; $Z_D = 1000 \Omega$.

Problem 15. The voltage and current in a circuit are $v = 200 \sin(314t + 30^\circ) \text{ V}$ and $i = 10 \sin(314t - 15^\circ) \text{ A}$. Find the power factor, active power and reactive power.

$$\phi = 30^\circ - (-15^\circ) = 45^\circ \text{ (current lags). } \cos \phi = 0.707.$$

$$V_{\text{rms}} = 141.4 \text{ V, } I_{\text{rms}} = 7.07 \text{ A. } P = 141.4 \times 7.07 \times 0.707 = 707 \text{ W. } Q = 141.4 \times 7.07 \times 0.707 = 707 \text{ VAR.}$$

Answer: pf = 0.707 lag; P = 707 W; Q = 707 VAR.

2.19 More solved numericals: Module II

Problem 1. A 230 V, 50 Hz supply is connected to a 50Ω resistor in series with a 0.2 H inductor and a $50 \mu\text{F}$ capacitor. Find the current and state whether the circuit is inductive or capacitive.

$$X_L = 62.83 \Omega; X_C = 63.66 \Omega; X = -0.83 \Omega. Z = \sqrt{(2500 + 0.69)} = 50.01 \Omega.$$

$$I = 230/50.01 = 4.6 \text{ A, slightly leading.}$$

Answer: $I \approx 4.6 \text{ A}$; slightly capacitive (almost at resonance).

Problem 2. A choke coil takes 5 A at 0.2 pf lagging from a 230 V , 50 Hz supply. Find its resistance and inductance.

$$Z = 230/5 = 46 \Omega. R = Z \cos \phi = 9.2 \Omega. X_L = Z \sin \phi = 46 \times 0.98 = 45.07 \Omega.$$

$$L = 45.07/314.16 = 0.1435 \text{ H.}$$

Answer: $R = 9.2 \Omega$; $L \approx 0.144 \text{ H}$.

Problem 3. A 100 W , 110 V lamp is to be operated from a 230 V , 50 Hz supply. What value of pure inductance must be placed in series?

$$\text{Lamp current} = 100/110 = 0.909 \text{ A; } R = 110/0.909 = 121 \Omega.$$

$$V_L = \sqrt{(230^2 - 110^2)} = \sqrt{(52900 - 12100)} = 202 \text{ V. } X_L = 202/0.909 = 222.2 \Omega.$$

$$L = 222.2/314.16 = 0.707 \text{ H.}$$

Answer: $L \approx 0.71 \text{ H}$ (an inductor wastes no power, unlike a series resistor).

Problem 4. Two impedances $Z_1 = 3 + j4 \Omega$ and $Z_2 = 6 - j8 \Omega$ are in parallel across 230 V . Find the total current and power factor.

$$Z = 4.64 + j2.06 = 5.077 \angle 23.96^\circ \Omega \text{ (Section 2.16).}$$

$$I = 230/5.077 = 45.3 \text{ A, lagging by } 23.96^\circ.$$

Answer: $I \approx 45.3 \text{ A}$; pf = $\cos 23.96^\circ = 0.914$ lagging.

Problem 5. A series circuit has $R = 20 \Omega$, $L = 50 \text{ mH}$ and a variable C , supplied at 100 V , 1 kHz . Find C for resonance and the voltage across L at resonance.

$$C = 1/(\omega^2 L) = 1/((2\pi \times 1000)^2 \times 0.05) = 1/(3.948 \times 10^7 \times 0.05) = 0.507 \mu\text{F.}$$

$$I = 100/20 = 5 \text{ A. } V_L = I\omega L = 5 \times 314.2 = 1571 \text{ V (} Q = 15.7\text{)}.$$

Answer: $C \approx 0.507 \mu\text{F}$; $V_L \approx 1571 \text{ V}$.

Problem 6. A balanced delta-connected load of $(8 + j6) \Omega$ per phase is connected to a 400 V, 3-phase supply. Find phase current, line current, pf, active, reactive and apparent power.

$$Z_{ph} = 10 \Omega. I_{ph} = 400/10 = 40 \text{ A. } I_L = 69.28 \text{ A. pf} = 0.8 \text{ lag.}$$

$$P = \sqrt{3} \times 400 \times 69.28 \times 0.8 = 38.4 \text{ kW. } Q = \sqrt{3} \times 400 \times 69.28 \times 0.6 = 28.8 \text{ kVAR. } S = 48 \text{ kVA.}$$

Answer: $I_{ph} = 40 \text{ A}$; $I_L = 69.3 \text{ A}$; pf 0.8; $P = 38.4 \text{ kW}$; $Q = 28.8 \text{ kVAR}$; $S = 48 \text{ kVA}$.

Problem 7. Three identical coils, each of $R = 20 \Omega$ and $L = 0.05 \text{ H}$, are connected in star to a 400 V, 50 Hz, 3-phase supply. Find the line current and total power. What readings would two wattmeters give?

$$X_L = 15.71 \Omega; Z = \sqrt{(400 + 246.7)} = 25.43 \Omega. I_L = 230.9/25.43 = 9.08 \text{ A. } \cos \phi = 20/25.43 = 0.786, \phi = 38.1^\circ.$$

$$P = \sqrt{3} \times 400 \times 9.08 \times 0.786 = 4.95 \text{ kW (} = 3 \times 9.08^2 \times 20 \text{)}.$$

$$W_1 = 400 \times 9.08 \times \cos(30^\circ - 38.1^\circ) = 3632 \times \cos(-8.1^\circ) = 3596 \text{ W; } W_2 = 3632 \times \cos(68.1^\circ) = 1352 \text{ W.}$$

Answer: $I_L \approx 9.08 \text{ A}$; $P \approx 4.95 \text{ kW}$; $W_1 \approx 3.60 \text{ kW}$, $W_2 \approx 1.35 \text{ kW}$.

Problem 8. A 3-phase, 400 V motor takes 20 kW at 0.75 pf lagging. Find the capacitance per phase (delta-connected) needed to raise the pf to 0.95.

$$Q_1 = 20 \times \tan(41.41^\circ) = 17.64 \text{ kVAR; } Q_2 = 20 \times \tan(18.19^\circ) = 6.57 \text{ kVAR. } Q_C = 11.07 \text{ kVAR total, } 3.69 \text{ kVAR per phase.}$$

$$\text{For delta, } V_{ph} = 400 \text{ V: } C = 3690/(314.16 \times 400^2) = 73.4 \mu\text{F per phase.}$$

Answer: $C \approx 73 \mu\text{F}$ per phase (delta).

Problem 9. A voltage $v = 100 \sin 314t$ is applied to a circuit and the current is $i = 5 \sin(314t - \pi/6)$. Find R and L (series).

$$Z = 100/5 = 20 \Omega; \phi = 30^\circ. R = 20 \cos 30^\circ = 17.32 \Omega; X_L = 20 \sin 30^\circ = 10 \Omega.$$

$$L = 10/314 = 31.8 \text{ mH.}$$

Answer: $R \approx 17.3 \Omega$; $L \approx 31.8 \text{ mH}$.

2.20 Practice problems with solutions: Module II

Q1. Find the peak value of the 230 V (rms) household supply.

$$V_m = \sqrt{2} \times 230.$$

Answer: 325 V.

Q2. A resistance of 20Ω and a capacitance of $100 \mu\text{F}$ are in series across 200 V, 50 Hz. Find the current and power factor.



$X_C = 31.83 \Omega$. $Z = \sqrt{400 + 1013} = 37.6 \Omega$. $I = 200/37.6 = 5.32 \text{ A}$. $\cos \phi = 20/37.6 = 0.53$ leading.

Answer: $I \approx 5.32 \text{ A}$; $\text{pf} \approx 0.53$ lead.

Q3. At what frequency will a 0.1 H inductor have a reactance of 100 Ω ?

$$f = X_L / (2\pi L) = 100 / (0.628).$$

Answer: $f \approx 159 \text{ Hz}$.

Q4. A circuit draws 10 A at 230 V with a power of 1840 W. Find pf, reactive power and apparent power.

$$S = 2300 \text{ VA}. \cos \phi = 1840/2300 = 0.8. Q = \sqrt{(2300^2 - 1840^2)} = 1380 \text{ VAR}.$$

Answer: $\text{pf} = 0.8$; $Q = 1380 \text{ VAR}$; $S = 2300 \text{ VA}$.

Q5. Find the resonant frequency of a series circuit with $L = 50 \text{ mH}$ and $C = 0.5 \mu\text{F}$.

$$f_0 = 1 / (2\pi\sqrt{0.05 \times 5 \times 10^{-7}}) = 1 / (2\pi \times 1.581 \times 10^{-4}).$$

Answer: $f_0 \approx 1007 \text{ Hz}$.

Q6. A series RLC circuit has $f_0 = 1 \text{ kHz}$ and $Q = 50$. Find its bandwidth and half-power frequencies.

$$\text{BW} = f_0 / Q = 20 \text{ Hz}. f_1 \approx 990 \text{ Hz}, f_2 \approx 1010 \text{ Hz}.$$

Answer: $\text{BW} = 20 \text{ Hz}$; 990 Hz and 1010 Hz .

Q7. Three 20 Ω resistors are connected in star across a 400 V, 3-phase supply. Find the line current and total power.

$$V_{\text{ph}} = 230.9 \text{ V}; I_L = 11.55 \text{ A}. P = 3 \times 11.55^2 \times 20 = 8000 \text{ W}.$$

Answer: $I_L \approx 11.5 \text{ A}$; $P = 8 \text{ kW}$.

Q8. A delta-connected load has phase current 10 A. Find the line current.

$$I_L = \sqrt{3} \times 10.$$

Answer: 17.32 A .

Q9. In the two-wattmeter method for a balanced load, both wattmeters read 2 kW. Find the power factor.

$$W_1 = W_2 \Rightarrow \tan \phi = 0 \Rightarrow \phi = 0.$$

Answer: $\text{pf} = 1$ (purely resistive load); $P = 4 \text{ kW}$.

Q10. Two wattmeters read 4 kW and 0 kW. Find the pf.

$$W_2 = 0 \Rightarrow \phi = 60^\circ.$$

Answer: $\text{pf} = 0.5$; $P = 4 \text{ kW}$.

2.21 MCQs: Module II



1. The rms value of a sinusoidal current of peak I_m is
(a) $\sqrt{2} I_m$ (b) $I_m/\sqrt{2}$ (c) $2I_m/\pi$ (d) $I_m/2$
□ (b) — $0.707 I_m$.
2. The average value of a sine wave over a half cycle is
(a) $0.637 I_m$ (b) $0.707 I_m$ (c) $0.5 I_m$ (d) I_m
□ (a) — $2I_m/\pi$.
3. The form factor of a sine wave is
(a) 1.414 (b) 1.0 (c) 1.11 (d) 0.637
□ (c) — rms/average.
4. The peak factor of a sine wave is
(a) 1.11 (b) 1.414 (c) 2 (d) 1.732
□ (b) — peak/rms.
5. AC voltmeters and ammeters read
(a) average values (b) peak values (c) instantaneous values (d) rms values
□ (d) — Effective values.
6. In a pure inductor, the current
(a) is in phase (b) leads the voltage by 90° (c) lags the voltage by 90° (d) leads by 45°
□ (c) — ELI.
7. In a pure capacitor, the current
(a) is in phase (b) lags by 90° (c) leads the voltage by 90° (d) is zero
□ (c) — ICE.
8. The average power consumed by a pure inductor is
(a) VI (b) $VI/2$ (c) I^2X_L (d) zero
□ (d) — Energy stored and returned.
9. Inductive reactance is
(a) proportional to frequency (b) independent of frequency (c) zero at high frequency (d) inversely proportional to frequency
□ (a) — $X_L = 2\pi fL$.
10. Capacitive reactance at DC is
(a) zero (b) equal to R (c) infinite (d) 1Ω
□ (c) — $X_C = 1/(2\pi fC)$, $f = 0$.
11. The power factor of a circuit is
(a) R/Z (b) Z/R (c) X/Z (d) X/R
□ (a) — $\cos \phi$.
12. The unit of reactive power is
(a) J (b) VA (c) W (d) VAR
□ (d) — Volt-ampere reactive.
13. Apparent power S equals
(a) $P - Q$ (b) $P + Q$ (c) PQ (d) $\sqrt{P^2 + Q^2}$
□ (d) — Power triangle.
14. Power factor of a purely resistive circuit is
(a) 0.707 (b) 1 (c) 0 (d) 0.5
□ (b) — $\phi = 0$.
15. In a series RLC circuit at resonance, the impedance is
(a) zero (b) minimum and equal to R (c) infinite (d) maximum
□ (b) — $X_L = X_C$.



- 16.** The resonant frequency of a series RLC circuit is
(a) $1/(LC)$ (b) $1/(2\pi\sqrt{LC})$ (c) $2\pi\sqrt{LC}$ (d) $\sqrt{L/C}$
☐ (b) — $\omega_0^2 = 1/LC$.
- 17.** At series resonance, the power factor is
(a) zero (b) 0.5 leading (c) 0.5 lagging (d) unity
☐ (d) — Resistive.
- 18.** Below the resonant frequency, a series RLC circuit behaves as
(a) open circuit (b) inductive (c) resistive (d) capacitive
☐ (d) — $X_C > X_L$.
- 19.** Voltage magnification occurs in
(a) pure resistive circuits (b) parallel resonance (c) series resonance (d) DC circuits
☐ (c) — $V_L = QV$.
- 20.** The Q-factor of a series resonant circuit is
(a) $R/\omega_0 L$ (b) $\omega_0 CR$ (c) $\omega_0 L/R$ (d) L/CR
☐ (c) — Selectivity.
- 21.** Bandwidth of a series RLC circuit is
(a) $1/(2\pi RC)$ (b) $2\pi L/R$ (c) L/R (d) $R/(2\pi L)$
☐ (d) — $f_2 - f_1$.
- 22.** A parallel resonant circuit is called
(a) amplifier (b) filter-free (c) rejector (d) acceptor
☐ (c) — Minimum current.
- 23.** Dynamic impedance of a parallel resonant circuit is
(a) LC/R (b) R/LC (c) L/CR (d) CR/L
☐ (c) — Maximum impedance.
- 24.** In a three-phase system, the phase difference between two phases is
(a) 90° (b) 60° (c) 180° (d) 120°
☐ (d) — $360^\circ/3$.
- 25.** The sum of three balanced phase voltages at any instant is
(a) $3V_{ph}$ (b) $\sqrt{3} V_{ph}$ (c) zero (d) V_{ph}
☐ (c) — Symmetric phasors.
- 26.** In a star connection, line voltage equals
(a) phase voltage/ $\sqrt{3}$ (b) $3 \times$ phase voltage (c) phase voltage (d) $\sqrt{3} \times$ phase voltage
☐ (d) — $V_L = \sqrt{3}V_{ph}$.
- 27.** In a star connection, line current equals
(a) phase current/ $\sqrt{3}$ (b) $3 \times$ phase current (c) phase current (d) $\sqrt{3} \times$ phase current
☐ (c) — Same current.
- 28.** In a delta connection, line current equals
(a) $\sqrt{3} \times$ phase current (b) phase current/ $\sqrt{3}$ (c) $3 \times$ phase current (d) phase current
☐ (a) — $I_L = \sqrt{3}I_{ph}$.
- 29.** In a delta connection, line voltage equals
(a) phase voltage/ $\sqrt{3}$ (b) phase voltage (c) $\sqrt{3} \times$ phase voltage (d) zero
☐ (b) — Each phase across two lines.
- 30.** Total power in a balanced three-phase circuit is
(a) $\sqrt{3} V_{ph} I_{ph}$ (b) $V_L I_L \cos \phi$ (c) $\sqrt{3} V_L I_L \cos \phi$ (d) $3V_L I_L \cos \phi$
☐ (c) — Line quantities.



- 31.** The same load connected in delta takes how much power compared with star?
(a) $1/3$ (b) 3 times (c) same (d) $\sqrt{3}$ times
□ (b) — V_{ph} increases by $\sqrt{3}$.
- 32.** In the two-wattmeter method, the total power is
(a) $W_1 W_2$ (b) $\sqrt{3}(W_1 + W_2)$ (c) $W_1 - W_2$ (d) $W_1 + W_2$
□ (d) — Algebraic sum.
- 33.** In the two-wattmeter method, if the pf is 0.5, one wattmeter reads
(a) equal to the other (b) zero (c) maximum (d) negative
□ (b) — $\cos(30^\circ + 60^\circ) = 0$.
- 34.** If the two wattmeter readings are equal, the pf is
(a) 1 (b) 0.866 (c) 0 (d) 0.5
□ (a) — $\phi = 0$.
- 35.** In the two-wattmeter method, $\tan \phi$ equals
(a) $(W_1 + W_2)/\sqrt{3}(W_1 - W_2)$ (b) $\sqrt{3}(W_1 - W_2)/(W_1 + W_2)$ (c) $(W_1 - W_2)/(W_1 + W_2)$ (d) $\sqrt{3}W_1/W_2$
□ (b) — Standard formula.
- 36.** A three-phase four-wire system is available with
(a) delta connection (b) both (c) star connection (d) neither
□ (c) — Neutral wire.
- 37.** The advantage of a three-phase supply over single-phase is
(a) lower voltage (b) no need for insulation (c) pulsating power (d) constant power and less conductor material
□ (d) — Main advantages.
- 38.** Low power factor causes
(a) higher current and higher losses (b) less voltage drop (c) lower current (d) higher efficiency
□ (a) — $I = P/(V \cos \phi)$.
- 39.** Power factor of an inductive load is improved by connecting
(a) an inductor in series (b) a capacitor in parallel (c) a diode (d) a resistor in parallel
□ (b) — Supplies leading VAR.
- 40.** The frequency of the Indian power supply is
(a) 100 Hz (b) 25 Hz (c) 60 Hz (d) 50 Hz
□ (d) — Standard.

Module III: Magnetic Circuits and DC Machines

Electrical machines convert mechanical energy into electrical energy (generators) and electrical energy into mechanical energy (motors). Both use a magnetic field as the coupling medium. This module reviews electromagnetic induction, analyses magnetic circuits, and studies DC machines.

3.1 Review of fundamental laws of electromagnetic induction

Faraday's first law. Whenever the magnetic flux linked with a circuit changes, an emf is induced in it.

Faraday's second law. The magnitude of the induced emf equals the rate of change of flux linkage:

$$e = -N d\Phi/dt$$

Lenz's law. The direction of the induced emf is such that the current it produces opposes the change that causes it (hence the minus sign). Lenz's law is a consequence of conservation of energy.

Dynamically induced emf. A conductor of length l moving with velocity v perpendicular to a field B has an induced emf $e = Blv$ (or $Blv \sin \theta$ at angle θ). Direction:

Fleming's right-hand rule (thumb = motion, first finger = field, middle finger = induced current). This is the principle of the generator.

Statically induced emf.

- **Self-induced:** a changing current in a coil induces an emf in the same coil: $e = -L di/dt$. Self-inductance $L = N\Phi/I = N^2/S$ (S = reluctance) $= \mu_0 \mu_r N^2 A/l$.
- **Mutually induced:** a changing current in one coil induces an emf in a neighbouring coil: $e_2 = -M di_1/dt$. $M = kv(L_1 L_2)$, where k is the coefficient of coupling ($0 \leq k \leq 1$). This is the principle of the transformer.

Force on a current-carrying conductor. $F = BIl$. Direction: **Fleming's left-hand rule** (first finger = field, middle finger = current, thumb = force/motion). This is the principle of the motor.

Energy stored in an inductor: $W = (1/2)LI^2$.

3.2 Magnetic circuit concepts

A **magnetic circuit** is the closed path followed by magnetic flux, usually through an iron core (e.g. transformer core, machine yoke and poles). It is analysed like an electric circuit.

Magnetic circuit	Electric circuit
Magnetomotive force (mmf) $F = NI$ (AT)	Electromotive force E (V)
Flux Φ (Wb)	Current I (A)
Reluctance $S = l/(\mu_0 \mu_r A)$ (AT/Wb)	Resistance $R = \rho l/A$ (Ω)
Permeance $= 1/S$	Conductance $= 1/R$
Flux density $B = \Phi/A$ (T)	Current density $J = I/A$
Magnetising force $H = NI/l$ (AT/m)	Electric field $E = V/l$
Permeability μ	Conductivity σ

Magnetic circuit	Electric circuit
$\Phi = F/S$ (Ohm's law of magnetic circuits)	$I = E/R$

Differences. Flux does not actually "flow"; energy is needed only to establish the flux (not to maintain it), whereas current in a resistor dissipates energy continuously. There is no perfect magnetic insulator, so leakage flux always exists. μ of iron varies with B (saturation), so the magnetic circuit is non-linear.

$$B = \mu_0 \mu_r H; \mu_0 = 4\pi \times 10^{-7} \text{ H/m}; \text{mmf} = NI = \Phi \times S = H \times l$$

Series magnetic circuit (e.g. iron core with an air gap): total mmf = sum of mmfs for each part: $NI = H_1 l_1 + H_2 l_2 + H_g l_g$, where for the air gap $H_g = B_g / \mu_0$. Even a small air gap needs a large mmf because $\mu_r = 1$ for air.

Parallel magnetic circuit (e.g. shell-type core): the flux divides between the side limbs like current in parallel resistors; the mmf across parallel paths is the same.

Leakage and fringing. Leakage flux is the part of the flux that does not follow the intended path. Leakage coefficient $\lambda = \text{total flux/useful flux}$ (1.1–1.25). **Fringing** is the spreading of flux lines at an air gap, which increases the effective area of the gap.

3.3 B-H curve and hysteresis

The **B-H (magnetisation) curve** of a ferromagnetic material is non-linear: B rises steeply at first, then more slowly (knee), and finally saturates. $\mu_r = B/(\mu_0 H)$ is therefore not constant.

When H is taken through a complete cycle, B lags behind H , tracing a closed **hysteresis loop**.

- **Retentivity (residual flux density B_r):** B remaining when H is reduced to zero.
- **Coercivity (coercive force H_c):** reverse H needed to reduce B to zero.
- The area of the loop equals the energy lost per unit volume per cycle, dissipated as heat.

$$\text{Hysteresis loss (Steinmetz): } P_h = \eta B_m^{1.6} f V \text{ (W)}$$

$$\text{Eddy-current loss: } P_e = k_e B_m^2 f^2 t^2 V \text{ (W)}, t = \text{lamination thickness}$$

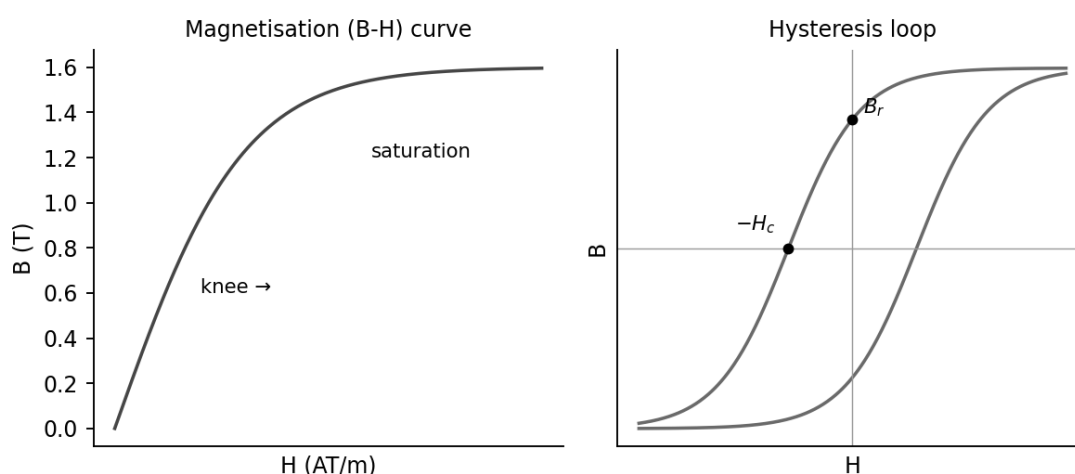


Figure 3.1: Magnetisation curve and hysteresis loop of a ferromagnetic material.

- **Soft magnetic materials** (silicon steel): narrow loop, low hysteresis loss; used for transformer and machine cores.
- **Hard magnetic materials** (Alnico, ferrite): wide loop, high retentivity and coercivity; used for permanent magnets.
- Eddy-current loss is reduced by **laminating** the core (thin insulated sheets) and using high-resistivity silicon steel.
- Hysteresis and eddy-current losses together are called **iron (core) losses**. (See BEE Lab, Experiment 5: B-H curve.)

3.4 Principle of electromechanical energy conversion

In both motors and generators, the same two effects occur simultaneously:

- A conductor moving in a magnetic field has an emf induced in it ($e = Blv$).
- A current-carrying conductor in a magnetic field experiences a force ($F = BIl$).

In a **generator**, mechanical input moves the conductors; the induced emf drives current into the load; this current interacts with the field to produce a force (torque) that opposes the motion (Lenz), so mechanical work must be done: mechanical energy → electrical energy.

In a **motor**, the supply drives current through the conductors; the force produces torque and rotation; the rotation induces a **back emf** that opposes the supply, so the supply does work against it: electrical energy → mechanical energy.

Energy balance (motor): $V I_a = E_b I_a + I_a^2 R_a \Rightarrow$ electrical input = mechanical power developed + copper loss.

3.5 Construction of a DC machine

The same machine can work as a generator or a motor. Main parts:

- **Yoke (frame)**: outer cast-iron or steel frame; provides mechanical support and carries the magnetic flux between poles.
- **Poles and pole shoes**: laminated steel cores bolted to the yoke; carry the field windings. Pole shoes spread the flux over the air gap and support the field coils.
- **Field winding**: coils on the poles, excited by DC, producing the main flux.
- **Armature core**: cylindrical, made of laminated silicon-steel discs (to reduce eddy losses), with slots on its surface for the armature conductors.
- **Armature winding**: copper conductors in the slots, where emf is induced (generator) or force is produced (motor). **Lap winding**: number of parallel paths $A = P$ (used for high current, low voltage). **Wave winding**: $A = 2$ (used for high voltage, low current).
- **Commutator**: a cylinder of wedge-shaped copper segments insulated by mica, connected to the armature coils. It converts the alternating emf in the armature coils into unidirectional (DC) voltage at the brushes (generator), or reverses the current in the coils at the right moment to keep the torque unidirectional (motor). It acts as a mechanical rectifier/inverter.
- **Brushes**: carbon or graphite blocks pressing on the commutator, which collect current from (or supply current to) the armature.
- **Bearings and shaft**.

3.6 EMF equation of a DC generator

Let P = number of poles, Φ = flux per pole (Wb), Z = total number of armature conductors, A = number of parallel paths, N = speed in rpm.

- In one revolution, each conductor cuts $P\Phi$ webers of flux, in time $60/N$ seconds.
- Average emf per conductor = $P\Phi/(60/N) = P\Phi N/60$ volts.
- The conductors in each parallel path are in series: Z/A conductors per path.

$$E_g = P\Phi ZN/(60A) \text{ (volts); } A = P \text{ for lap winding, } A = 2 \text{ for wave winding}$$

For a given machine, $E_g = k\Phi N$ ($k = PZ/60A$): the generated emf is proportional to flux and speed.

3.7 Types of DC machines (by method of excitation)

- **Separately excited:** field winding supplied from an independent DC source.
- **Self-excited:** field winding supplied by the machine's own armature.
- **Shunt:** field winding (many turns of thin wire, high resistance R_{sh}) in parallel with the armature.
- **Series:** field winding (few turns of thick wire, low resistance R_{se}) in series with the armature.
- **Compound:** both shunt and series windings. Long-shunt (shunt field across armature + series field) or short-shunt (shunt field across armature only). Cumulative (series field aids shunt field) or differential (opposes).

Current and voltage relations (generator).

Type	Currents	Terminal voltage
Separately excited	$I_a = I_L$	$V = E_g - I_a R_a$
Shunt	$I_a = I_L + I_{sh}$, $I_{sh} = V/R_{sh}$	$V = E_g - I_a R_a$
Series	$I_a = I_{se} = I_L$	$V = E_g - I_a (R_a + R_{se})$
Long-shunt compound	$I_a = I_{se} = I_L + I_{sh}$	$V = E_g - I_a (R_a + R_{se})$

(Include brush drop, typically 1-2 V, if given.)

Open-circuit characteristic (OCC, magnetisation curve). Graph of generated emf E_0 (no load) against field current I_f at constant speed. It starts from a small residual voltage (residual magnetism), rises linearly, then bends and saturates like the B-H curve. For a self-excited shunt generator to build up voltage: (1) there must be residual magnetism; (2) the field winding must be connected so that its flux aids the residual flux; (3) the field-circuit resistance must be less than the **critical field resistance** (slope of the OCC's linear part); (4) the speed must be above the critical speed. (See BEE Lab, Experiment 6.)

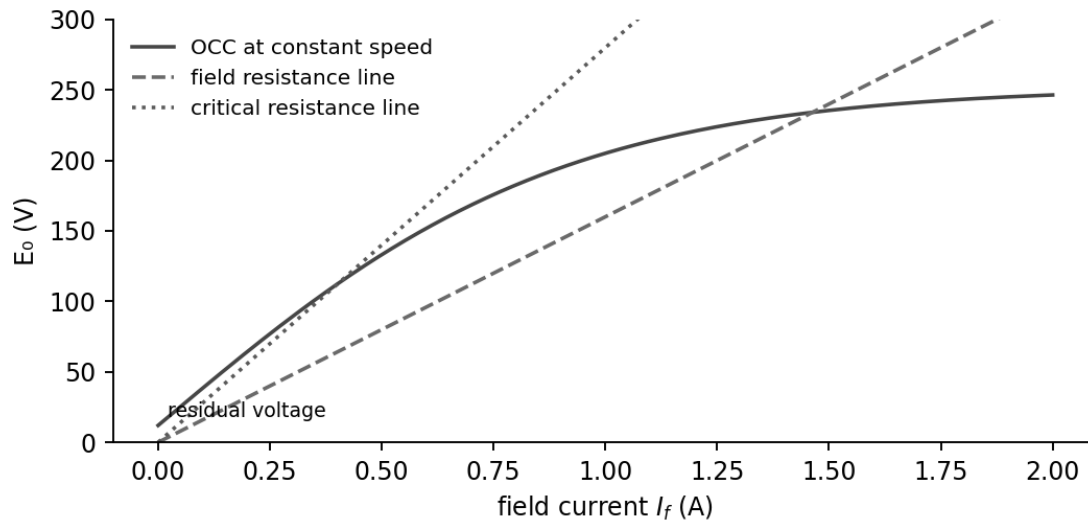


Figure 3.2: Open-circuit characteristic of a DC shunt generator. Voltage builds up to the intersection of the OCC and the field resistance line.

3.8 DC motor: principle and back emf

When a current-carrying armature is placed in the magnetic field of the poles, each conductor experiences a force $F = BIl$ (Fleming's left-hand rule). The forces on conductors under N and S poles produce a torque in the same direction (because the commutator reverses the current as conductors pass from one pole to the next), so the armature rotates.

Back emf. The rotating armature conductors cut the flux and have an emf induced in them, which by Lenz's law opposes the applied voltage. This is the **back (counter) emf**:

$$E_b = P\Phi ZN/(60A), V = E_b + I_a R_a \Rightarrow I_a = (V - E_b)/R_a$$

Significance of back emf. It makes the motor self-regulating: when the load increases, the speed falls slightly, E_b falls, and I_a increases to supply the extra torque. At starting, $N = 0$ so $E_b = 0$, and the armature current V/R_a would be dangerously large (R_a is small); hence a **starter** (series resistance) is needed.

Power equation. Multiplying by I_a : $V I_a = E_b I_a + I_a^2 R_a$. $E_b I_a$ = gross mechanical power developed. It is maximum when $E_b = V/2$, but then efficiency is below 50%, so motors never run in this condition.

3.9 Torque equation of a DC motor

Mechanical power developed = $E_b I_a = T_a \omega = T_a \times 2\pi N/60$. Substituting $E_b = P\Phi ZN/60A$:

$$T_a = (P\Phi Z I_a)/(2\pi A) = 0.159 P\Phi Z I_a/A \text{ (N m)} \Rightarrow T_a \propto \Phi I_a$$

Also $T_a = 9.55 E_b I_a/N \text{ (N m)}$; shaft torque $T_{sh} = 9.55 \times \text{output}/N$

Speed equation. $N = (V - I_a R_a)/(k\Phi) \Rightarrow N \propto E_b/\Phi$.

Characteristics of DC motors.

Motor	Torque vs armature current	Speed vs load	Applications
Shunt ($\Phi \approx \text{constant}$)	$T \propto I_a$ (straight line)	Nearly constant speed (falls slightly)	Lathes, fans, blowers, centrifugal pumps, machine tools
Series ($\Phi \propto I_a$)	$T \propto I_a^2$ (before saturation): very high starting torque	Speed falls sharply with load; dangerously high speed at no load	Electric traction, cranes, hoists, trolleys
Cumulative compound	Between shunt and series	Speed falls more than shunt	Rolling mills, presses, shears, elevators

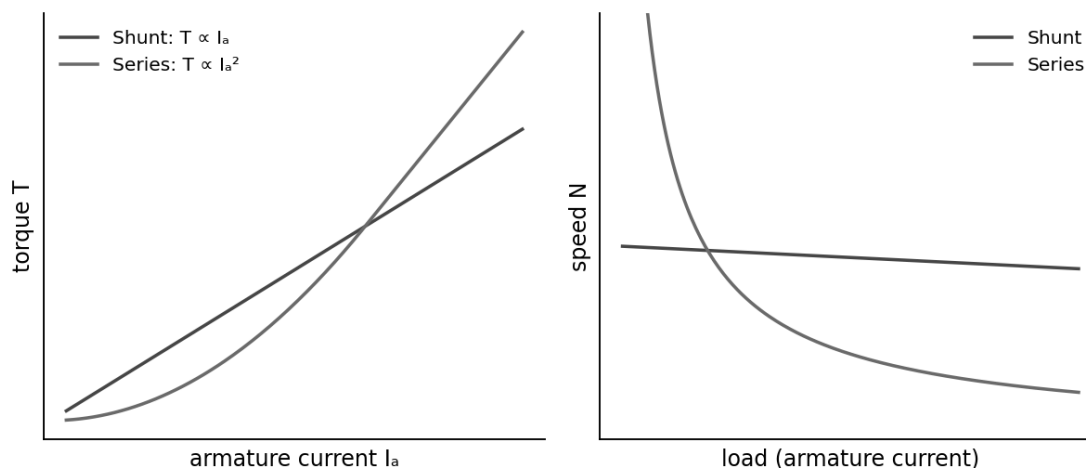


Figure 3.3: Torque and speed characteristics of DC shunt and series motors.

Note: A DC series motor must never be started or run without load, because Φ is very small and the speed rises to dangerous values ($N \propto 1/\Phi$).

3.10 Speed control of DC motors

From $N \propto (V - I_a R_a)/\Phi$, speed can be controlled by changing Φ , R_a or V .

(1) Flux (field) control. A variable resistance (field rheostat) in series with the shunt field reduces I_{sh} and Φ , so speed **increases**. Gives speeds **above** rated speed. Simple, cheap and efficient (the field current is small, so losses are small). For series motors, a **diverter** resistance in parallel with the series field is used.

(2) Armature (rheostatic) control. A variable resistance in series with the armature increases the voltage drop, reducing E_b and the speed. Gives speeds **below** rated speed. Wasteful (large $I_a^2 R$ loss in the rheostat) and speed regulation is poor. Used for small motors and short periods.

(3) Voltage control (Ward-Leonard system / modern power-electronic drives). The armature voltage is varied (using a variable-voltage DC generator or a controlled rectifier/chopper) while the field is kept constant. Gives smooth, wide-range speed control in both directions. Used in lifts, rolling mills, paper machines and electric vehicles.

Starting. A 3-point starter (for shunt motors) inserts a resistance in series with the armature at starting and cuts it out as the speed builds up; it also has a no-volt release and an overload release for protection. (See BEE Lab, Experiment 4.)

3.11 Losses and efficiency of DC machines

- **Copper losses:** armature $I_a^2 R_a$, field $I_{sh}^2 R_{sh}$ or $I_{se}^2 R_{se}$, brush contact loss.
- **Iron (core) losses:** hysteresis and eddy currents in the armature core (nearly constant at constant speed).
- **Mechanical losses:** friction and windage.
- **Constant losses** = iron + mechanical + shunt field copper loss. **Variable losses** = armature copper loss.

Efficiency η = output/input = output/(output + losses); maximum efficiency when variable loss = constant loss

3.12 Derivations and explanations for long answers

(A) Self-inductance of a coil in terms of its dimensions. For a coil of N turns on a core of length l and area A : $H = NI/l$, $B = \mu_0 \mu_r NI/l$, $\Phi = BA = \mu_0 \mu_r NIA/l$. $L = N\Phi/I = \mu_0 \mu_r N^2 A/l = N^2/S$. Inductance is proportional to N^2 and to the permeability of the core.

(B) Energy stored in a magnetic field. While the current rises from 0 to I , the source does work against the self-induced emf: $W = \int e i dt = \int L (di/dt) i dt = \int_0^I L i di = (1/2)LI^2$. Energy per unit volume in the field = $B^2/(2\mu) = (1/2)BH$.

(C) Why armature reaction occurs (DC machines). When the armature carries current, it produces its own mmf, which distorts and weakens the main field (cross-magnetising and demagnetising effects). This shifts the magnetic neutral axis and can cause sparking at the brushes. Remedies: interpoles (commutating poles) and compensating windings. Interpoles also help **commutation**, the process of reversing the current in a coil while it is short-circuited by a brush.

(D) Self-regulation of a DC shunt motor (step by step). Load torque increases $\rightarrow$ motor slows slightly $\rightarrow E_b = k\Phi N$ falls $\rightarrow I_a = (V - E_b)/R_a$ rises sharply (R_a is small) $\rightarrow$ developed torque $k\Phi I_a$ rises until it equals the new load torque $\rightarrow$ the speed settles at a slightly lower value. Because R_a is small, the speed drop from no load to full load is only 5–10%.

(E) Speed regulation. % Speed regulation = $(N_0 - N_{FL})/N_{FL} \times 100$. Shunt motors: 5–10% (nearly constant speed). Series motors: very large (speed falls sharply with load).

(F) Why a series motor has high starting torque. At start, the current is large; since the field winding carries the same current, the flux is also large. $T \propto \Phi I_a \propto I_a^2$, so the starting torque is much higher than that of a shunt motor with the same current. This suits traction, cranes and hoists.

3.13 Losses and efficiency: power stages of DC machines

Generator. Mechanical input $\rightarrow$ (iron and mechanical losses) $\rightarrow$ electrical power developed $E_g I_a \rightarrow$ (copper losses) $\rightarrow$ electrical output $V_L I_L$.

Motor. Electrical input $V_L I_L \rightarrow$ (copper losses) $\rightarrow$ mechanical power developed $E_b I_a \rightarrow$ (iron and mechanical losses) $\rightarrow$ mechanical output (shaft power).

Mechanical efficiency = output/(power developed); Electrical efficiency = (power developed)/input; Overall η = output/input

Maximum efficiency occurs when variable (armature copper) loss equals constant losses: $I_a^2 R_a = W_c$, i.e. at $I_a = \sqrt{W_c/R_a}$.

3.14 Applications of DC machines

Machine	Typical applications
Separately excited generator	Laboratory testing, Ward-Leonard speed control, electroplating
Shunt generator	Battery charging, lighting, excitation of alternators
Compound generator	Constant-voltage supply to distant loads (over-compounded)
Shunt motor	Lathes, drilling machines, fans, blowers, conveyors, spinning machines
Series motor	Electric traction, cranes, hoists, lifts, mixers, vacuum cleaners (universal motors)
Compound motor	Rolling mills, presses, shears, punching machines, elevators

Today DC motors are increasingly replaced by induction motors with variable-frequency drives and by brushless DC (BLDC) motors, which need less maintenance, but DC machines remain important for understanding all electrical machines.

3.15 Solved numericals: Module III

Problem 1. A coil of 500 turns carries a current that changes from 5 A to 1 A in 0.1 s, and the flux changes from 2 mWb to 0.4 mWb. Find the average induced emf and the self-inductance.

$$e = N d\Phi/dt = 500 \times (2 - 0.4) \times 10^{-3}/0.1 = 8 \text{ V.}$$

$$L = N d\Phi/dI = 500 \times 1.6 \times 10^{-3}/4 = 0.2 \text{ H (check: } e = L di/dt = 0.2 \times 4/0.1 = 8 \text{ V)}.$$

Answer: e = 8 V; L = 0.2 H.

Problem 2. A conductor 0.5 m long moves at 20 m/s perpendicular to a field of 0.8 T. Find the induced emf. If it carries 10 A, what force acts on it?

$$e = Blv = 0.8 \times 0.5 \times 20 = 8 \text{ V. } F = BIl = 0.8 \times 10 \times 0.5 = 4 \text{ N.}$$

Answer: e = 8 V; F = 4 N.

Problem 3. Two coils have self-inductances of 0.4 H and 0.9 H and a coefficient of coupling of 0.5. Find the mutual inductance.

$$M = k\sqrt{L_1 L_2} = 0.5 \times \sqrt{0.36} = 0.5 \times 0.6.$$

Answer: M = 0.3 H.

Problem 4. An iron ring of mean length 50 cm and cross-section 4 cm² ($\mu_r = 800$) is wound with 500 turns. Find the current needed to produce a flux of 0.4 mWb.

$$S = l/(\mu_0 \mu_r A) = 0.5/(4\pi \times 10^{-7} \times 800 \times 4 \times 10^{-4}) = 1.243 \times 10^6 \text{ AT/Wb.}$$

$$\text{mmf} = \Phi S = 0.4 \times 10^{-3} \times 1.243 \times 10^6 = 497 \text{ AT. } I = 497/500 = 0.995 \text{ A.}$$

Answer: $I \approx 1.0 \text{ A}$ ($B = 1 \text{ T}$).

Problem 5. If a 1 mm air gap is cut in the ring of the previous problem, find the new current for the same flux (ignore fringing).

$$S_{\text{gap}} = 10^{-3} / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) = 1.989 \times 10^6 \text{ AT/Wb (iron length change is negligible).}$$

$$\text{Total } S = 3.232 \times 10^6 \text{ AT/Wb; mmf} = 0.4 \times 10^{-3} \times 3.232 \times 10^6 = 1293 \text{ AT. } I = 1293/500 = 2.59 \text{ A.}$$

Answer: $I \approx 2.6 \text{ A}$. The 1 mm gap needs more mmf than the 50 cm of iron.

Problem 6. A 4-pole, lap-wound DC generator has 500 armature conductors and a flux of 0.02 Wb per pole. Find the emf generated at 1000 rpm. What would it be if wave wound?

$$\text{Lap: } E = P\Phi ZN / (60A) = 4 \times 0.02 \times 500 \times 1000 / (60 \times 4) = 166.7 \text{ V.}$$

$$\text{Wave (} A = 2 \text{): } E = 333.3 \text{ V.}$$

Answer: 166.7 V (lap); 333.3 V (wave).

Problem 7. A shunt generator supplies 100 A at a terminal voltage of 250 V. The armature and shunt field resistances are 0.1 Ω and 125 Ω . Find the generated emf.

$$I_{\text{sh}} = 250/125 = 2 \text{ A. } I_a = I_L + I_{\text{sh}} = 102 \text{ A.}$$

$$E_g = V + I_a R_a = 250 + 102 \times 0.1 = 260.2 \text{ V.}$$

Answer: $E_g = 260.2 \text{ V}$.

Problem 8. A 220 V DC shunt motor has $R_a = 0.5 \Omega$ and takes an armature current of 20 A at 1000 rpm. Find the back emf and the speed when the armature current rises to 40 A (flux constant).

$$E_{b1} = 220 - 20 \times 0.5 = 210 \text{ V. } E_{b2} = 220 - 40 \times 0.5 = 200 \text{ V.}$$

$$N \propto E_b \text{ (}\Phi \text{ constant): } N_2 = 1000 \times 200/210 = 952 \text{ rpm.}$$

Answer: $E_b = 210 \text{ V}$; $N_2 \approx 952 \text{ rpm}$.

Problem 9. A 4-pole, lap-wound DC motor has 400 conductors, a flux of 25 mWb per pole and takes an armature current of 50 A. Find the armature torque.

$$T_a = 0.159 P\Phi Z I_a / A = 0.159 \times 4 \times 0.025 \times 400 \times 50 / 4 = 79.5 \text{ N m.}$$

Answer: $T_a \approx 79.6 \text{ N m}$.

Problem 10. A 220 V series motor ($R_a + R_{se} = 0.5 \Omega$) runs at 800 rpm taking 20 A. Find the speed when the load torque is doubled (assume $\Phi \propto I$).

$$T \propto \Phi I \propto I^2. \text{ Doubling } T \Rightarrow I_2 = 20\sqrt{2} = 28.28 \text{ A.}$$

$$E_{b1} = 220 - 10 = 210 \text{ V; } E_{b2} = 220 - 14.14 = 205.86 \text{ V.}$$

$$N_2/N_1 = (E_{b2}/E_{b1})(\Phi_1/\Phi_2) = (205.86/210)(20/28.28) = 0.6933.$$

Answer: $N_2 \approx 555$ rpm.

Problem 11. A 220 V shunt motor ($R_a = 0.2 \Omega$) runs at 1000 rpm with $I_a = 50$ A. The field is weakened so that the flux falls by 20%. Find the new speed if the load torque is constant.

$$T \propto \Phi I_a \text{ constant} \Rightarrow I_{a2} = 50/0.8 = 62.5 \text{ A.}$$

$$E_{b1} = 220 - 10 = 210 \text{ V; } E_{b2} = 220 - 12.5 = 207.5 \text{ V.}$$

$$N_2 = 1000 \times (207.5/210)/0.8 = 1235 \text{ rpm.}$$

Answer: $N_2 \approx 1235$ rpm (field weakening raises the speed).

Problem 12. A 220 V shunt motor with $R_a = 0.25 \Omega$ runs at 1000 rpm at $I_a = 40$ A. What resistance must be added in the armature circuit to reduce the speed to 800 rpm at the same torque?

$$E_{b1} = 220 - 10 = 210 \text{ V. Torque and flux constant} \Rightarrow I_a = 40 \text{ A. } E_{b2} = 0.8 \times 210 = 168 \text{ V.}$$

$$168 = 220 - 40(0.25 + R) \Rightarrow 0.25 + R = 1.3 \Rightarrow R = 1.05 \Omega.$$

Answer: $R_{\text{ext}} = 1.05 \Omega$.

Problem 13. A 220 V shunt motor takes 20 A from the supply. $R_{\text{sh}} = 220 \Omega$, $R_a = 0.4 \Omega$, and the iron plus mechanical losses are 300 W. Find the output and efficiency.

$$I_{\text{sh}} = 1 \text{ A; } I_a = 19 \text{ A. Input} = 220 \times 20 = 4400 \text{ W.}$$

$$\text{Armature copper loss} = 19^2 \times 0.4 = 144.4 \text{ W; field loss} = 220 \times 1 = 220 \text{ W.}$$

$$\text{Output} = 4400 - 144.4 - 220 - 300 = 3735.6 \text{ W.}$$

Answer: Output ≈ 3.74 kW; $\eta \approx 84.9\%$.

Problem 14. Why does a DC motor need a starter? Calculate the starting current of a 220 V motor with $R_a = 0.5 \Omega$ without a starter, and the starter resistance needed to limit it to $2 \times$ full-load current of 20 A.

$$\text{At start } E_b = 0 \Rightarrow I = 220/0.5 = 440 \text{ A (22 times full load).}$$

$$\text{To limit to 40 A: } 220/(0.5 + R) = 40 \Rightarrow R = 5 \Omega.$$

Answer: 440 A without a starter; a 5Ω starter resistance is needed.

3.16 More solved numericals: Module III

Problem 1. A coil of 1000 turns is wound on a non-magnetic ring of mean diameter 20 cm and cross-section 5 cm^2 . Find its self-inductance.

$$l = \pi \times 0.2 = 0.6283 \text{ m. } L = \mu_0 N^2 A / l = 4\pi \times 10^{-7} \times 10^6 \times 5 \times 10^{-4} / 0.6283.$$

Answer: $L \approx 1.0$ mH.

Problem 2. If the ring of the previous problem is replaced by iron of $\mu_r = 1000$, find the inductance and the energy stored at 2 A.

$$L = 1000 \times 1 \text{ mH} = 1 \text{ H. } W = (1/2) \times 1 \times 4 = 2 \text{ J.}$$

Answer: $L = 1 \text{ H}$; $W = 2 \text{ J}$.

Problem 3. A magnetic circuit consists of an iron path 80 cm long ($\mu_r = 1200$) and an air gap 2 mm long, both of area 10 cm^2 . How many ampere-turns are needed for a flux of 1 mWb?

$$B = 1 \text{ T. Iron: } H_i = B/(\mu_0 \mu_r) = 1/(4\pi \times 10^{-7} \times 1200) = 663 \text{ AT/m; } AT_i = 663 \times 0.8 = 531 \text{ AT.}$$

$$\text{Gap: } H_g = 1/(4\pi \times 10^{-7}) = 7.96 \times 10^5 \text{ AT/m; } AT_g = 7.96 \times 10^5 \times 0.002 = 1592 \text{ AT.}$$

Answer: Total $\approx 2123 \text{ AT}$ (the 2 mm gap needs three times the mmf of the 80 cm iron path).

Problem 4. A 4-pole DC generator with a lap winding has 51 slots, each containing 20 conductors. The flux per pole is 7 mWb. Find the emf at 1500 rpm.

$$Z = 51 \times 20 = 1020. E = 4 \times 0.007 \times 1020 \times 1500/(60 \times 4) = 178.5 \text{ V.}$$

Answer: $E = 178.5 \text{ V}$.

Problem 5. A long-shunt compound generator supplies 50 A at 250 V. $R_a = 0.05 \Omega$, $R_{se} = 0.03 \Omega$, $R_{sh} = 125 \Omega$. Find the generated emf (brush drop 2 V).

$$I_{sh} = 250/125 = 2 \text{ A; } I_a = I_{se} = 52 \text{ A.}$$

$$E_g = 250 + 52(0.05 + 0.03) + 2 = 250 + 4.16 + 2 = 256.2 \text{ V.}$$

Answer: $E_g \approx 256.2 \text{ V}$.

Problem 6. A 230 V DC shunt motor has $R_a = 0.4 \Omega$ and $R_{sh} = 230 \Omega$. At no load it takes 5 A and runs at 1000 rpm. Find its speed when it takes 30 A from the supply.

$$\text{No load: } I_{a0} = 5 - 1 = 4 \text{ A; } E_{b0} = 230 - 1.6 = 228.4 \text{ V.}$$

$$\text{Load: } I_a = 29 \text{ A; } E_b = 230 - 11.6 = 218.4 \text{ V.}$$

$$N = 1000 \times 218.4/228.4 = 956 \text{ rpm.}$$

Answer: $N \approx 956 \text{ rpm}$ (speed regulation $\approx 4.6\%$).

Problem 7. For the motor of the previous problem, find the efficiency at 30 A input, assuming the no-load input covers the constant losses.

$$\text{Constant losses} = \text{no-load input} - \text{no-load armature Cu loss} = 230 \times 5 - 4^2 \times 0.4 = 1150 - 6.4 = 1143.6 \text{ W.}$$

$$\text{At 30 A: input} = 6900 \text{ W; armature Cu loss} = 29^2 \times 0.4 = 336.4 \text{ W. Output} = 6900 - 1143.6 - 336.4 = 5420 \text{ W.}$$

Answer: $\eta = 5420/6900 \approx 78.6\%$.

Problem 8. A 6-pole, wave-wound DC motor has 500 conductors, flux 20 mWb per pole, and takes an armature current of 60 A. Find the torque and the speed if the supply is 440 V and $R_a = 0.5 \Omega$.

$$T = 0.159 \times 6 \times 0.02 \times 500 \times 60/2 = 286 \text{ N m.}$$

$$E_b = 440 - 30 = 410 \text{ V. } N = 60AE_b/(P\Phi Z) = 60 \times 2 \times 410/(6 \times 0.02 \times 500) = 820 \text{ rpm.}$$

Answer: $T \approx 286 \text{ N m}$; $N = 820 \text{ rpm}$.

Problem 9. A DC series motor runs at 1000 rpm taking 40 A from a 230 V supply ($R_a + R_{se} = 0.3 \Omega$). A diverter reduces the field current to 60% of the armature current. Find the new speed if the torque is unchanged ($\Phi \propto$ field current).

Initially $\Phi_1 \propto 40$. With the diverter, $\Phi_2 \propto 0.6I_{a2}$. Torque: $\Phi_1 I_{a1} = \Phi_2 I_{a2} \Rightarrow 40 \times 40 = 0.6I_{a2}^2 \Rightarrow I_{a2} = 51.64 \text{ A}$.

$E_{b1} = 230 - 12 = 218 \text{ V}$. Neglecting the small change in circuit resistance due to the diverter, $E_{b2} \approx 230 - 51.64 \times 0.3 = 214.5 \text{ V}$.

$$N_2 = 1000 \times (214.5/218) \times (40/(0.6 \times 51.64)) = 1000 \times 0.984 \times 1.291 = 1270 \text{ rpm.}$$

Answer: $N_2 \approx 1270 \text{ rpm}$ (field weakening raises the speed).

Problem 10. The hysteresis loop of a transformer core has an area equivalent to 250 J/m^3 per cycle. Find the hysteresis loss in a core of volume 0.01 m^3 at 50 Hz.

$$P_h = \text{energy per cycle} \times \text{volume} \times f = 250 \times 0.01 \times 50.$$

Answer: $P_h = 125 \text{ W}$.

3.17 Practice problems with solutions: Module III

Q1. A coil of 200 turns links a flux of 5 mWb when carrying 2 A. Find its inductance.

$$L = N\Phi/I = 200 \times 5 \times 10^{-3}/2.$$

Answer: $L = 0.5 \text{ H}$.

Q2. Find the energy stored in a 0.5 H inductor carrying 4 A.

$$W = (1/2)LI^2 = 0.5 \times 0.5 \times 16.$$

Answer: 4 J.

Q3. Find the reluctance of an air gap 2 mm long and 10 cm^2 in area.

$$S = l/(\mu_0 A) = 2 \times 10^{-3}/(4\pi \times 10^{-7} \times 10^{-3}).$$

Answer: $S \approx 1.59 \times 10^6 \text{ AT/Wb}$.

Q4. An 8-pole wave-wound generator has 960 conductors and a flux of 40 mWb. Find the emf at 400 rpm.

$$E = 8 \times 0.04 \times 960 \times 400/(60 \times 2).$$

Answer: $E = 1024 \text{ V}$.

Q5. A 6-pole lap-wound generator has 720 conductors and generates 250 V at 500 rpm. Find the flux per pole.



$$\Phi = 60AE/(PZN) = 60 \times 6 \times 250/(6 \times 720 \times 500).$$

Answer: $\Phi \approx 41.7 \text{ mWb}$.

Q6. A 250 V shunt motor has $R_a = 0.5 \Omega$ and draws $I_a = 30 \text{ A}$ at 1200 rpm. Find the gross mechanical power and torque.

$$E_b = 250 - 15 = 235 \text{ V. } P = E_b I_a = 7050 \text{ W. } T = 9.55 \times 7050/1200.$$

Answer: $P = 7.05 \text{ kW}$; $T \approx 56.1 \text{ N m}$.

Q7. A DC series motor takes 30 A at 600 rpm. At what speed will it run when it takes 15 A (neglect resistance drops, $\Phi \propto I$)?

$$N \propto 1/\Phi \propto 1/I \text{ (} E_b \approx V \text{)}.$$

Answer: 1200 rpm.

Q8. The speed of a shunt motor is to be increased. Which control method should be used?

Flux control (field rheostat): reducing the field current reduces Φ and raises the speed.

Answer: Field (flux) control.

3.18 MCQs: Module III

1. The induced emf $e = -N d\Phi/dt$ is known as
(a) Coulomb's law (b) Ampère's law (c) Ohm's law (d) Faraday's law
☐ (d) — Law of induction.
2. The minus sign in Faraday's law is due to
(a) Kirchhoff (b) Fleming (c) Ohm (d) Lenz
☐ (d) — Opposition to change.
3. Lenz's law is a consequence of the law of conservation of
(a) mass (b) energy (c) momentum (d) charge
☐ (b) — Opposing force requires work.
4. Fleming's right-hand rule gives the direction of
(a) magnetic field (b) induced emf/current (c) force on a conductor (d) torque
☐ (b) — Generator rule.
5. Fleming's left-hand rule is used for
(a) generators (b) rectifiers (c) transformers (d) motors
☐ (d) — Motor rule.
6. The unit of reluctance is
(a) Wb/AT (b) H/m (c) T (d) AT/Wb
☐ (d) — $S = \text{mmf}/\Phi$.
7. Magnetomotive force is measured in
(a) volt (b) tesla (c) weber (d) ampere-turn
☐ (d) — NI .
8. The magnetic equivalent of Ohm's law is
(a) $\Phi = S/\text{mmf}$ (b) $\Phi = \text{mmf}/S$ (c) $\Phi = \text{mmf} \times S$ (d) $\Phi = NI \times \mu$
☐ (b) — Flux = mmf/reluctance.



- 9.** Relative permeability of air is
(a) 1 (b) 0 (c) 1000 (d) $4\pi \times 10^{-7}$
□ (a) — Reference medium.
- 10.** The area of the hysteresis loop represents
(a) permeability (b) residual magnetism (c) energy loss per cycle per unit volume (d) flux
□ (c) — Heat loss.
- 11.** Eddy-current loss is reduced by
(a) using copper core (b) increasing frequency (c) using a solid core (d) laminating the core
□ (d) — Breaks eddy paths.
- 12.** Material with a narrow hysteresis loop is used for
(a) magnetic tapes (b) transformer cores (c) compass needles (d) permanent magnets
□ (b) — Low loss.
- 13.** Hysteresis loss is proportional to
(a) f^2 (b) f (c) $1/f$ (d) $\sqrt{f}$
□ (b) — $P_h = \eta B_m^{1.6} fV$.
- 14.** Eddy-current loss is proportional to
(a) $1/f$ (b) f (c) f^2 (d) $\sqrt{f}$
□ (c) — $P_e \propto f^2$.
- 15.** Coercivity is the value of H needed to
(a) saturate the material (b) reduce B to zero (c) double B (d) produce residual flux
□ (b) — Definition.
- 16.** The commutator in a DC generator acts as
(a) an amplifier (b) a transformer (c) a mechanical rectifier (d) an inverter only
□ (c) — AC in coils → DC at brushes.
- 17.** The armature core is laminated to reduce
(a) eddy-current loss (b) copper loss (c) hysteresis loss only (d) friction loss
□ (a) — Laminations.
- 18.** The number of parallel paths in a lap winding is
(a) 2 (b) 4 always (c) P (d) P/2
□ (c) — $A = P$.
- 19.** The number of parallel paths in a wave winding is
(a) P (b) P/2 (c) 2 (d) 4
□ (c) — $A = 2$.
- 20.** The emf equation of a DC generator is
(a) $P\Phi Z/60AN$ (b) $60A/P\Phi ZN$ (c) $P\Phi ZN/60A$ (d) $P\Phi N/60$
□ (c) — Standard.
- 21.** The field winding of a shunt machine has
(a) many turns of thin wire (b) one turn (c) no turns (d) few turns of thick wire
□ (a) — High resistance.
- 22.** The series field winding has
(a) no resistance (b) high resistance (c) low resistance (d) infinite resistance
□ (c) — Carries full armature current.
- 23.** For voltage build-up, a self-excited shunt generator needs
(a) no residual magnetism (b) residual magnetism and field resistance below critical (c) very high field resistance (d) zero speed
□ (b) — Build-up conditions.



24. Back emf in a DC motor

- (a) opposes the applied voltage (b) is zero at full speed (c) aids the applied voltage (d) equals supply voltage at start
□ (a) — Lenz's law.

25. At the instant of starting a DC motor, the back emf is

- (a) zero (b) maximum (c) half of V (d) equal to V
□ (a) — $N = 0$.

26. A starter is used with a DC motor to

- (a) limit starting current (b) increase speed (c) increase torque (d) reverse direction
□ (a) — $E_b = 0$ at start.

27. The torque of a DC motor is proportional to

- (a) I_a/Φ (b) ΦI_a (c) Φ^2 (d) Φ/I_a
□ (b) — $T \propto \Phi I_a$.

28. The speed of a DC motor is proportional to

- (a) I_a (b) ΦE_b (c) Φ/E_b (d) E_b/Φ
□ (d) — $N \propto E_b/\Phi$.

29. A DC series motor should never be started on no load because

- (a) it may reach dangerous speed (b) it draws no current (c) it will not start (d) torque is zero
□ (a) — Φ very small.

30. For traction (electric trains), the preferred DC motor is

- (a) separately excited (b) differential compound (c) shunt (d) series
□ (d) — High starting torque.

31. A DC shunt motor is essentially a

- (a) constant speed motor (b) high starting torque motor (c) variable speed motor (d) zero speed motor
□ (a) — Φ nearly constant.

32. Torque of a DC series motor before saturation is proportional to

- (a) I_a^2 (b) $\sqrt{I_a}$ (c) I_a (d) $1/I_a$
□ (a) — $\Phi \propto I_a$.

33. Field control of a DC shunt motor gives speeds

- (a) above rated (b) zero (c) below rated (d) only at rated
□ (a) — Weakening field raises speed.

34. Armature resistance control gives speeds

- (a) infinite (b) above rated (c) below rated (d) constant
□ (c) — Reduces E_b .

35. The Ward-Leonard system controls speed by varying

- (a) field resistance (b) number of poles (c) armature voltage (d) frequency
□ (c) — Variable-voltage supply.

36. The mechanical power developed by a DC motor is

- (a) $E_b I_a$ (b) $V I_{sh}$ (c) $I_a^2 R_a$ (d) $V I_a$
□ (a) — Power equation.

37. Maximum mechanical power is developed when E_b equals

- (a) zero (b) V (c) $2V$ (d) $V/2$
□ (d) — But efficiency $< 50\%$.



38. Iron losses in a DC machine are

- (a) copper losses (b) variable losses (c) zero (d) constant losses
☐ (d) — Depend on flux and speed.

39. A 3-point starter is used with

- (a) shunt motors (b) induction motors (c) series motors (d) alternators
☐ (a) — Has no-volt release.

40. The coefficient of coupling k lies between

- (a) -1 and 1 (b) 0 and ∞ (c) 1 and 2 (d) 0 and 1
☐ (d) — $M = k\sqrt{L_1 L_2}$.

Module IV: AC Machines

This module covers the three most important AC machines: the single-phase transformer (which makes AC transmission possible), the three-phase induction motor (the workhorse of industry, driving about 70% of all industrial loads) and the three-phase alternator (which generates almost all electrical power).

4.1 Single-phase transformer: principle and construction

A transformer is a static device that transfers electrical energy from one circuit to another by **mutual induction**, without change of frequency, usually changing the voltage level.

Principle. An alternating voltage V_1 applied to the primary winding (N_1 turns) drives a magnetising current that sets up an alternating flux Φ in the iron core. This flux links the secondary winding (N_2 turns) and induces an emf E_2 in it (Faraday's law). A transformer does not work on DC, because a steady flux induces no emf (and the primary would draw a very large current limited only by its resistance).

Construction.

- **Core:** laminated silicon steel (0.35–0.5 mm sheets, insulated by varnish) to reduce hysteresis and eddy-current losses.
- **Windings:** primary and secondary coils of copper, insulated from each other and from the core. The low-voltage winding is placed nearer the core.
- **Core type:** windings surround the core limbs; one magnetic path; easier to insulate; used for high voltages.
- **Shell type:** the core surrounds the windings; two parallel magnetic paths (central limb carries full flux); better mechanical support and less leakage; used for low voltages and high currents.
- Large transformers have a tank filled with transformer oil (insulation and cooling), a conservator, a breather, bushings and cooling radiators.

4.2 EMF equation of a transformer

The flux is sinusoidal: $\Phi = \Phi_m \sin \omega t$. The emf induced in the primary is $e_1 = -N_1 d\Phi/dt = -N_1 \omega \Phi_m \cos \omega t$, with maximum value $N_1 \omega \Phi_m = 2\pi f N_1 \Phi_m$. The rms value is $2\pi f N_1 \Phi_m / \sqrt{2}$:

$$E_1 = 4.44 f N_1 \Phi_m, E_2 = 4.44 f N_2 \Phi_m \quad (\Phi_m = B_m A)$$

Transformation ratio.

$$K = E_2/E_1 = N_2/N_1 = V_2/V_1 \approx I_1/I_2$$

- $K > 1$: step-up transformer; $K < 1$: step-down transformer.
- In an ideal transformer (no losses), input VA = output VA: $V_1 I_1 = V_2 I_2$, so the current ratio is the inverse of the voltage ratio.
- The emf per turn is the same in both windings: $E_1/N_1 = E_2/N_2 = 4.44 f \Phi_m$.
- An impedance Z_2 on the secondary appears on the primary side as Z_2/K^2 (impedance transformation, used for matching).

4.3 Transformer on no load

With the secondary open, the primary draws a small **no-load current** I_0 (2-5% of full-load current), lagging V_1 by a large angle ϕ_0 (pf 0.1-0.2). I_0 has two components:

Magnetising component $I_{\mu} = I_0 \sin \phi_0$ (produces the flux, in phase with Φ); Working (loss) component $I_w = I_0 \cos \phi_0$ (supplies iron loss)
No-load input $W_0 = V_1 I_0 \cos \phi_0 \approx$ iron loss

In the phasor diagram, Φ is taken as reference; E_1 and E_2 lag Φ by 90° ; V_1 is equal and opposite to E_1 ; I_0 lags V_1 by ϕ_0 .

4.4 Transformer on load

When a load is connected, the secondary current I_2 produces an mmf $N_2 I_2$ that tends to reduce the core flux. This momentarily reduces E_1 , so the primary draws an extra current I_2' (the **load component**) such that $N_1 I_2' = N_2 I_2$, cancelling the secondary mmf. The net flux stays almost constant from no load to full load (it depends only on V_1 and f).

$$I_1 = I_0 + I_2' \text{ (phasor sum), } I_2' = K I_2$$

This is how the primary "knows" the load: the transformer is self-regulating.

Winding resistance and leakage reactance. Real windings have resistances R_1, R_2 and leakage reactances X_1, X_2 (from flux that links only one winding). Referred to the primary:

$$R_{01} = R_1 + R_2/K^2, X_{01} = X_1 + X_2/K^2, Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

Referred to the secondary: $R_{02} = R_2 + K^2 R_1, X_{02} = X_2 + K^2 X_1$.

Equivalent circuit. Series R_{01}, X_{01} with a shunt branch R_0 (iron loss) in parallel with X_0 (magnetising reactance) across the supply (approximate circuit).

4.5 Voltage regulation

As load increases, the secondary terminal voltage falls because of the voltage drop in the winding impedances.

$$\% \text{ Regulation} = (E_2 - V_2)/E_2 \times 100 = (V_0 - V_{FL})/V_0 \times 100 \text{ (} E_2 = \text{no-load secondary voltage)}$$

$$\text{Approximate: } \% \text{ Reg} = I_2(R_{02} \cos \phi \pm X_{02} \sin \phi)/E_2 \times 100 \text{ (+ for lagging pf, - for leading pf)}$$

- A good transformer has low regulation (2-5%): the output voltage changes little with load.
- With a leading pf the regulation can be zero or negative (the voltage rises on load).
- Zero regulation occurs when $\tan \phi = R_{02}/X_{02}$ (leading).

4.6 Losses and efficiency

Losses.

- **Iron (core) loss P_i :** hysteresis + eddy-current loss; depends on V and f , practically constant at all loads. Found by the **open-circuit test** (rated voltage on LV side, HV open): wattmeter reads P_i .
- **Copper loss P_{cu} :** $I_1^2 R_1 + I_2^2 R_2 = I_2^2 R_{02}$; varies as the square of the load. Found by the **short-circuit test** (reduced voltage on HV side, LV shorted, rated current):

wattmeter reads full-load P_{cu} .

At a fraction x of full load: copper loss = $x^2 P_{cu,FL}$.

$$\text{Efficiency } \eta = \text{output} / (\text{output} + \text{losses}) = x S \cos \phi / (x S \cos \phi + P_i + x^2 P_{cu,FL}) \quad (S = \text{rated kVA})$$

Condition for maximum efficiency. η is maximum when copper loss = iron loss:

$$x^2 P_{cu,FL} = P_i \Rightarrow \text{load for } \eta_{\max} = S \times \sqrt{(P_i / P_{cu,FL})}$$

Transformers are very efficient (95–99%) because they have no moving parts.

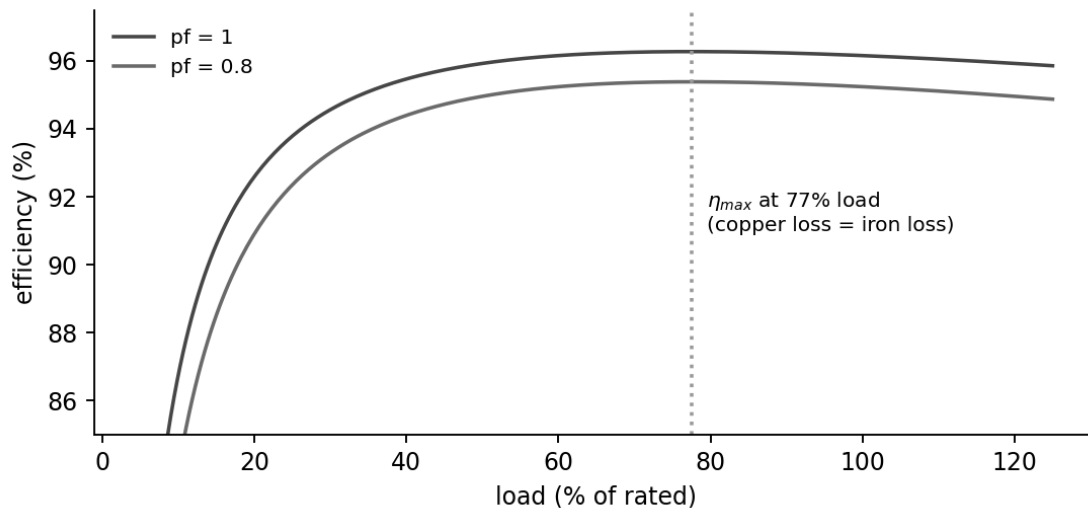


Figure 4.1: Efficiency of a 20 kVA transformer (iron loss 300 W, full-load copper loss 500 W) against load.

All-day efficiency (for distribution transformers that are energised all day) = output kWh/input kWh over 24 hours; they are designed with low iron loss.

4.7 Three-phase induction motor: construction

Stator. A laminated steel core with slots carrying a three-phase winding (star or delta), wound for P poles and fed from the three-phase supply.

Rotor. A laminated cylindrical core mounted on the shaft, of two types:

- **Squirrel-cage rotor:** heavy copper or aluminium bars in the slots, short-circuited at both ends by end rings (it looks like a squirrel cage). The bars are often skewed to reduce noise and cogging. Simple, rugged, cheap, almost maintenance-free; used in about 90% of induction motors (pumps, fans, compressors, machine tools).
- **Slip-ring (wound) rotor:** a three-phase winding (usually star) whose ends are brought out to three slip rings. External resistances can be added through brushes to increase starting torque and control speed. Used for cranes, hoists and elevators needing high starting torque.

Air gap: very small (0.4–4 mm) to keep the magnetising current low and the power factor high.

4.8 Rotating magnetic field and principle of working

When a balanced three-phase current flows in a three-phase winding displaced by 120° in space, the resultant magnetic field has a **constant magnitude** ($1.5 \Phi_m$) and **rotates** at synchronous speed:

$$N_s = 120f/P \text{ (rpm)}$$

(At 50 Hz: 2 poles → 3000 rpm, 4 poles → 1500 rpm, 6 poles → 1000 rpm, 8 poles → 750 rpm.) Interchanging any two supply leads reverses the direction of rotation.

Working. The rotating field cuts the stationary rotor conductors and induces emfs in them (Faraday). Since the rotor circuit is closed, currents flow. These currents in the magnetic field experience forces ($F = BIl$), producing torque. By Lenz's law, the rotor turns in the same direction as the field to reduce the relative speed. The motor is **self-starting**. Because torque is produced by induction, it is called an induction motor (or asynchronous motor).

Why the rotor never reaches synchronous speed. If it did, there would be no relative motion between field and rotor, no induced emf, no current and no torque. So the rotor always runs slightly slower than N_s .

4.9 Slip

$$\text{Slip } s = (N_s - N)/N_s, N = N_s(1 - s)$$

- $s = 1$ at standstill (starting); $s \approx 0$ at no load. Full-load slip is typically 2-5%.
- **Frequency of rotor currents:** $f_r = s f$ (e.g. at 4% slip and 50 Hz, $f_r = 2$ Hz).
- **Rotor emf:** $E_{2r} = s E_2$ (E_2 = standstill rotor emf per phase). **Rotor reactance:** $X_{2r} = s X_2$.
- Rotor current $I_{2r} = sE_2/\sqrt{(R_2^2 + (sX_2)^2)}$.

4.10 Torque and torque-speed (torque-slip) relation

The torque is proportional to the rotor power, and the analysis gives

$$T = k s E_2^2 R_2 / (R_2^2 + s^2 X_2^2)$$

Starting torque ($s = 1$): $T_{st} = k E_2^2 R_2 / (R_2^2 + X_2^2)$.

Condition for maximum torque: $dT/ds = 0$ gives

$$s_m = R_2/X_2; T_{max} = k E_2^2 / (2X_2) \text{ (independent of } R_2)$$

Torque-slip characteristic.

- **Low slip region (normal operation, s small):** $sX_2 \ll R_2$, so $T \propto s$. The curve is a straight line; speed falls only slightly as load increases (nearly constant-speed motor, like a DC shunt motor).
- **High slip region ($s > s_m$):** $sX_2 \gg R_2$, so $T \propto 1/s$. Torque decreases as slip increases; this is the unstable region.
- The maximum torque is called the **breakdown (pull-out) torque**. If the load exceeds it, the motor stalls.
- **Effect of rotor resistance:** increasing R_2 (possible in slip-ring motors) shifts the maximum-torque point to higher slip without changing T_{max} . Choosing $R_2 = X_2$ gives maximum torque at starting.

$$T \propto V^2 \text{ (torque is very sensitive to supply voltage)}$$

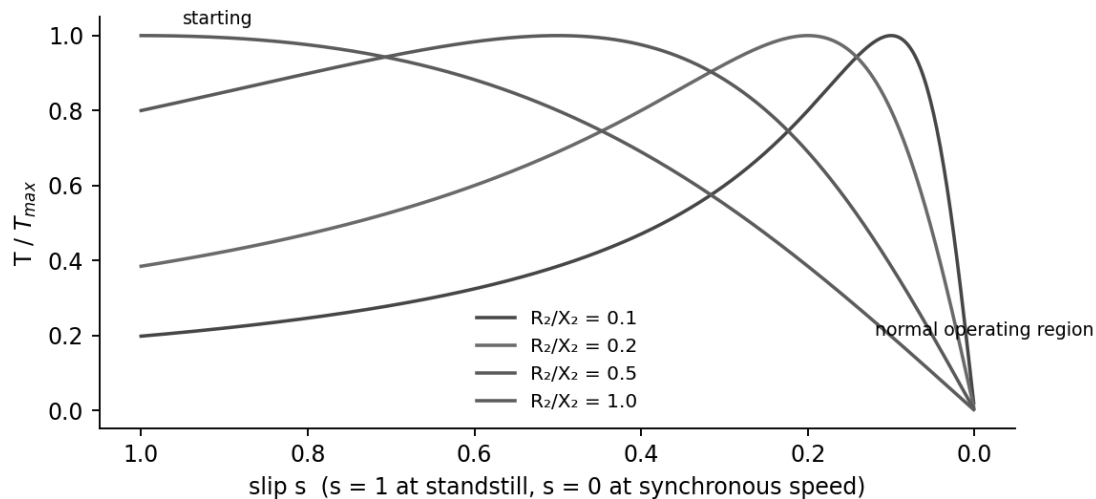


Figure 4.2: Torque-slip curves for different rotor resistances. Maximum torque is the same; its slip $s_m = R_2/X_2$ increases with R_2 .

Power flow. Stator input $\rightarrow$ stator losses $\rightarrow$ air-gap power $P_g \rightarrow$ rotor copper loss ($= sP_g$) $\rightarrow$ mechanical power developed $(1 - s)P_g \rightarrow$ friction and windage $\rightarrow$ output. So P_g : rotor Cu loss : $P_{mech} = 1 : s : (1 - s)$.

Starting methods. Direct-on-line (small motors), star-delta starter (applies $V/\sqrt{3}$ per phase at start, reducing starting current and torque to $1/3$), autotransformer starter, and rotor-resistance starter (slip-ring motors).

4.11 Three-phase alternator: principle of operation

An alternator (synchronous generator) converts mechanical energy into three-phase AC electrical energy. Almost all power stations use alternators.

Construction.

- **Stator (armature):** laminated core with slots carrying a three-phase winding displaced 120° electrically. The emf is induced here. Keeping the armature stationary makes it easy to insulate high voltages and take out large currents without sliding contacts.
- **Rotor (field):** carries the DC field winding, supplied through two slip rings from an exciter. Two types:
 - **Salient-pole rotor:** large diameter, short axial length, projecting poles; used with low-speed prime movers (hydraulic turbines, diesel engines), many poles.
 - **Cylindrical (non-salient, smooth) rotor:** small diameter, long axial length; used with high-speed steam and gas turbines (turbo-alternators), 2 or 4 poles, 3000 or 1500 rpm.

Principle. The rotor is driven by the prime mover and its DC field produces a rotating magnetic field. This field cuts the stationary armature conductors and induces emfs in them (Faraday's law). The three windings, displaced by 120° , produce three emfs 120° apart: a three-phase supply.

Frequency. The alternator must run at synchronous speed to generate the standard frequency:

$$f = PN_s/120$$

EMF equation.

$$E_{ph} = 4.44 f \Phi T_{ph} k_p k_d \quad (T_{ph} = \text{turns per phase; } k_p = \text{pitch factor; } k_d = \text{distribution factor, each slightly less than 1})$$

Voltage regulation of an alternator = $(E_0 - V)/V \times 100$, due to armature resistance, leakage reactance and armature reaction. It is positive for lagging and unity pf loads and can be negative for leading pf.

Comparison: induction motor and alternator.

Induction motor	Alternator
Converts electrical to mechanical energy	Converts mechanical to electrical energy
Stator fed with 3-phase AC; rotor currents induced	Rotor fed with DC; stator emf induced
Runs below synchronous speed (slip)	Runs exactly at synchronous speed
Squirrel-cage or slip-ring rotor	Salient-pole or cylindrical rotor

4.12 Common mistakes to avoid in Module IV

- Using 4.44 with the rms flux. Φ_m is the maximum flux.
- Forgetting that the current ratio is the inverse of the voltage ratio in a transformer.
- Taking full-load copper loss at part loads. Copper loss varies as x^2 .
- Writing slip in rpm. Slip is a fraction (or %); the slip speed $N_s - N$ is in rpm.
- Saying an induction motor can run at synchronous speed.

4.13 Transformer: ideal vs practical and phasor diagrams

Ideal transformer assumptions: no winding resistance, no leakage flux, no core losses, infinite core permeability (zero magnetising current). Then $V_1 I_1 = V_2 I_2$, $V_1/V_2 = N_1/N_2$, and $\eta = 100\%$.

Practical transformer phasor diagram on load (lagging pf), step by step:

- Draw Φ horizontally (reference). E_1 and E_2 lag Φ by 90° .
- Draw I_2 lagging V_2 by ϕ_2 . $E_2 = V_2 + I_2 R_2 + j I_2 X_2$.
- Draw $I_2' = K I_2$ in antiphase with I_2 . Add I_0 (leading Φ slightly) to get $I_1 = I_0 + I_2'$.
- $V_1 = -E_1 + I_1 R_1 + j I_1 X_1$.

Open-circuit test details. Connect rated voltage to the LV side with the HV side open. Instruments: voltmeter, ammeter, wattmeter. Readings V_0 , I_0 , W_0 . Results: iron loss $P_i = W_0$; $\cos \phi_0 = W_0/(V_0 I_0)$; $I_w = I_0 \cos \phi_0$, $I_\mu = I_0 \sin \phi_0$; $R_0 = V_0/I_w$, $X_0 = V_0/I_\mu$.

Short-circuit test details. Short the LV side and apply a reduced voltage (5-10% of rated) to the HV side until rated current flows. Readings V_{sc} , I_{sc} , W_{sc} . Results: full-load copper loss = W_{sc} ; $Z_{01} = V_{sc}/I_{sc}$; $R_{01} = W_{sc}/I_{sc}^2$; $X_{01} = \sqrt{(Z_{01}^2 - R_{01}^2)}$. Core loss is negligible because the voltage (and hence flux) is very low.

Autotransformer. A single winding with a tapping; part of the winding is common to primary and secondary. It saves copper (saving = K fraction for ratios near 1) and is used for starting motors, variacs and interconnecting systems of nearly equal voltages. It does not provide electrical isolation.

Instrument transformers. Current transformers (CT) step down large currents for ammeters and relays; potential transformers (PT) step down high voltages for voltmeters. The secondary of a CT must never be opened while current flows in the primary.

4.14 Induction motor: more details for long answers

Production of the rotating field (graphical proof). At $\omega t = 0^\circ, 60^\circ, 120^\circ, \dots$ the three phase currents have values such that the resultant mmf has the same magnitude ($1.5 \times$ maximum mmf of one phase) but its axis has moved through $60^\circ, 120^\circ, \dots$. So the field rotates once per cycle for a 2-pole winding ($N_s = 60f$ rpm), and once every $P/2$ cycles for a P-pole winding ($N_s = 120f/P$).

Torque equation derivation (outline). Rotor emf per phase at slip s : sE_2 ; impedance $\sqrt{R_2^2 + (sX_2)^2}$. Rotor current $I_2 = sE_2/\sqrt{R_2^2 + s^2X_2^2}$; rotor pf $\cos \phi_2 = R_2/\sqrt{R_2^2 + s^2X_2^2}$. Torque $\propto \Phi I_2 \cos \phi_2$, and $\Phi \propto E_2$, so $T = ksE_2^2 R_2/(R_2^2 + s^2X_2^2)$.

Maximum torque derivation. Write $T = kE_2^2 R_2/(R_2^2/s + sX_2^2)$. T is maximum when the denominator $y = R_2^2/s + sX_2^2$ is minimum: $dy/ds = -R_2^2/s^2 + X_2^2 = 0 \Rightarrow s = R_2/X_2$. Substituting, $T_{\max} = kE_2^2/(2X_2)$.

Useful ratios.

$$T/T_{\max} = 2 s s_m/(s^2 + s_m^2); T_{st}/T_{\max} = 2 s_m/(1 + s_m^2)$$

Comparison: squirrel-cage vs slip-ring motor.

Squirrel-cage	Slip-ring (wound rotor)
Rotor bars short-circuited permanently	Three-phase rotor winding brought out to slip rings
Simple, rugged, cheap, low maintenance	Costlier, needs brush maintenance
Moderate starting torque; high starting current	High starting torque with external resistance; lower starting current
Speed control only from the stator side	Speed control by rotor resistance also possible
Pumps, fans, compressors, machine tools	Cranes, hoists, lifts, compressors with heavy starting load

Single-phase induction motors (awareness). A single-phase winding produces a pulsating (not rotating) field, so the motor is not self-starting. Auxiliary windings with capacitors (split-phase, capacitor-start, capacitor-run) or shaded poles create a starting torque. Used in fans, pumps, washing machines and refrigerators.

4.15 Alternator: more details

Advantages of stationary armature and rotating field. High-voltage armature windings are easier to insulate when stationary; the load current is taken out through fixed terminals (no sliding contacts); only low-voltage DC (a few hundred volts) is fed to the rotor through slip rings; the rotor is lighter and can run at high speed; the stationary armature can be cooled more easily.

Excitation systems. DC exciter on the same shaft, static excitation (rectified AC from the alternator itself), or brushless excitation (AC exciter with rotating rectifiers).

Synchronisation. To connect an alternator to the grid (bus-bars), its voltage, frequency and phase sequence must match those of the bus-bars, and the switch is closed when the

voltages are in phase (checked with synchroscope or lamps).

Synchronous motor (awareness). The same machine connected to a 3-phase supply runs at exactly synchronous speed. It is not self-starting; over-excited, it draws a leading current and is used for power-factor correction (synchronous condenser).

4.16 Solved numericals: Module IV

Problem 1. A 25 kVA, 2200/220 V, 50 Hz single-phase transformer has 56 secondary turns. Find the primary turns, the full-load currents and the maximum core flux.

$$N_1 = N_2 \times V_1/V_2 = 56 \times 10 = 560.$$

$$I_1 = 25000/2200 = 11.36 \text{ A}; I_2 = 25000/220 = 113.6 \text{ A}.$$

$$\Phi_m = E_2/(4.44fN_2) = 220/(4.44 \times 50 \times 56) = 0.0177 \text{ Wb}.$$

Answer: $N_1 = 560$; $I_1 = 11.4 \text{ A}$, $I_2 = 113.6 \text{ A}$; $\Phi_m = 17.7 \text{ mWb}$.

Problem 2. A single-phase transformer has 400 primary and 1000 secondary turns. The net core area is 60 cm². If the primary is connected to 500 V, 50 Hz, find the peak flux density and the secondary voltage.

$$\Phi_m = 500/(4.44 \times 50 \times 400) = 5.63 \times 10^{-3} \text{ Wb}. B_m = \Phi_m/A = 5.63 \times 10^{-3}/60 \times 10^{-4} = 0.94 \text{ T}.$$

$$V_2 = 500 \times 1000/400 = 1250 \text{ V}.$$

Answer: $B_m \approx 0.94 \text{ T}$; $V_2 = 1250 \text{ V}$.

Problem 3. A transformer takes a no-load current of 1.5 A at 0.2 pf from a 230 V supply. Find the magnetising and working components and the iron loss.

$$I_w = I_0 \cos \phi_0 = 0.3 \text{ A}. I_\mu = I_0 \sin \phi_0 = 1.5 \times 0.98 = 1.47 \text{ A}.$$

$$\text{Iron loss} = V I_0 \cos \phi_0 = 230 \times 1.5 \times 0.2 = 69 \text{ W}.$$

Answer: $I_w = 0.3 \text{ A}$; $I_\mu = 1.47 \text{ A}$; $P_i = 69 \text{ W}$.

Problem 4. A 2000/200 V transformer has $R_1 = 2 \Omega$, $R_2 = 0.02 \Omega$, $X_1 = 6 \Omega$, $X_2 = 0.06 \Omega$. Find R_{02} , X_{02} and Z_{02} .

$$K = 0.1. R_{02} = R_2 + K^2 R_1 = 0.02 + 0.01 \times 2 = 0.04 \Omega. X_{02} = 0.06 + 0.01 \times 6 = 0.12 \Omega.$$

$$Z_{02} = \sqrt{(0.04^2 + 0.12^2)} = 0.126 \Omega.$$

Answer: $R_{02} = 0.04 \Omega$; $X_{02} = 0.12 \Omega$; $Z_{02} = 0.126 \Omega$.

Problem 5. A 10 kVA, 2000/200 V transformer has $R_{02} = 0.02 \Omega$ and $X_{02} = 0.06 \Omega$. Find the full-load regulation at (a) 0.8 pf lagging, (b) 0.8 pf leading.

$$I_2 = 10000/200 = 50 \text{ A}. \cos \phi = 0.8, \sin \phi = 0.6.$$

$$(a) \% \text{Reg} = 50(0.02 \times 0.8 + 0.06 \times 0.6)/200 \times 100 = 50 \times 0.052/2 = 1.3\%.$$

$$(b) \% \text{Reg} = 50(0.016 - 0.036)/200 \times 100 = -0.5\%.$$

Answer: (a) 1.3%; (b) -0.5% (the voltage rises on load).

Problem 6. A 20 kVA transformer has an iron loss of 300 W and full-load copper loss of 500 W. Find the efficiency at (a) full load, 0.8 pf; (b) half load, 0.8 pf; (c) full load, unity pf.

(a) Output = 16000 W. $\eta = 16000/(16000 + 300 + 500) = 95.24\%$.

(b) Output = 8000 W; Cu loss = $0.25 \times 500 = 125$ W. $\eta = 8000/8425 = 94.96\%$.

(c) Output = 20000 W. $\eta = 20000/20800 = 96.15\%$.

Answer: 95.2%, 95.0%, 96.2%.

Problem 7. For the transformer of the previous problem, find the load at which efficiency is maximum and the maximum efficiency at unity pf.

$$x = \sqrt{P_i/P_{cu}} = \sqrt{(300/500)} = 0.775. \text{ Load} = 0.775 \times 20 = 15.49 \text{ kVA.}$$

At unity pf: output = 15492 W; losses = 300 + 300 = 600 W. $\eta_{\max} = 15492/16092 = 96.27\%$.

Answer: At 15.5 kVA (77.5% of full load); $\eta_{\max} \approx 96.3\%$.

Problem 8. In OC and SC tests on a 5 kVA, 230/115 V transformer: OC test (LV side): 115 V, 1 A, 50 W; SC test (HV side): 12 V, 21.7 A, 120 W. Find the efficiency at full load, 0.8 pf.

Iron loss = 50 W. Rated HV current = $5000/230 = 21.7$ A, so the SC test is at full load: $P_{cu,FL} = 120$ W.

Output = 4000 W. $\eta = 4000/(4000 + 50 + 120) = 95.9\%$.

Answer: $\eta \approx 95.9\%$.

Problem 9. A 4-pole, 50 Hz induction motor runs at 1440 rpm. Find the synchronous speed, slip and rotor-current frequency.

$$N_s = 120 \times 50/4 = 1500 \text{ rpm. } s = (1500 - 1440)/1500 = 0.04 \text{ (4\%).}$$

$$f_r = sf = 0.04 \times 50 = 2 \text{ Hz.}$$

Answer: $N_s = 1500$ rpm; $s = 4\%$; $f_r = 2$ Hz.

Problem 10. A 6-pole, 50 Hz induction motor has a full-load slip of 3%. Find its full-load speed. What is the speed of the rotor field relative to the rotor and relative to the stator?

$$N_s = 1000 \text{ rpm; } N = 1000(1 - 0.03) = 970 \text{ rpm.}$$

Rotor field speed relative to rotor = $sN_s = 30$ rpm; relative to stator = $970 + 30 = 1000$ rpm = N_s .

Answer: 970 rpm; 30 rpm relative to rotor; 1000 rpm relative to stator (stator and rotor fields are stationary relative to each other).

Problem 11. A 3-phase induction motor has rotor resistance 0.2Ω and standstill reactance 1Ω per phase. Find the slip at maximum torque and the ratio of starting torque to maximum torque.

$$s_m = R_2/X_2 = 0.2.$$

$$T_{st}/T_{max} = 2s_m/(1 + s_m^2) = 0.4/1.04 = 0.385.$$

Answer: $s_m = 0.2$; $T_{st}/T_{max} \approx 0.385$.

Problem 12. What external rotor resistance per phase is needed in the motor of the previous problem to obtain maximum torque at starting?

$$\text{For } T_{max} \text{ at } s = 1: R_2 + R_{ext} = X_2 = 1 \Omega \Rightarrow R_{ext} = 0.8 \Omega.$$

Answer: 0.8Ω per phase.

Problem 13. The power input to the rotor of a 3-phase induction motor is 48 kW and the slip is 4%. Find the rotor copper loss and the mechanical power developed.

$$\text{Rotor Cu loss} = sP_g = 0.04 \times 48 = 1.92 \text{ kW. } P_{mech} = (1 - s)P_g = 46.08 \text{ kW.}$$

Answer: 1.92 kW; 46.08 kW.

Problem 14. If the supply voltage of an induction motor falls by 10%, by what percentage does the torque change (at the same slip)?

$$T \propto V^2 \Rightarrow T_2/T_1 = 0.9^2 = 0.81.$$

Answer: Torque falls by 19%.

Problem 15. Find the speed at which a 12-pole alternator must be driven to generate 50 Hz. What speed is needed for a 2-pole alternator?

$$N_s = 120f/P = 6000/12 = 500 \text{ rpm; for } P = 2: 3000 \text{ rpm.}$$

Answer: 500 rpm (salient-pole, hydro); 3000 rpm (cylindrical rotor, steam turbine).

Problem 16. A 3-phase, 50 Hz, star-connected alternator has 200 turns per phase, flux per pole 0.05 Wb, pitch factor 0.95 and distribution factor 0.96. Find the phase and line emf.

$$E_{ph} = 4.44 \times 50 \times 0.05 \times 200 \times 0.95 \times 0.96 = 2025 \text{ V.}$$

$$E_L = \sqrt{3} \times 2025 = 3507 \text{ V.}$$

Answer: $E_{ph} \approx 2.02 \text{ kV}$; $E_L \approx 3.51 \text{ kV}$.

4.17 More solved numericals: Module IV

Problem 1. The OC test on a 230/115 V transformer (LV side) gives 115 V, 1.2 A, 36 W. Find the no-load pf, magnetising and working components, R_0 and X_0 .

$$\cos \phi_0 = 36/(115 \times 1.2) = 0.261. \sin \phi_0 = 0.965.$$

$$I_w = 1.2 \times 0.261 = 0.313 \text{ A; } I_\mu = 1.2 \times 0.965 = 1.158 \text{ A.}$$

$$R_0 = 115/0.313 = 367 \Omega; X_0 = 115/1.158 = 99.3 \Omega.$$

Answer: pf 0.261; $I_w = 0.31 \text{ A}$; $I_\mu = 1.16 \text{ A}$; $R_0 \approx 367 \Omega$; $X_0 \approx 99 \Omega$ (referred to LV).

Problem 2. The SC test on the HV side of a 10 kVA, 2000/400 V transformer gives 60 V, 5 A, 100 W. Find R_{01} , X_{01} and the full-load regulation at 0.8 pf lagging.

Rated HV current = $10000/2000 = 5$ A (test is at full load). $Z_{01} = 60/5 = 12 \Omega$; $R_{01} = 100/25 = 4 \Omega$; $X_{01} = \sqrt{144 - 16} = 11.31 \Omega$.

%Reg = $I_1(R_{01} \cos \phi + X_{01} \sin \phi)/V_1 \times 100 = 5(3.2 + 6.79)/2000 \times 100 = 2.5\%$.

Answer: $R_{01} = 4 \Omega$; $X_{01} \approx 11.3 \Omega$; regulation $\approx 2.5\%$.

Problem 3. A 100 kVA distribution transformer has an iron loss of 1 kW and full-load copper loss of 1.5 kW. It supplies: full load at 0.8 pf for 6 h, half load at 0.8 pf for 10 h, and no load for 8 h daily. Find the all-day efficiency.

Output energy = $80 \times 6 + 40 \times 10 = 880$ kWh.

Iron loss energy = $1 \times 24 = 24$ kWh. Copper loss energy = $1.5 \times 6 + 1.5 \times 0.25 \times 10 = 9 + 3.75 = 12.75$ kWh.

All-day $\eta = 880/(880 + 24 + 12.75)$.

Answer: $\approx 96.0\%$.

Problem 4. A 50 kVA transformer has a full-load efficiency of 98% at unity pf, and maximum efficiency occurs at 80% of full load. Find the iron and full-load copper losses.

Total losses at full load = $50/0.98 - 50 = 1.0204$ kW = $P_i + P_{cu}$.

At 80% load: $0.64P_{cu} = P_i$. So $1.64P_{cu} = 1.0204 = P_{cu} = 0.622$ kW, $P_i = 0.398$ kW.

Answer: $P_i \approx 398$ W; $P_{cu,FL} \approx 622$ W.

Problem 5. An 8-pole induction motor is supplied from a 50 Hz source and runs at 720 rpm. Find the slip, the rotor frequency and the rotor emf per phase if the standstill value is 100 V.

$N_s = 750$ rpm; $s = 30/750 = 0.04$. $f_r = 2$ Hz. $E_{2r} = 0.04 \times 100 = 4$ V.

Answer: $s = 4\%$; $f_r = 2$ Hz; $E_{2r} = 4$ V.

Problem 6. A 3-phase induction motor has $R_2 = 0.02 \Omega$ and $X_2 = 0.1 \Omega$ per phase. Find the ratio of full-load torque to maximum torque if the full-load slip is 4%.

$s_m = 0.2$. $T/T_{max} = 2 \times 0.04 \times 0.2/(0.04^2 + 0.2^2) = 0.016/0.0416$.

Answer: $T_{FL}/T_{max} \approx 0.385$.

Problem 7. A 6-pole, 50 Hz induction motor delivers 15 kW at 960 rpm. Friction and windage losses are 500 W. Find the rotor copper loss.

$N_s = 1000$; $s = 0.04$. Mechanical power developed = $15 + 0.5 = 15.5$ kW = $(1 - s)P_g$
 $\Rightarrow P_g = 16.15$ kW.

Rotor Cu loss = $sP_g = 0.646$ kW.

Answer: ≈ 646 W.

Problem 8. A 3-phase, 400 V, 50 Hz induction motor has a full-load efficiency of 88% and pf 0.85 while delivering 7.5 kW. Find the line current and the full-load torque at 1450 rpm.

$$\text{Input} = 7500/0.88 = 8523 \text{ W. } I_L = 8523/(\sqrt{3} \times 400 \times 0.85) = 14.5 \text{ A.}$$

$$T = 9.55 \times 7500/1450 = 49.4 \text{ N m.}$$

Answer: $I_L \approx 14.5 \text{ A}$; $T \approx 49.4 \text{ N m}$.

Problem 9. A 4-pole alternator has 36 slots and 10 conductors per slot, star-connected, with flux 0.03 Wb per pole at 50 Hz. Assuming full-pitch coils and $k_d = 0.96$, find the phase and line emf.

$$\text{Conductors per phase} = 360/3 = 120 \Rightarrow \text{turns per phase } T_{ph} = 60. k_p = 1.$$

$$E_{ph} = 4.44 \times 50 \times 0.03 \times 60 \times 0.96 = 383.6 \text{ V. } E_L = 664 \text{ V.}$$

Answer: $E_{ph} \approx 384 \text{ V}$; $E_L \approx 664 \text{ V}$.

Problem 10. A step-down transformer 11 kV/415 V supplies a load of 100 kW at 0.8 pf. Find the primary and secondary currents (ideal, single-phase equivalent).

$$S = 100/0.8 = 125 \text{ kVA. } I_2 = 125000/415 = 301 \text{ A. } I_1 = 125000/11000 = 11.4 \text{ A.}$$

Answer: $I_1 \approx 11.4 \text{ A}$; $I_2 \approx 301 \text{ A}$.

4.18 Practice problems with solutions: Module IV

Q1. A transformer has a turns ratio of 1 : 5 and the primary voltage is 230 V. Find the secondary voltage and the primary current if the secondary supplies 2 A.

$$V_2 = 230 \times 5 = 1150 \text{ V. } I_1 = 2 \times 5 = 10 \text{ A (ideal).}$$

Answer: 1150 V; 10 A.

Q2. Find the number of primary turns needed for a 230 V, 50 Hz transformer with a core of 25 cm^2 and $B_m = 1.2 \text{ T}$.

$$\Phi_m = 1.2 \times 25 \times 10^{-4} = 3 \times 10^{-3} \text{ Wb. } N_1 = 230/(4.44 \times 50 \times 3 \times 10^{-3}).$$

Answer: $N_1 \approx 345$ turns.

Q3. A 500 kVA transformer has iron loss 2 kW and full-load copper loss 5 kW. Find the copper loss at 60% load.

$$P_{cu} = 0.6^2 \times 5 = 1.8 \text{ kW.}$$

Answer: 1.8 kW.

Q4. The no-load secondary voltage of a transformer is 240 V and the full-load voltage is 230 V. Find the regulation.

$$(240 - 230)/240 \times 100.$$

Answer: 4.17%.



Q5. Find the synchronous speed of an 8-pole, 60 Hz induction motor and its speed at 5% slip.

$$N_s = 120 \times 60/8 = 900 \text{ rpm. } N = 855 \text{ rpm.}$$

Answer: 900 rpm; 855 rpm.

Q6. An induction motor runs at 960 rpm on a 50 Hz supply. How many poles does it have and what is the slip?

The nearest synchronous speed above 960 rpm is 1000 rpm $\Rightarrow P = 6$. $s = 40/1000$.

Answer: 6 poles; s = 4%.

Q7. The rotor of a 4-pole, 50 Hz motor has a rotor emf frequency of 1.5 Hz. Find the rotor speed.

$$s = 1.5/50 = 0.03. N = 1500 \times 0.97.$$

Answer: 1455 rpm.

Q8. A star-delta starter is used with a delta-connected motor. By what factor are the starting line current and torque reduced compared with direct-on-line starting?

Phase voltage at start = $V/\sqrt{3} \Rightarrow$ phase current reduces by $\sqrt{3}$; line current (star) reduces by 3 compared with DOL delta. Torque $\propto V^2$ reduces by 3.

Answer: Both reduced to one-third.

4.19 MCQs: Module IV

- A transformer works on the principle of
(a) electrostatic induction (b) rectification (c) self-induction (d) mutual induction
☐ (d) — Flux links both windings.
- A transformer cannot be used on DC because
(a) the core saturates at zero current (b) it would get too cold (c) DC has no voltage (d) no emf is induced by steady flux
☐ (d) — $d\Phi/dt = 0$.
- The transformer core is laminated to reduce
(a) eddy-current loss (b) leakage (c) copper loss (d) hysteresis loss only
☐ (a) — Laminations break eddy paths.
- The emf equation of a transformer is
(a) $E = 2.22fN\Phi_m$ (b) $E = 4.44fN\Phi_m$ (c) $E = fN\Phi_m$ (d) $E = 4.44N\Phi_m/f$
☐ (b) — Standard.
- In a step-up transformer,
(a) $N_2 = N_1$ (b) $N_1 = 0$ (c) $N_2 < N_1$ (d) $N_2 > N_1$
☐ (d) — $K > 1$.
- In an ideal transformer, V_1/V_2 equals
(a) I_2/I_1 (b) N_2/N_1 (c) 1 (d) I_1/I_2
☐ (a) — Power conserved.
- The frequency of the secondary voltage of a transformer is
(a) same as primary (b) lower than primary (c) zero (d) higher than primary
☐ (a) — No frequency change.



- 8.** The no-load current of a transformer is about
(a) 2-5% of full load (b) zero (c) 100% of full load (d) 50% of full load
□ (a) — Small magnetising current.
- 9.** The no-load power factor of a transformer is
(a) leading (b) high (0.9) (c) unity (d) low (0.1-0.2)
□ (d) — Mostly magnetising.
- 10.** Iron loss of a transformer is measured by
(a) open-circuit test (b) load test (c) short-circuit test (d) polarity test
□ (a) — OC test at rated voltage.
- 11.** Copper loss of a transformer is measured by
(a) short-circuit test (b) no-load test (c) open-circuit test (d) megger test
□ (a) — SC test at rated current.
- 12.** Copper loss in a transformer varies as
(a) load (b) independent of load (c) square root of load (d) square of load
□ (d) — I^2R .
- 13.** A transformer has maximum efficiency when
(a) copper loss = iron loss (b) iron loss = 0 (c) copper loss = 0 (d) copper loss = $2 \times$ iron loss
□ (a) — Condition for $\eta_{\max}$.
- 14.** The efficiency of large transformers is about
(a) 97-99% (b) 75% (c) 50% (d) 100%
□ (a) — No moving parts.
- 15.** Voltage regulation of a transformer can be negative for
(a) leading pf (b) unity pf (c) no load (d) lagging pf
□ (a) — Capacitive load raises voltage.
- 16.** Shell-type transformers have
(a) one magnetic path (b) air core (c) two parallel magnetic paths (d) no core
□ (c) — Central limb carries full flux.
- 17.** An impedance Z_2 on the secondary referred to the primary becomes
(a) KZ_2 (b) K^2Z_2 (c) Z_2/K (d) Z_2/K^2
□ (d) — $K = N_2/N_1$.
- 18.** Transformer oil is used for
(a) increasing flux (b) lubrication (c) insulation and cooling (d) reducing frequency
□ (c) — Dual role.
- 19.** The synchronous speed of a 4-pole, 50 Hz motor is
(a) 3000 rpm (b) 750 rpm (c) 1000 rpm (d) 1500 rpm
□ (d) — $120 \times 50/4$.
- 20.** Slip of an induction motor is
(a) N_s/N (b) $(N_s - N)/N_s$ (c) $(N - N_s)/N$ (d) N/N_s
□ (b) — Definition.
- 21.** At starting, the slip of an induction motor is
(a) ∞ (b) 1 (c) 0.5 (d) 0
□ (b) — $N = 0$.
- 22.** The frequency of rotor current is
(a) f (b) f/s (c) $(1 - s)f$ (d) sf
□ (d) — $f_r = sf$.



- 23.** An induction motor cannot run at synchronous speed because
(a) friction is too high (b) the stator would overheat (c) no emf would be induced in the rotor (d) the supply frequency changes
☐ (c) — No relative motion.
- 24.** The direction of rotation of a 3-phase induction motor can be reversed by
(a) reversing the rotor (b) reducing voltage (c) interchanging any two supply leads (d) adding resistance
☐ (c) — Reverses phase sequence.
- 25.** Squirrel-cage rotor bars are
(a) connected to DC (b) insulated from the core and open-circuited (c) short-circuited by end rings (d) connected to slip rings
☐ (c) — Permanently shorted.
- 26.** External resistance can be added to the rotor of
(a) a DC shunt motor (b) a slip-ring motor (c) a squirrel-cage motor (d) a transformer
☐ (b) — Through slip rings.
- 27.** Slip at maximum torque is
(a) X_2/R_2 (b) R_2/X_2 (c) 1 (d) R_2/X_2
☐ (d) — $s_m = R_2/X_2$.
- 28.** Maximum torque of an induction motor is
(a) zero (b) inversely proportional to R_2 (c) proportional to R_2 (d) independent of R_2
☐ (d) — $T_{\max} = kE_2^2/2X_2$.
- 29.** In the normal operating region, torque of an induction motor is
(a) inversely proportional to slip (b) proportional to slip (c) independent of slip (d) proportional to s^2
☐ (b) — Low-slip region.
- 30.** Torque of an induction motor is proportional to
(a) V^2 (b) V (c) $\sqrt{V}$ (d) $1/V$
☐ (a) — $T \propto V^2$.
- 31.** Rotor copper loss equals
(a) $(1 - s) \times$ air-gap power (b) zero (c) $s \times$ air-gap power (d) air-gap power
☐ (c) — Power flow ratio.
- 32.** A star-delta starter reduces starting current to
(a) $1/3$ (b) $1/9$ (c) $1/\sqrt{3}$ (d) $1/2$
☐ (a) — Of DOL value.
- 33.** The rotating magnetic field of a 3-phase winding has a magnitude of
(a) zero (b) $1.5\Phi_m$ (c) Φ_m (d) $3\Phi_m$
☐ (b) — Constant resultant.
- 34.** The armature of a large alternator is usually
(a) on the rotor (b) in the exciter (c) on the stator (d) absent
☐ (c) — Easier insulation.
- 35.** The rotor field of an alternator is excited by
(a) both (b) AC (c) permanent magnets always (d) DC
☐ (d) — Through slip rings.
- 36.** Salient-pole alternators are used with
(a) no prime mover (b) hydraulic turbines (low speed) (c) steam turbines (d) gas turbines only
☐ (b) — Many poles, low speed.



- 37.** Cylindrical-rotor alternators are used for
(a) diesel engines only (b) low-speed operation (c) hydro plants (d) high-speed turbo-generators
☐ (d) — 2 or 4 poles.
- 38.** The frequency generated by an alternator is
(a) $120N/P$ (b) $PN/60$ (c) $PN/120$ (d) $120P/N$
☐ (c) — $f = PN/120$.
- 39.** A 2-pole alternator must run at what speed to generate 50 Hz?
(a) 1000 rpm (b) 1500 rpm (c) 750 rpm (d) 3000 rpm
☐ (d) — $120 \times 50/2$.
- 40.** About what fraction of industrial motors are induction motors?
(a) 30% (b) 10% (c) most (70–90%) (d) none
☐ (c) — Rugged and cheap.

Module V: Introduction to Power Systems

A power system is the network that generates electrical energy, transmits it over long distances and distributes it to consumers. It must supply power reliably, at constant voltage and frequency, as economically as possible and with minimum harm to the environment.

5.1 General structure of an electrical power system

A typical AC power system has three main parts connected by transformers and substations:

- **Generation:** power stations generate three-phase AC at 11–25 kV (in India typically 11 kV, 13.8 kV, 15.75 kV, 21 kV).
- **Transmission:** a generating-station step-up transformer raises the voltage to 132 kV, 220 kV, 400 kV or 765 kV (EHV/UHV). Power is carried over long distances by overhead lines. **Primary transmission** (220–765 kV) links power stations and major receiving stations; **secondary transmission** (66–132 kV) feeds sub-transmission substations.
- **Distribution:** step-down substations reduce the voltage to 33 kV or 11 kV (**primary distribution**), and distribution transformers reduce it to 400 V three-phase / 230 V single-phase (**secondary distribution**) for consumers.

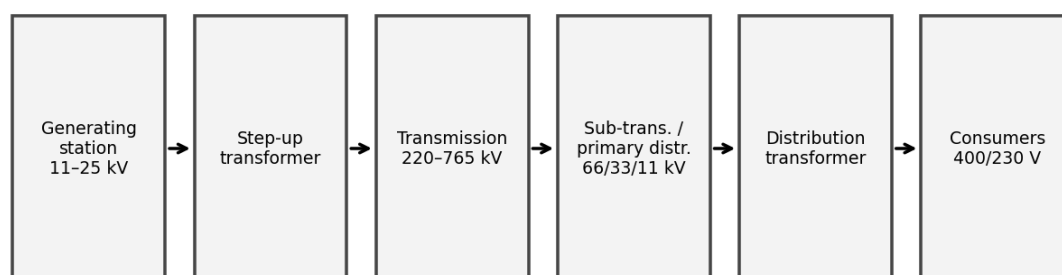


Figure 5.1: Structure of an AC power system from generation to consumers.

Standard voltages in India. Generation 11–25 kV; transmission 765, 400, 220, 132 kV AC (and ± 500 kV, ± 800 kV HVDC); sub-transmission 66, 33 kV; primary distribution 11 kV; secondary distribution 400/230 V, 50 Hz.

Grid. Power stations and loads are interconnected into regional and national grids.

Advantages of interconnection: greater reliability, sharing of reserve capacity, use of the most economical plants, and exchange of power between regions. India has one synchronous national grid ("One Nation, One Grid, One Frequency") operated by Grid Controller of India (Grid-India).

5.2 Why transmit at high voltage?

For a given power P transmitted at voltage V and power factor $\cos \phi$, the current is $I = P/(\sqrt{3} V \cos \phi)$. The line loss is $3I^2R$.

Line loss $\propto 1/V^2$; conductor volume (for same loss) $\propto 1/V^2$; voltage drop % $\propto 1/V^2$

- Doubling the voltage reduces the current to half and the I^2R loss to one-quarter.

- Higher voltage needs more insulation, taller towers and costlier transformers and switchgear, so there is an economic optimum voltage that increases with distance and power (rule of thumb: $kV \approx 5.5 \sqrt{(L/1.6 + kVA/100)}$, L in km).

AC vs DC transmission. AC is used for most transmission because voltage can be changed easily with transformers. **HVDC** is used for very long distances (above about 600–800 km for overhead lines), undersea cables, and interconnecting grids of different frequencies: it has no reactance or skin effect, needs only two conductors, and has no stability limit, but converter stations are expensive.

5.3 Concepts of generation

Electrical energy is produced by converting some other form of energy, usually by driving an alternator with a **prime mover** (turbine or engine).

Load curve terms. The variation of demand with time is shown by a **load curve**.

Load factor = average load/maximum demand; Demand factor = maximum demand/connected load

Plant capacity factor = average load/rated plant capacity; Diversity factor = sum of individual maximum demands/maximum demand of the group

- **Base-load plants** run continuously at nearly full output (large thermal, nuclear, run-of-river hydro).
- **Peak-load plants** are started only at times of high demand (gas turbines, pumped storage, storage hydro), because they can start and change output quickly.

5.4 Sources of electrical power

Conventional (non-renewable) sources.

- **Thermal (coal-fired steam) power station.** Coal is burnt in a boiler to produce high-pressure steam, which drives a steam turbine coupled to an alternator. The exhaust steam is condensed and recycled. Energy chain: chemical → heat → mechanical → electrical. Overall efficiency about 30–40%. Located near coal mines or water sources. Advantages: reliable base load, can be built anywhere with coal and water. Disadvantages: air pollution (CO_2 , SO_2 , fly ash), coal transport, low efficiency. Coal is the largest source of electricity in India, supplying roughly three-quarters of the energy generated, although its share of installed capacity is falling as renewables grow.
- **Hydro-electric power station.** Water stored at a height in a dam flows through a penstock to a water turbine (Pelton for high head, Francis for medium head, Kaplan for low head), driving an alternator. $P = \rho g Q H \eta$ (W). Advantages: no fuel cost, no pollution, quick starting (good for peak load), long life, also provides irrigation and flood control. Disadvantages: high initial cost, depends on rainfall, submergence of land, located far from load centres. Example in Odisha: Hirakud, Balimela, Rengali, Upper Indravati.
- **Nuclear power station.** Heat from the fission of uranium-235 (or plutonium) in a reactor produces steam, which drives a turbine-alternator. 1 kg of U-235 produces as much energy as about 2500 tonnes of coal. Advantages: small fuel quantity, no CO_2 emission, large base-load capacity. Disadvantages: high capital cost, radioactive waste disposal, safety concerns. Examples: Tarapur, Kudankulam, Kaiga.

- **Gas turbine and diesel power stations.** Burn natural gas or diesel; quick starting; used for peak load and emergency/standby supply; combined-cycle gas plants reach about 55-60% efficiency.

Non-conventional (renewable) sources.

- **Solar energy.** Photovoltaic (PV) cells convert sunlight directly into DC electricity (semiconductor p-n junction); inverters convert it to AC for the grid. Solar thermal plants concentrate sunlight to produce steam. Clean and abundant, but intermittent (no output at night) and needs large area and storage. India has rapidly growing solar capacity (large solar parks, rooftop solar).
- **Wind energy.** Wind turbines convert the kinetic energy of wind into mechanical energy that drives a generator. Power $\propto$ (wind speed)³: $P = (1/2)\rho A v^3 C_p$ ($C_p \leq 0.593$, the Betz limit). Clean, but intermittent and site-dependent; used in Tamil Nadu, Gujarat, Karnataka and offshore.
- **Biomass and bio-gas:** combustion of agricultural waste or gas from anaerobic digestion.
- **Geothermal, tidal and ocean energy; small hydro; fuel cells.**

Source	Energy conversion	Main advantage	Main limitation
Thermal (coal)	Chemical $\rightarrow$ heat $\rightarrow$ mechanical $\rightarrow$ electrical	Reliable base load	Pollution, low efficiency
Hydro	Potential $\rightarrow$ kinetic $\rightarrow$ mechanical $\rightarrow$ electrical	No fuel, quick start	High capital cost, site-specific
Nuclear	Nuclear $\rightarrow$ heat $\rightarrow$ mechanical $\rightarrow$ electrical	Huge energy from little fuel	Radioactive waste, cost
Solar PV	Light $\rightarrow$ electrical (direct)	Clean, no moving parts	Intermittent, needs storage
Wind	Kinetic $\rightarrow$ mechanical $\rightarrow$ electrical	Clean, low running cost	Intermittent, site-dependent

5.5 Transmission system components

- **Overhead lines:** conductors (ACSR, aluminium conductor steel reinforced: aluminium for conductivity, steel core for strength), supports (steel lattice towers for high voltage, RCC/steel poles for distribution), insulators (pin, suspension/disc strings, strain), cross-arms, earth wire (shields against lightning), lightning arresters.
- **Underground cables:** used in cities and where overhead lines are not possible; costlier but safe and unaffected by weather.
- **Substations:** contain transformers, circuit breakers, isolators, busbars, current and potential transformers, relays and lightning arresters, to change voltage and switch and protect circuits.
- **Line parameters:** resistance R, inductance L, capacitance C and conductance G, distributed along the line. Short lines (< 80 km) are represented by series R and L only; medium lines (80-240 km) add lumped capacitance (nominal-T or π); long lines (> 240 km) are treated with distributed parameters.

- **Corona:** at very high voltages, air around the conductor ionises, producing a faint violet glow, hissing noise, power loss and radio interference. Reduced by using larger-diameter or bundled conductors.
- **Ferranti effect:** at light load or no load, the receiving-end voltage of a long line can exceed the sending-end voltage, because of the line capacitance.

5.6 Distribution system

- **Feeders:** conductors from the substation to the distribution area; no tappings; designed for current capacity.
- **Distributors:** conductors from which tappings are taken for consumers; designed for voltage drop (should be within $\pm 6\%$ of declared voltage).
- **Service mains:** the connection from the distributor to the consumer's premises.

Types of distribution systems.

- **Radial system:** separate feeders radiate from a substation. Simple and cheap, but a fault on a feeder cuts off all consumers beyond it; consumers at the far end get lower voltage. Used in rural and small areas.
- **Ring main system:** feeders form a closed loop, so each consumer can be fed from two directions. Less voltage fluctuation and better reliability. Used in cities.
- **Interconnected system:** a ring main fed by two or more substations; highest reliability.

Secondary distribution. 3-phase 4-wire, 400 V between lines and 230 V between line and neutral. Single-phase loads (lights, fans, domestic appliances) are connected between a line and neutral; 3-phase loads (motors) across the three lines. Loads are distributed evenly among the phases to keep the system balanced.

5.7 Electricity tariffs and energy conservation (awareness)

- **Tariff:** the rate at which electricity is sold. Types: simple (flat) rate, block rate (different rates for blocks of units), two-part tariff (fixed charge per kW of maximum demand + energy charge per kWh), maximum demand tariff, power factor tariff (penalty for low pf), and time-of-day (ToD) tariff.
- **Energy conservation:** use of efficient equipment (LED lamps, BEE star-rated appliances, energy-efficient motors, variable-speed drives), power factor correction, reduction of standby power, and good maintenance.
- **Smart grid:** use of communication and automation (smart meters, remote control, integration of renewables and storage) to make the grid efficient and reliable.

5.8 Electrical safety and earthing (awareness)

- **Electric shock** occurs when current passes through the body; currents above about 30 mA can be fatal. Wet skin lowers body resistance.
- **Earthing (grounding):** connecting metal bodies of equipment to earth through a low-resistance path, so that in case of insulation failure the fault current flows to earth and blows the fuse or trips the breaker, keeping the body at earth potential. Types: pipe earthing, plate earthing. Earth resistance should be low (below about $1\ \Omega$ for substations, a few ohms for domestic installations). (See BEE Lab, Experiment 9: earth and insulation resistance.)
- **Protective devices:** fuses, miniature circuit breakers (MCB), earth-leakage circuit breakers (ELCB/RCCB, trip at 30 mA leakage), and circuit breakers in substations.

- **Safety rules:** switch off supply before working on circuits; use insulated tools and rubber mats; never touch live parts; use proper-rated fuses; do not overload sockets.

5.9 Common mistakes to avoid in Module V

- Stating that transmission is at high voltage "to increase current". It is done to **reduce** current and hence I^2R losses.
- Confusing feeders (no tappings) with distributors (tappings taken).
- Writing wind power $\propto v^2$. It is $\propto v^3$.
- Saying hydro stations are best for base load only. They are excellent for peak load because they start quickly.

5.10 Thermal power station: layout and working (long answer)

Main circuits.

- **Coal and ash circuit:** coal from the mines is crushed, pulverised and fed to the boiler furnace; the ash is removed to ash ponds; fly ash is captured by electrostatic precipitators (ESP).
- **Air and flue-gas circuit:** forced-draught fans supply air through the air pre-heater; induced-draught fans draw the flue gases through the economiser, air pre-heater and ESP to the chimney.
- **Feed water and steam circuit (Rankine cycle):** feed water $\rightarrow$ economiser (pre-heated by flue gas) $\rightarrow$ boiler drum and water tubes (converted to steam) $\rightarrow$ superheater $\rightarrow$ high-pressure turbine $\rightarrow$ reheater $\rightarrow$ IP/LP turbines $\rightarrow$ condenser (exhaust steam condensed under vacuum) $\rightarrow$ condensate pump $\rightarrow$ feed-water heaters $\rightarrow$ boiler feed pump $\rightarrow$ economiser.
- **Cooling water circuit:** water from a river or cooling tower circulates through the condenser tubes and returns heat to the atmosphere.

Energy conversion: chemical energy of coal $\rightarrow$ heat in the boiler $\rightarrow$ kinetic energy of steam $\rightarrow$ mechanical energy of the turbine $\rightarrow$ electrical energy in the alternator.

Why efficiency is low (30-40%): most of the heat is rejected in the condenser (the Carnot limit for the steam temperatures used), plus boiler, turbine and generator losses. Supercritical and ultra-supercritical boilers (higher pressure and temperature) raise the efficiency to about 40-45%.

Site selection: availability of coal (or good rail transport), plenty of water, cheap land, distance from populated areas, ash disposal facilities, and nearness to load centres.

5.11 Hydro-electric power station: layout and working

Main parts. Catchment area $\rightarrow$ reservoir $\rightarrow$ dam $\rightarrow$ intake with trash rack $\rightarrow$ penstock (large pipe carrying water to the turbine) $\rightarrow$ surge tank (absorbs pressure surges when the load changes suddenly) $\rightarrow$ valve house $\rightarrow$ water turbine $\rightarrow$ draft tube $\rightarrow$ tail race.

Energy conversion: potential energy of stored water $\rightarrow$ kinetic energy in the penstock $\rightarrow$ mechanical energy of the turbine $\rightarrow$ electrical energy.

Classification by head: high head (above 300 m, Pelton wheel), medium head (30-300 m, Francis turbine), low head (below 30 m, Kaplan turbine).

Pumped-storage plants: during off-peak hours, surplus power pumps water from a lower reservoir to an upper one; during peak hours the water generates electricity. They act as large energy stores and help integrate solar and wind power.

5.12 Nuclear power station: layout and working

Nuclear fission. A slow neutron splits a U-235 nucleus into two lighter nuclei, releasing about 200 MeV of energy and 2-3 fast neutrons, which cause further fissions (chain reaction).

Reactor components. Fuel rods (enriched uranium or natural uranium); moderator (heavy water, graphite or light water) to slow neutrons; control rods (cadmium or boron) that absorb neutrons to control the reaction rate; coolant (water, heavy water, CO₂ or liquid sodium) to remove heat; reflector; thick concrete biological shield.

Working (pressurised water reactor): heat from fission is carried by the pressurised primary coolant to a heat exchanger (steam generator), where secondary water is turned into steam; the steam drives a turbine-alternator as in a thermal station.

India: uses Pressurised Heavy Water Reactors (PHWR) with natural uranium as the first stage of a three-stage programme leading to fast breeder reactors and thorium-based reactors (India has large thorium reserves).

5.13 Solar photovoltaic systems: more details

- **Solar cell:** a large-area p-n junction of silicon. Photons with energy greater than the band gap create electron-hole pairs, which are separated by the junction field, producing a DC voltage of about 0.5-0.6 V per cell.
- **Module and array:** cells are connected in series (to raise voltage) and parallel (to raise current) to form modules (typically 300-550 W) and arrays.
- **System types:** stand-alone (with batteries, for remote areas), grid-connected (with inverter feeding the grid, net metering), hybrid (with a diesel or wind generator).
- **Components:** PV array, charge controller (with maximum power point tracking, MPPT), battery bank, inverter (DC to AC), protection and metering.
- **Efficiency:** commercial silicon modules about 18-23%. Output depends on sunlight intensity, temperature (output falls as temperature rises) and orientation (in India, panels face south, tilted at about the latitude angle).
- **Solar thermal:** flat-plate collectors for water heating; concentrating collectors (parabolic trough, tower) for steam generation.

5.14 Wind energy systems: more details

- **Horizontal-axis wind turbines (HAWT):** the most common type; 2 or 3 blades facing the wind; nacelle houses gearbox, generator and controls; tower 80-150 m high.
- **Vertical-axis wind turbines (VAWT):** Darrieus and Savonius types; accept wind from any direction; used for small applications.
- **Operation:** cut-in wind speed about 3-4 m/s; rated speed about 12-15 m/s; cut-out speed about 25 m/s (turbine is stopped to prevent damage).
- **Generators:** induction generators, doubly-fed induction generators (DFIG) or permanent-magnet synchronous generators with power-electronic converters.
- **Advantages:** free, clean, renewable fuel; low running cost. **Disadvantages:** variable output, needs windy sites, noise, visual impact, effect on birds.

5.15 Environmental aspects and energy mix

- Burning fossil fuels emits CO₂ (greenhouse gas), SO₂ and NO_x (acid rain) and particulates (health effects). Flue-gas desulphurisation, low-NO_x burners and ESPs

reduce emissions.

- Hydro projects affect ecosystems and displace people; nuclear plants produce long-lived radioactive waste.
- Renewable sources (solar, wind, small hydro, biomass) have very low operating emissions and are growing fast; they require energy storage (batteries, pumped storage) and a flexible grid to handle their variability.
- A reliable, affordable and clean power system uses a mix of sources: base-load, peaking and variable renewable generation, balanced by storage and demand management.

5.16 Solved numericals: Module V

Problem 1. 10 MW is to be transmitted at 0.8 pf over a 3-phase line of resistance 2 Ω per phase. Compare the line losses at 11 kV, 33 kV and 132 kV.

$I = P/(\sqrt{3} V \cos \phi)$. At 11 kV: $I = 10^7/(1.732 \times 11000 \times 0.8) = 656$ A; loss = $3 \times 656^2 \times 2 = 2.58$ MW.

At 33 kV: $I = 218.7$ A; loss = $3 \times 218.7^2 \times 2 = 287$ kW.

At 132 kV: $I = 54.7$ A; loss = 17.9 kW.

Answer: 2.58 MW (25.8%!), 287 kW (2.9%) and 17.9 kW (0.18%). Loss falls as $1/V^2$, which is why high voltages are used.

Problem 2. A hydro plant has a head of 100 m and a discharge of 50 m³/s. If the overall efficiency is 85%, find the power output.

$P = \rho g Q H \eta = 1000 \times 9.81 \times 50 \times 100 \times 0.85$ W.

Answer: $P \approx 41.7$ MW.

Problem 3. A wind turbine has a rotor diameter of 80 m and a power coefficient of 0.4. Find the power at a wind speed of 10 m/s ($\rho = 1.225$ kg/m³). What happens if the wind speed halves?

$A = \pi \times 40^2 = 5027$ m². $P = 0.5 \times 1.225 \times 5027 \times 10^3 \times 0.4 = 1.23$ MW.

At 5 m/s, $P \propto v^3 \Rightarrow P/8 = 0.154$ MW.

Answer: 1.23 MW; it falls to one-eighth (0.15 MW).

Problem 4. A power station has a maximum demand of 100 MW and generates 525,600 MWh in a year. Find the load factor.

Average load = $525600/8760 = 60$ MW. Load factor = $60/100$.

Answer: Load factor = 0.6 (60%).

Problem 5. A consumer has a connected load of 10 kW and a maximum demand of 6 kW. Find the demand factor.

Demand factor = $6/10$.

Answer: 0.6.



Problem 6. The individual maximum demands of four consumers are 5, 8, 6 and 11 kW, and the maximum demand of the group is 20 kW. Find the diversity factor.

Diversity factor = $(5 + 8 + 6 + 11)/20 = 30/20$.

Answer: 1.5.

Problem 7. A 500 MW coal-fired station has an overall efficiency of 35%. Coal has a calorific value of 5000 kcal/kg. Find the coal burnt per hour at full load.

Heat input per hour = $500 \times 10^3 \text{ kW} \times 3600 \text{ s}/0.35 = 5.143 \times 10^9 \text{ kJ}$.

Coal = $5.143 \times 10^9 / (5000 \times 4.186) \text{ kg} = 2.457 \times 10^5 \text{ kg}$.

Answer: About 246 tonnes per hour.

Problem 8. A 3 kW rooftop solar plant receives the equivalent of 5 full-sun hours per day. Estimate its annual energy output.

Daily energy = $3 \times 5 = 15 \text{ kWh}$. Annual = 15×365 .

Answer: About 5475 kWh (units) per year (in practice slightly less because of losses).

Problem 9. If the transmission voltage is doubled for the same power and losses, how does the required conductor cross-section change?

Current halves; loss = I^2R must stay the same $\Rightarrow R$ can be 4 times larger $\Rightarrow$ area (and volume) of conductor is one-quarter.

Answer: The conductor volume reduces to 25%.

5.17 MCQs: Module V

1. In India, power is generated at about

- (a) 220 kV (b) 765 kV (c) 400 V (d) 11-25 kV

☐ (d) — Alternator voltage.

2. The highest AC transmission voltage used in India is

- (a) 220 kV (b) 400 kV (c) 765 kV (d) 1200 kV

☐ (c) — EHV/UHV lines.

3. Primary distribution in India is at

- (a) 230 V (b) 11 kV (c) 400 V (d) 132 kV

☐ (b) — Also 33 kV.

4. Secondary distribution voltage (line) in India is

- (a) 400 V (b) 11 kV (c) 110 V (d) 230 V

☐ (a) — 3-phase 4-wire.

5. Power is transmitted at high voltage to

- (a) reduce insulation (b) increase frequency (c) reduce line losses (d) increase current

☐ (c) — Loss $\propto 1/V^2$.

6. Doubling the transmission voltage reduces line loss to

- (a) 1/2 (b) 1/8 (c) 2 times (d) 1/4

☐ (d) — I halves.



- 7. ACSR conductor means**
(a) aluminium cable single run (b) aluminium conductor steel reinforced (c) all copper steel rod (d) alloy conductor steel reinforced
☐ (b) — Standard overhead conductor.
- 8. The Ferranti effect refers to**
(a) power factor (b) skin effect (c) receiving-end voltage exceeding sending-end voltage at light load (d) corona loss
☐ (c) — Line capacitance.
- 9. Corona can be reduced by**
(a) using DC only (b) increasing voltage (c) reducing conductor diameter (d) using bundled or larger conductors
☐ (d) — Lower surface field.
- 10. HVDC transmission is preferred for**
(a) household wiring (b) rural distribution (c) very long distances and undersea cables (d) short distances
☐ (c) — No reactance.
- 11. A feeder differs from a distributor in that a feeder has**
(a) lower voltage (b) tappings (c) no tappings (d) no current
☐ (c) — Designed for current capacity.
- 12. The most reliable distribution system is**
(a) radial (b) single feeder (c) ring main / interconnected (d) none
☐ (c) — Fed from two sides.
- 13. Load factor is**
(a) average load/maximum demand (b) connected load/max demand (c) maximum demand/average load (d) always 1
☐ (a) — Definition.
- 14. Demand factor is**
(a) average load/connected load (b) always > 1 (c) connected load/maximum demand (d) maximum demand/connected load
☐ (d) — Definition.
- 15. Diversity factor is usually**
(a) equal to 0 (b) less than 1 (c) greater than 1 (d) negative
☐ (c) — Maximum demands occur at different times.
- 16. Which plant is best suited for peak load?**
(a) Hydro/gas turbine (b) Nuclear (c) Solar only (d) Coal thermal
☐ (a) — Quick starting.
- 17. Which plant is best suited for base load?**
(a) Pumped storage (b) Gas turbine (c) Nuclear/large thermal (d) Diesel
☐ (c) — Runs continuously.
- 18. The energy conversion in a thermal power station is**
(a) kinetic $\rightarrow$ electrical (b) chemical $\rightarrow$ heat $\rightarrow$ mechanical $\rightarrow$ electrical (c) nuclear $\rightarrow$ electrical (d) light $\rightarrow$ electrical
☐ (b) — Coal combustion.
- 19. The overall efficiency of a coal thermal plant is about**
(a) 30-40% (b) 95% (c) 80% (d) 10%
☐ (a) — Losses in condenser.



- 20.** A Pelton wheel turbine is used for
(a) high head (b) medium head (c) no head (d) low head
☐ (a) — Impulse turbine.
- 21.** A Kaplan turbine is used for
(a) high head (b) medium head only (c) steam (d) low head
☐ (d) — Propeller turbine.
- 22.** The power of a hydro plant is
(a) $\rho g Q/H$ (b) $\rho Q/gH$ (c) gH/Q (d) ρgQH
☐ (d) — Times efficiency.
- 23.** The fuel used in most nuclear reactors is
(a) diesel (b) hydrogen (c) U-235 (d) coal
☐ (c) — Fissile isotope.
- 24.** Solar PV cells convert
(a) light directly to electricity (b) chemical to electricity (c) heat to electricity (d) wind to electricity
☐ (a) — Photovoltaic effect.
- 25.** The output of solar PV cells is
(a) AC (b) DC (c) three-phase AC (d) pulsating AC
☐ (b) — Inverter needed for grid.
- 26.** Wind power is proportional to
(a) $\sqrt{v}$ (b) v (c) v^3 (d) v^2
☐ (c) — $P = (1/2)\rho A v^3 C_p$.
- 27.** The Betz limit for wind turbines is about
(a) 90% (b) 59.3% (c) 100% (d) 25%
☐ (b) — Maximum C_p .
- 28.** Which is a renewable source?
(a) Natural gas (b) Diesel (c) Wind (d) Coal
☐ (c) — Replenished naturally.
- 29.** A disadvantage of hydro power is
(a) high initial cost and dependence on rainfall (b) pollution (c) slow starting (d) high fuel cost
☐ (a) — Site and rainfall.
- 30.** A disadvantage of nuclear power is
(a) radioactive waste disposal (b) large fuel quantity (c) slow response only (d) CO_2 emission
☐ (a) — Safety and waste.
- 31.** Earthing is done to
(a) improve power factor (b) reduce frequency (c) protect people from electric shock (d) increase voltage
☐ (c) — Low-resistance path to earth.
- 32.** An RCCB/ELCB typically trips at a leakage current of
(a) 300 A (b) 1 μA (c) 30 mA (d) 3 A
☐ (c) — Protects people.
- 33.** The substation equipment used to step voltage up or down is
(a) transformer (b) circuit breaker (c) isolator (d) lightning arrester
☐ (a) — Changes voltage.



- 34.** The equipment that interrupts fault currents automatically is
(a) isolator (b) insulator (c) circuit breaker (d) busbar
□ (c) — Opens on fault.
- 35.** The frequency of the Indian grid is
(a) 50 Hz (b) 60 Hz (c) 400 Hz (d) 25 Hz
□ (a) — National standard.

5.18 Practice problems with solutions: Module V

Q1. A town's peak demand is 50 MW and its annual energy consumption is 262,800 MWh. Find the load factor.

$$\text{Average} = 262800/8760 = 30 \text{ MW. LF} = 30/50.$$

Answer: 0.6.

Q2. A power station of 100 MW capacity generates 438,000 MWh per year. Find its plant capacity factor.

$$\text{Average} = 438000/8760 = 50 \text{ MW. CF} = 50/100.$$

Answer: 0.5.

Q3. Find the power that can be generated from a stream with a flow of 10 m³/s and a head of 30 m ($\eta = 80\%$).

$$P = 1000 \times 9.81 \times 10 \times 30 \times 0.8 = 2.354 \times 10^6 \text{ W.}$$

Answer: ≈ 2.35 MW.

Q4. A wind turbine produces 500 kW at 8 m/s. Estimate its output at 12 m/s (below rated speed).

$$P \propto v^3 \Rightarrow 500 \times (12/8)^3 = 500 \times 3.375.$$

Answer: ≈ 1.69 MW (limited in practice by the turbine rating).

Q5. A solar module has 60 cells in series, each giving 0.5 V and 8 A at maximum power. Find the module voltage and power.

$$V = 60 \times 0.5 = 30 \text{ V; } P = 30 \times 8 = 240 \text{ W.}$$

Answer: 30 V; 240 W.

Q6. How many such modules are needed for a 3 kW rooftop plant, and how should 10 modules be arranged for about 300 V?

$$3000/240 = 12.5 \Rightarrow 13 \text{ modules. For 300 V, connect 10 modules in series (10} \times 30 \text{ V).}$$

Answer: 13 modules; 10 in series give 300 V.

Q7. 100 kW is transmitted at 400 V (3-phase, pf 1) over a feeder of resistance 0.05 Ω per conductor. Find the loss. What would it be at 11 kV?

$$\text{At 400 V: } I = 10^5/(\sqrt{3} \times 400) = 144.3 \text{ A; loss} = 3 \times 144.3^2 \times 0.05 = 3.12 \text{ kW.}$$

$$\text{At 11 kV: } I = 5.25 \text{ A; loss} = 3 \times 5.25^2 \times 0.05 = 4.1 \text{ W.}$$

Answer: 3.12 kW at 400 V vs 4.1 W at 11 kV.



Q8. An ELCB rated 30 mA protects a circuit. If a person of body resistance 1500 Ω touches a 230 V live wire, will it trip?

Leakage current = $230/1500 = 153 \text{ mA} > 30 \text{ mA}$.

Answer: Yes, it trips (in a fraction of a second), protecting the person.

Q9. A consumer uses a 1.5 kW geyser for 1 h, a 100 W fan for 10 h and ten 10 W LED lamps for 6 h daily. Find the monthly (30-day) energy consumption and cost at ₹6 per unit.

Daily energy = $1.5 + 1.0 + 0.6 = 3.1 \text{ kWh}$. Monthly = 93 kWh .

Answer: 93 units; ₹558.



Short-Answer Questions (2 marks each) with Answers

Module I

Q1. State Kirchhoff's laws.

KCL: the algebraic sum of currents at a node is zero. KVL: the algebraic sum of emfs and voltage drops around any closed loop is zero.

Q2. Differentiate between a mesh and a loop.

A loop is any closed path; a mesh is a loop that does not contain any other loop inside it. Every mesh is a loop, but not every loop is a mesh.

Q3. What is a supermesh?

When a current source is common to two meshes, the two meshes are combined into a supermesh; KVL is written around the outer path, and the source gives a constraint equation between the mesh currents.

Q4. State the superposition theorem.

In a linear bilateral network with several sources, the response in any element is the algebraic sum of the responses due to each source acting alone, with the other sources replaced by their internal resistances.

Q5. State Thevenin's theorem.

Any linear two-terminal network can be replaced by a voltage source V_{th} (open-circuit voltage) in series with R_{th} (resistance seen from the terminals with sources replaced by internal resistances).

Q6. State Norton's theorem.

Any linear two-terminal network can be replaced by a current source I_N (short-circuit current) in parallel with $R_N (= R_{th})$.

Q7. State the maximum power transfer theorem.

Maximum power is delivered to a load when the load resistance equals the Thevenin resistance of the source network; $P_{max} = V_{th}^2 / 4R_{th}$.

Q8. Write the delta-to-star conversion formula for R_A .

$$R_A = R_{AB}R_{CA} / (R_{AB} + R_{BC} + R_{CA}).$$

Q9. What is the efficiency at maximum power transfer? Why is it not used in power systems?

50%. Half the power is wasted in the source resistance, which is unacceptable for large power.

Q10. Define linear and bilateral elements.



Linear: V-I relation is a straight line (obeys Ohm's law). Bilateral: behaves the same for current in either direction.

Module II

Q1. Define rms value of an alternating current.

The steady DC current that produces the same heat in a given resistance in the same time as the AC. For a sine wave $I_{\text{rms}} = I_m/\sqrt{2}$.

Q2. Define form factor and peak factor. Give their values for a sine wave.

Form factor = rms/average = 1.11; peak factor = peak/rms = 1.414.

Q3. What is power factor? Why is a low power factor undesirable?

Power factor = $\cos \phi = P/S$. Low pf needs larger current for the same power, causing larger losses, voltage drop and bigger equipment.

Q4. Define active, reactive and apparent power.

Active $P = VI \cos \phi$ (W), reactive $Q = VI \sin \phi$ (VAR), apparent $S = VI$ (VA); $S^2 = P^2 + Q^2$.

Q5. What is resonance in a series RLC circuit? Give its frequency.

The condition $X_L = X_C$, when the circuit is purely resistive, impedance is minimum and current maximum. $f_0 = 1/(2\pi\sqrt{LC})$.

Q6. Define Q-factor and bandwidth of a series resonant circuit.

$Q = \omega_0 L/R$ (voltage magnification). Bandwidth = $f_2 - f_1 = R/(2\pi L) = f_0/Q$.

Q7. Why is a parallel resonant circuit called a rejector circuit?

At resonance its impedance is maximum (L/CR), so it draws minimum current, rejecting the resonant frequency.

Q8. Write the relation between line and phase quantities in star and delta.

Star: $V_L = \sqrt{3}V_{\text{ph}}$, $I_L = I_{\text{ph}}$. Delta: $V_L = V_{\text{ph}}$, $I_L = \sqrt{3}I_{\text{ph}}$.

Q9. Write the expression for power in a balanced 3-phase circuit.

$P = \sqrt{3}V_L I_L \cos \phi$, where ϕ is the angle between phase voltage and phase current.

Q10. In the two-wattmeter method, what does a negative reading indicate?

The power factor of the load is less than 0.5 ($\phi > 60^\circ$).

Q11. Give two advantages of a three-phase system over single phase.

Less conductor material for the same power; constant (non-pulsating) power; self-starting 3-phase motors.



Module III

Q1. State Faraday's laws of electromagnetic induction.

(1) An emf is induced when the flux linked with a circuit changes. (2) Its magnitude equals the rate of change of flux linkage, $e = N d\Phi/dt$.

Q2. State Lenz's law.

The induced emf acts in a direction that opposes the change producing it.

Q3. Define reluctance. Write its unit.

The opposition of a magnetic circuit to the setting up of flux, $S = l/(\mu_0\mu_r A)$. Unit AT/Wb.

Q4. Compare electric and magnetic circuits (any two similarities and one difference).

Similar: mmf $\leftrightarrow$ emf, flux $\leftrightarrow$ current, reluctance $\leftrightarrow$ resistance. Different: current flows and dissipates energy continuously; flux does not.

Q5. What is hysteresis loss? How is it reduced?

Energy lost as heat in each cycle of magnetisation, proportional to the loop area. Reduced by using soft magnetic materials like silicon steel.

Q6. What is the function of the commutator in a DC machine?

It converts the alternating emf in the armature coils into DC at the brushes (generator), and reverses coil currents to keep torque unidirectional (motor).

Q7. Write the emf equation of a DC generator.

$E = P\Phi ZN/(60A)$; $A = P$ for lap, 2 for wave winding.

Q8. What is back emf? What is its significance?

The emf induced in a motor's armature opposing the supply voltage. It regulates the armature current according to load and makes the motor self-regulating.

Q9. Why is a starter needed for a DC motor?

At starting, back emf is zero, so the armature current V/R_a would be very large and damage the motor.

Q10. Why should a DC series motor never be started without load?

At no load the current and flux are very small, so the speed ($N \propto 1/\Phi$) rises to dangerously high values.

Q11. Name the methods of speed control of a DC shunt motor.

Flux (field) control, armature resistance control, and armature voltage control (Ward-Leonard).



Module IV

Q1. State the working principle of a transformer.

Mutual induction: an alternating flux produced by the primary current links the secondary and induces an emf in it.

Q2. Write the emf equation of a transformer.

$$E = 4.44 f N \Phi_m.$$

Q3. Why is the transformer core laminated?

To reduce eddy-current loss by increasing the resistance of the eddy-current paths.

Q4. Define voltage regulation of a transformer.

The change in secondary terminal voltage from no load to full load, expressed as a percentage of the no-load voltage.

Q5. What is the condition for maximum efficiency of a transformer?

Copper loss = iron loss.

Q6. Why is transformer rating in kVA and not in kW?

Losses depend on voltage (iron loss) and current (copper loss), not on the load power factor, so the rating is given in kVA.

Q7. Define slip of an induction motor.

$s = (N_s - N)/N_s$, the fractional difference between synchronous speed and rotor speed.

Q8. Why does an induction motor never run at synchronous speed?

At synchronous speed there is no relative motion between the rotating field and the rotor, so no emf, no current and no torque.

Q9. Why is a squirrel-cage motor widely used?

It is simple, rugged, cheap, needs little maintenance and runs at nearly constant speed.

Q10. Write the expression for the frequency of an alternator.

$$f = PN/120.$$

Q11. Why is the field of a large alternator placed on the rotor?

The high-voltage armature is easier to insulate and connect when stationary; only low-voltage DC needs slip rings; the rotor is lighter.

Module V

Q1. Draw (describe) the general structure of a power system.



Generation (11-25 kV) → step-up transformer → transmission (132-765 kV) → sub-transmission and primary distribution (66/33/11 kV) → distribution transformer → consumers (400/230 V).

Q2. Why is power transmitted at high voltage?

For the same power, higher voltage means lower current, so I^2R losses, voltage drop and conductor material are reduced.

Q3. Distinguish between feeders and distributors.

Feeders carry power from the substation to the distribution area without tapplings (designed for current); distributors have tapplings for consumers (designed for voltage drop).

Q4. Name two conventional and two non-conventional sources of electrical energy.

Conventional: thermal (coal), hydro, nuclear. Non-conventional: solar, wind.

Q5. Define load factor.

Load factor = average load/maximum demand over a given period.

Q6. What is earthing? Why is it necessary?

Connecting the non-current-carrying metal parts of equipment to earth through a low-resistance path, to protect people from electric shock and to allow protective devices to operate on faults.

Supplementary Topics: Measuring Instruments, Wiring and Safety

These topics support the Basic Electrical Engineering Laboratory and are frequently asked as short questions.

Electrical measuring instruments

Instrument	Measures	Connection	Ideal internal resistance
Ammeter	Current	In series with the load	Zero (very low)
Voltmeter	Voltage	In parallel with the element	Infinite (very high)
Wattmeter	Power	Current coil in series, pressure coil in parallel	—
Energy meter	Energy (kWh)	Like a wattmeter, with a counting mechanism	—
Ohmmeter / multimeter	Resistance (also V, I)	Across the component (circuit de-energised)	—
Megger (insulation tester)	Insulation resistance (MΩ)	Between conductor and earth/another conductor	—
Earth tester	Earth-electrode resistance	Three-electrode method	—
Frequency meter	Frequency (Hz)	Across the supply	—

Types of instruments.

- **Permanent magnet moving coil (PMMC):** DC only; uniform scale; very accurate; the deflecting torque is proportional to current.
- **Moving iron (MI):** AC and DC; non-uniform (square-law) scale; rugged and cheap; reads rms values.
- **Electrodynamometer:** AC and DC; used as a wattmeter (current coil fixed, pressure coil moving).
- **Induction type:** AC only; used in energy meters (now largely replaced by electronic/smart meters).
- **Digital multimeter (DMM):** electronic; high input resistance; measures V, I, R, continuity, sometimes capacitance and frequency.

Essential torques in an analogue instrument: deflecting torque (moves the pointer), controlling torque (spring or gravity, brings it to a definite position and back to zero), damping torque (air friction or eddy current, brings the pointer to rest quickly without oscillation, ideally critically damped; compare Physics, Module I).

Extension of range. Ammeter: a low-resistance **shunt** in parallel: $R_{sh} = I_m R_m / (I - I_m)$. Voltmeter: a high-resistance **multiplier** in series: $R_{se} = V/I_m - R_m$.

Energy meter calibration. Meter constant (revolutions or impulses per kWh). % error = (meter reading – true energy)/true energy × 100. (See BEE Lab, Experiment 7.)

Domestic wiring

- **Supply:** single-phase 230 V, 50 Hz from the distribution transformer (phase and neutral) through the service main, energy meter, main switch with fuse or MCB, distribution board, and individual circuits (lighting and power sub-circuits). A separate earth wire runs to all sockets and metal appliances.
- **Types of wiring:** PVC conduit (concealed or surface), casing-capping (older), cleat (temporary), batten wiring.
- **Colour code (IS/IEC):** phase red/brown, neutral black/blue, earth green or green-yellow.
- **Switches are always connected in the phase wire**, so that the appliance is dead when switched off.
- **Staircase (two-way) wiring:** two two-way switches allow a lamp to be controlled from two places (the lamp is ON when the two switches are in "matching" positions).
- **Fluorescent / LED lamps, fans, sockets** (5 A for light loads, 15/16 A for heavy loads such as heaters and ACs).

Protective devices

- **Fuse:** a short piece of wire of low melting point in series with the circuit; melts when the current exceeds its rating. Rewirable and HRC (high rupturing capacity) cartridge fuses. Fusing factor = minimum fusing current/rated current (> 1).
- **Miniature circuit breaker (MCB):** thermal (bimetal) trip for overloads and magnetic trip for short circuits; can be reset; replaces fuses in modern installations.
- **Residual current circuit breaker (RCCB/ELCB):** compares phase and neutral currents; trips when the difference (leakage to earth) exceeds 30 mA (for human protection) or 100–300 mA (for fire protection).
- **Earthing:** pipe or plate electrode buried in moist soil with charcoal and salt; earth resistance should be low. The earth wire connects all metal bodies; a fault then causes a large current that operates the fuse, MCB or RCCB.

First aid for electric shock (awareness)

Switch off the supply immediately or separate the victim from the live conductor using a dry insulating object (wooden stick, rubber mat). Do not touch the victim directly while they are in contact with the supply. Check breathing and pulse; start artificial respiration/CPR if trained, and call for medical help at once.



Exam Practice Set: Mixed Solved Numericals

Problem 1. Find the current supplied by the battery in a network where a 10 V battery feeds 2 Ω in series with a parallel combination of 3 Ω and 6 Ω , and that combination is in series with 4 Ω back to the battery.

$$3 \parallel 6 = 2 \Omega. \text{ Total} = 2 + 2 + 4 = 8 \Omega.$$

Answer: $I = 10/8 = 1.25 \text{ A}.$

Problem 2. A network has two sources: 6 V with 1 Ω and 12 V with 2 Ω , both connected in parallel (same polarity) across a 3 Ω load. Find the load voltage by nodal analysis.

$$(V - 6)/1 + (V - 12)/2 + V/3 = 0 \Rightarrow \text{multiply by 6: } 6V - 36 + 3V - 36 + 2V = 0 \Rightarrow 11V = 72.$$

Answer: $V = 6.55 \text{ V}; \text{ load current } 2.18 \text{ A}.$

Problem 3. A resistor and a capacitor in series draw 2 A at 0.6 pf leading from 200 V, 50 Hz. Find R and C.

$$Z = 100 \Omega; R = 60 \Omega; X_C = 80 \Omega. C = 1/(314.16 \times 80).$$

Answer: $R = 60 \Omega; C \approx 39.8 \mu\text{F}.$

Problem 4. A series circuit takes 1 kW at 0.5 pf lagging from 230 V. What must be added to bring the pf to unity, and what will the new current be (supply voltage unchanged, same R)?

$$I = 1000/(230 \times 0.5) = 8.7 \text{ A}; Z = 26.44 \Omega; R = 13.22 \Omega; X_L = 22.9 \Omega.$$

Add a series capacitor with $X_C = 22.9 \Omega$: $C = 1/(314.16 \times 22.9) = 139 \mu\text{F}$. New current = $230/13.22 = 17.4 \text{ A}$ (power rises to 4 kW).

Answer: Series C $\approx 139 \mu\text{F}; I = 17.4 \text{ A}.$

Problem 5. A balanced 3-phase star load takes 12 kW at 0.6 pf from 400 V. Find the impedance per phase.

$$I_L = 12000/(\sqrt{3} \times 400 \times 0.6) = 28.87 \text{ A}. Z_{ph} = 230.9/28.87 = 8 \Omega. R = 4.8 \Omega, X = 6.4 \Omega.$$

Answer: $Z_{ph} = 4.8 + j6.4 \Omega.$

Problem 6. An iron ring with a 1 mm gap needs 2000 AT to produce 1.2 T in the gap. How much of this is used by the gap?

$$AT_{gap} = (B/\mu_0) \times l_g = (1.2/4\pi \times 10^{-7}) \times 10^{-3} = 955 \text{ AT}.$$

Answer: About 955 AT (48%) for the 1 mm gap.

Problem 7. A 230 V DC shunt motor has $R_a = 0.25 \Omega$. At full load it takes $I_a = 40 \text{ A}$ and runs at 1200 rpm. Find the speed at half torque (flux constant).

$$E_{b1} = 220 \text{ V}. \text{ Half torque} \Rightarrow I_{a2} = 20 \text{ A} \Rightarrow E_{b2} = 225 \text{ V}.$$

Answer: $N_2 = 1200 \times 225/220 \approx 1227 \text{ rpm}.$



Problem 8. A 4-pole, wave-wound DC generator has 400 conductors and runs at 750 rpm. Find the flux per pole needed to generate 240 V.

$$\Phi = 60AE/(PZN) = 60 \times 2 \times 240/(4 \times 400 \times 750).$$

Answer: $\Phi = 0.024$ Wb.

Problem 9. A 1-phase transformer takes 0.5 A at 0.25 pf on no load from 230 V. The secondary supplies 20 A at 0.8 pf lagging at 115 V. Find the primary current.

$$K = 0.5. I_2' = 10 \text{ A at } 0.8 \text{ lag: } 8 - j6 \text{ A. } I_0 = 0.5 \text{ at } 0.25 \text{ lag: } 0.125 - j0.484 \text{ A.}$$

$$I_1 = 8.125 - j6.484 \text{ A} \Rightarrow |I_1| = 10.4 \text{ A.}$$

Answer: $I_1 \approx 10.4$ A.

Problem 10. A transformer has maximum efficiency of 98% at 15 kVA, unity pf. Find its iron loss.

$$\text{Losses at } \eta_{\max} = 15/0.98 - 15 = 0.306 \text{ kW, equally divided between iron and copper.}$$

Answer: $P_i \approx 153$ W.

Problem 11. A 4-pole, 50 Hz induction motor has a rotor frequency of 1.5 Hz at full load. Find the full-load speed and slip.

$$s = 1.5/50 = 0.03; N = 1500 \times 0.97.$$

Answer: $s = 3\%$; $N = 1455$ rpm.

Problem 12. A 3-phase line transmits 5 MW at 33 kV and 0.9 pf over 20 km. The resistance is 0.2 Ω /km per conductor. Find the line loss and transmission efficiency.

$$I = 5 \times 10^6/(\sqrt{3} \times 33000 \times 0.9) = 97.2 \text{ A. } R = 4 \Omega. \text{ Loss} = 3 \times 97.2^2 \times 4 = 113.4 \text{ kW.}$$

Answer: Loss ≈ 113 kW; $\eta = 5/(5 + 0.113) \approx 97.8\%$.

Mixed MCQ Revision Test (all modules)

1. In a series circuit of 4 Ω and 6 Ω across 20 V, the voltage across 6 Ω is

- (a) 8 V (b) 20 V (c) 10 V (d) 12 V

☐ (d) — $20 \times 6/10$.

2. Thevenin resistance of a 10 V source with 2 Ω internal resistance in parallel with 2 Ω is

- (a) 0.5 Ω (b) 1 Ω (c) 4 Ω (d) 2 Ω

☐ (b) — $2 \parallel 2$.

3. The current through a 5 Ω load connected to a Norton source of 4 A with 20 Ω is

- (a) 0.8 A (b) 1 A (c) 4 A (d) 3.2 A

☐ (d) — $4 \times 20/25$.

4. A 230 V, 1 kW heater has a resistance of

- (a) 52.9 Ω (b) 4.35 Ω (c) 23 Ω (d) 230 Ω

☐ (a) — V^2/P .



5. If the frequency of supply to a pure capacitor is doubled, the current
(a) doubles (b) remains same (c) becomes zero (d) halves
□ (a) — X_C halves.
6. The impedance of $6\ \Omega$ resistance and $8\ \Omega$ reactance in series is
(a) $14\ \Omega$ (b) $10\ \Omega$ (c) $2\ \Omega$ (d) $48\ \Omega$
□ (b) — $\sqrt{36 + 64}$.
7. A circuit has $P = 3\ \text{kW}$ and $Q = 4\ \text{kVAR}$. The apparent power is
(a) $12\ \text{kVA}$ (b) $7\ \text{kVA}$ (c) $1\ \text{kVA}$ (d) $5\ \text{kVA}$
□ (d) — $\sqrt{9 + 16}$.
8. The power factor of the above circuit is
(a) 0.8 (b) 0.5 (c) 0.75 (d) 0.6
□ (d) — $3/5$.
9. At resonance, a series RLC circuit with $R = 20\ \Omega$ on $100\ \text{V}$ draws
(a) $0\ \text{A}$ (b) $5\ \text{A}$ (c) $2000\ \text{A}$ (d) $20\ \text{A}$
□ (b) — V/R .
10. Three-phase line voltage in India is
(a) $110\ \text{V}$ (b) $230\ \text{V}$ (c) $400\ \text{V}$ (d) $440\ \text{V}$
□ (c) — Standard LT supply.
11. A delta load has $20\ \text{A}$ phase current. The line current is
(a) $60\ \text{A}$ (b) $34.6\ \text{A}$ (c) $11.5\ \text{A}$ (d) $20\ \text{A}$
□ (b) — $\sqrt{3} \times 20$.
12. With $W_1 = W_2$ in the two-wattmeter method, the load is
(a) capacitive (b) purely resistive (c) purely inductive (d) zero pf
□ (b) — $\text{pf} = 1$.
13. The unit of magnetic flux density is
(a) weber (b) tesla (c) henry (d) AT/m
□ (b) — Wb/m^2 .
14. A $1\ \text{mm}$ air gap in a magnetic circuit
(a) reduces reluctance (b) reduces the required mmf (c) increases the required mmf greatly (d) has no effect
□ (c) — $\mu_r = 1$.
15. A 6-pole lap winding has how many parallel paths?
(a) 2 (b) 6 (c) 12 (d) 3
□ (b) — $A = P$.
16. The back emf of a $220\ \text{V}$ motor with $R_a = 1\ \Omega$ drawing $10\ \text{A}$ is
(a) $220\ \text{V}$ (b) $10\ \text{V}$ (c) $210\ \text{V}$ (d) $230\ \text{V}$
□ (c) — $V - I_a R_a$.
17. If the flux of a DC motor is reduced, its speed
(a) decreases (b) stays same (c) increases (d) becomes zero
□ (c) — $N \propto 1/\Phi$.
18. The emf per turn of a transformer is
(a) proportional to turns (b) zero (c) different in primary and secondary (d) same in both windings
□ (d) — $4.44f\Phi_m$.
19. A transformer with 500 primary and 100 secondary turns on $230\ \text{V}$ gives
(a) $230\ \text{V}$ (b) $23\ \text{V}$ (c) $1150\ \text{V}$ (d) $46\ \text{V}$
□ (d) — $230/5$.



- 20.** The losses that remain constant in a transformer from no load to full load are
(a) iron losses (b) copper losses (c) both (d) none
☐ (a) — Depend on V and f.
- 21.** A 2-pole, 50 Hz induction motor can never run at
(a) 2880 rpm (b) 3000 rpm (c) 2950 rpm (d) 2900 rpm
☐ (b) — Synchronous speed.
- 22.** If the slip of an induction motor increases, the rotor current frequency
(a) decreases (b) remains 50 Hz (c) increases (d) becomes zero
☐ (c) — $f_r = sf$.
- 23.** Hydro power is proportional to
(a) square of head (b) product of head and discharge (c) discharge only (d) head only
☐ (b) — $P = \rho gQH\eta$.
- 24.** The component that protects a transmission line against lightning is
(a) fuse (b) transformer (c) lightning arrester / earth wire (d) isolator
☐ (c) — Diverts surges to earth.
- 25.** Which is a good conductor used for overhead lines?
(a) ACSR (b) Steel only (c) Silver (d) Copper only
☐ (a) — Strength + conductivity.



Additional Practice Problems with Solutions (All Modules)

DC networks

Q1. Four resistors of $10\ \Omega$ each are connected in a square ABCD. Find the resistance between A and C (opposite corners).

Two paths A-B-C and A-D-C, each $20\ \Omega$, in parallel.

Answer: $10\ \Omega$.

Q2. In the same square, find the resistance between A and B (adjacent corners).

$10\ \Omega$ in parallel with $(10 + 10 + 10) = 30\ \Omega$: $10 \times 30/40$.

Answer: $7.5\ \Omega$.

Q3. A 6 V battery of internal resistance $0.5\ \Omega$ is connected to a $2.5\ \Omega$ resistor. Find the terminal voltage and the power in the load.

$I = 6/3 = 2\text{ A}$. $V = 6 - 1 = 5\text{ V}$. $P = 2^2 \times 2.5 = 10\text{ W}$.

Answer: 5 V ; 10 W .

Q4. Two resistors in series total $25\ \Omega$; in parallel they give $4\ \Omega$. Find them.

$R_1 + R_2 = 25$, $R_1 R_2 = 100 \Rightarrow$ roots of $x^2 - 25x + 100 = 0$.

Answer: $20\ \Omega$ and $5\ \Omega$.

Q5. Find V_{th} across $R_3 = 6\ \Omega$ in a circuit of 12 V source, $R_1 = 3\ \Omega$ in series, and $R_2 = 6\ \Omega$ across the source side of R_3 (R_2 and R_3 both from the same node to ground).

With R_3 removed: $V_{th} = 12 \times 6/(3 + 6) = 8\text{ V}$. $R_{th} = 3 \parallel 6 = 2\ \Omega$. $I_3 = 8/(2 + 6) = 1\text{ A}$.

Answer: $V_{th} = 8\text{ V}$; $R_{th} = 2\ \Omega$; $I_3 = 1\text{ A}$.

AC circuits

Q1. A 60 W , 230 V lamp and a pure inductor are in series on 230 V , 50 Hz . The current is 0.2 A . Find the inductance.

Lamp $R = 230^2/60 = 881.7\ \Omega$. $Z = 230/0.2 = 1150\ \Omega$. $X_L = \sqrt{(1150^2 - 881.7^2)} = 738.3\ \Omega$.

Answer: $L = 738.3/314.16 \approx 2.35\text{ H}$.

Q2. Find the resonant frequency and Q of a circuit with $R = 5\ \Omega$, $L = 20\text{ mH}$, $C = 2\ \mu\text{F}$.

$f_0 = 1/(2\pi\sqrt{(4 \times 10^{-8})}) = 1/(2\pi \times 2 \times 10^{-4}) = 796\text{ Hz}$. $Q = (1/5)\sqrt{(0.02/(2 \times 10^{-6}))} = (1/5)(100) = 20$.

Answer: $f_0 \approx 796\text{ Hz}$; $Q = 20$.

Q3. The current in a circuit is $i = 10 \sin(314t + 60^\circ)\text{ A}$ and the voltage is $v = 100 \sin(314t + 30^\circ)\text{ V}$. Find the nature of the circuit, the pf and the power.



The current leads the voltage by $30^\circ \Rightarrow$ capacitive (R-C). $\text{pf} = \cos 30^\circ = 0.866$ leading.

$$P = (100 \times 10/2) \times 0.866 = 433 \text{ W.}$$

Answer: Capacitive; pf 0.866 leading; P = 433 W.

Q4. A 3-phase, 400 V supply feeds three 100Ω resistors in delta. Find the line current and power. What happens if one resistor opens?

$$I_{ph} = 4 \text{ A}; I_L = 6.93 \text{ A}; P = 3 \times 16 \times 100 = 4.8 \text{ kW.}$$

With one resistor open, two resistors remain across two line voltages: $P = 2 \times 1600 = 3.2 \text{ kW.}$

Answer: $I_L = 6.93 \text{ A}$; $P = 4.8 \text{ kW}$; with one open, 3.2 kW .

Q5. A 3-phase load has a pf of 0.4. If W_1 reads 5 kW, what does W_2 read (balanced load)?

$$\phi = 66.42^\circ. W_1/W_2 = \cos(30^\circ - 66.42^\circ)/\cos(30^\circ + 66.42^\circ) = \cos(-36.42^\circ)/\cos(96.42^\circ) = 0.8047/(-0.1118) = -7.2.$$

Answer: $W_2 \approx -0.69 \text{ kW}$ (negative, since $\text{pf} < 0.5$); total $\approx 4.31 \text{ kW}$.

Machines

Q1. A 230 V shunt motor with $R_a = 0.5 \Omega$ runs at 1000 rpm at no load taking $I_a = 4 \text{ A}$. What is the full-load speed if $I_a = 40 \text{ A}$ and armature reaction weakens the flux by 4%?

$$E_{b0} = 228 \text{ V}; E_{b1} = 210 \text{ V. } N = 1000 \times (210/228)/0.96 = 959 \text{ rpm.}$$

Answer: $\approx 959 \text{ rpm}$.

Q2. A transformer's full-load copper loss is 1.2 kW and its iron loss is 0.3 kW. At what fraction of full load is efficiency maximum?

$$x = \sqrt{(0.3/1.2)} = 0.5.$$

Answer: At half load.

Q3. What is the full-load slip of an induction motor whose rotor copper loss is 500 W when the air-gap power is 20 kW?

$$s = \text{rotor Cu loss}/P_g = 500/20000.$$

Answer: 2.5%.

Q4. A 3-phase, 50 Hz alternator runs at 300 rpm. How many poles does it have?

$$P = 120f/N = 6000/300.$$

Answer: 20 poles (a low-speed hydro alternator).

Q5. An induction motor has $T_{st} = 1.5 T_{FL}$ and $T_{max} = 2.5 T_{FL}$. If the full-load slip is 4%, find the slip at maximum torque (neglect stator impedance).



$$\frac{T_{FL}}{T_{max}} = \frac{2s s_m}{(s^2 + s_m^2)} = 0.4 \Rightarrow 0.4(0.0016 + s_m^2) = 0.08s_m \Rightarrow s_m^2 - 0.2s_m + 0.0016 = 0 \Rightarrow s_m = \frac{0.2 \pm \sqrt{(0.04 - 0.0064)}}{2} = 0.1917.$$

Answer: $s_m \approx 0.19$ (the root greater than $s = 0.04$).



MCQ Practice Set 2 (Conceptual)

1. The SI unit of conductance is
(a) ohm (b) farad (c) henry (d) siemens
☐ (d) — $G = 1/R$.
2. A voltmeter should have
(a) zero resistance (b) resistance equal to the load (c) very high resistance (d) very low resistance
☐ (c) — Draws negligible current.
3. An ammeter is connected
(a) across the supply (b) between line and earth (c) in parallel (d) in series
☐ (d) — Carries circuit current.
4. A shunt is used to extend the range of
(a) a voltmeter (b) an ammeter (c) a wattmeter (d) an energy meter
☐ (b) — Low resistance in parallel.
5. A multiplier is used to extend the range of
(a) a megger (b) an ammeter (c) a voltmeter (d) an ohmmeter
☐ (c) — High resistance in series.
6. PMMC instruments can measure
(a) only frequency (b) both AC and DC (c) AC only (d) DC only
☐ (d) — Deflection reverses with current.
7. Moving iron instruments have a scale that is
(a) absent (b) non-uniform (square law) (c) logarithmic (d) uniform
☐ (b) — $\text{Torque} \propto I^2$.
8. Insulation resistance is measured with a
(a) wattmeter (b) megger (c) multimeter (d) ammeter
☐ (b) — High-voltage ohmmeter.
9. The damping torque in an instrument
(a) keeps the pointer at zero (b) brings the pointer to rest quickly (c) increases sensitivity (d) moves the pointer
☐ (b) — Prevents oscillation.
10. A switch should always be connected in the
(a) phase wire (b) earth wire (c) any wire (d) neutral wire
☐ (a) — Appliance dead when off.
11. Two-way (staircase) wiring allows a lamp to be controlled from
(a) one place (b) three places (c) two places (d) nowhere
☐ (c) — Two 2-way switches.
12. The colour of the earth wire is
(a) black (b) green or green-yellow (c) blue (d) red
☐ (b) — Standard code.
13. An MCB protects against
(a) only earth leakage (b) only overload (c) overload and short circuit (d) lightning
☐ (c) — Thermal and magnetic trips.
14. An RCCB operates on the difference between
(a) phase and neutral currents (b) two phases (c) earth and neutral voltage (d) voltage and current
☐ (a) — Residual current.



- 15.** The time constant of an R-L circuit is
(a) L/R (b) RL (c) R/L (d) $1/RL$
□ (a) — $\tau = L/R$.
- 16.** The time constant of an R-C circuit is
(a) R/C (b) C/R (c) RC (d) $1/RC$
□ (c) — $\tau = RC$.
- 17.** An uncharged capacitor at the instant of switching on behaves as
(a) a short circuit (b) an open circuit (c) an inductor (d) a resistor
□ (a) — Voltage cannot change instantly.
- 18.** In DC steady state, an inductor behaves as
(a) a resistor of infinite value (b) a capacitor (c) an open circuit (d) a short circuit
□ (d) — $di/dt = 0$.
- 19.** After one time constant, a charging capacitor reaches
(a) 100% (b) 63.2% (c) 50% (d) 36.8%
□ (b) — $1 - e^{-1}$.
- 20.** The energy stored in a capacitor is
(a) $(1/2)CV^2$ (b) V/C (c) CV^2 (d) CV
□ (a) — Electrostatic energy.
- 21.** Which quantity cannot change instantaneously in an inductor?
(a) current (b) resistance (c) power (d) voltage
□ (a) — $v = L di/dt$.
- 22.** Kirchhoff's laws are applicable to
(a) both AC and DC (b) DC circuits only (c) only linear circuits (d) AC circuits only
□ (a) — Conservation laws.
- 23.** The reciprocal of impedance is
(a) conductance (b) admittance (c) susceptance (d) reactance
□ (b) — $Y = 1/Z$.
- 24.** The imaginary part of admittance is
(a) susceptance (b) conductance (c) resistance (d) reactance
□ (a) — $Y = G - jB$.
- 25.** A capacitor bank improves power factor because it
(a) increases resistance (b) consumes active power (c) reduces voltage (d) supplies leading reactive power
□ (d) — Cancels lagging VAR.
- 26.** In a purely capacitive circuit, the power factor is
(a) 0.5 (b) 0 leading (c) 1 (d) 0 lagging
□ (b) — $\phi = 90^\circ$ leading.
- 27.** If a coil is connected to DC instead of AC at the same voltage, the current
(a) increases (only R limits it) (b) stays same (c) becomes zero (d) decreases
□ (a) — $X_L = 0$ at DC.
- 28.** The phase sequence of a 3-phase supply determines the
(a) voltage (b) direction of rotation of 3-phase motors (c) power factor (d) frequency
□ (b) — Reversal reverses rotation.
- 29.** Neutral current in a balanced 3-phase star system is
(a) equal to line current (b) $\sqrt{3}$ times line current (c) zero (d) maximum
□ (c) — Currents sum to zero.



- 30.** The magnetic field inside a long solenoid is
(a) uniform (b) maximum at ends (c) zero (d) radial
□ (a) — $B = \mu_0 nI$.
- 31.** Iron is used in magnetic circuits because it has
(a) high resistance (b) low density (c) high permeability (d) low permeability
□ (c) — Low reluctance.
- 32.** Interpoles in a DC machine are used to
(a) reduce losses in the field (b) improve commutation (c) increase flux (d) increase speed
□ (b) — Neutralise reactance voltage.
- 33.** A universal motor is a
(a) series motor that runs on AC and DC (b) shunt motor (c) induction motor (d) synchronous motor
□ (a) — Series field and armature.
- 34.** The speed of a DC motor can be reversed by reversing
(a) the load (b) either the field or the armature connections (c) the supply polarity to both field and armature (d) the brushes
□ (b) — Reverse one of them.
- 35.** The no-load test on a transformer is performed on the
(a) both (b) LV side (c) HV side (d) neither
□ (b) — Rated LV voltage is convenient.
- 36.** The short-circuit test on a transformer is performed on the
(a) open (b) both shorted (c) LV side (d) HV side (LV shorted)
□ (d) — Rated HV current is small.
- 37.** An autotransformer has
(a) a single winding with a tap (b) two separate windings (c) no core (d) no winding
□ (a) — Saves copper, no isolation.
- 38.** The secondary of a current transformer should never be
(a) earthed (b) connected to an ammeter (c) shorted (d) open-circuited while primary current flows
□ (d) — Dangerous high voltage.
- 39.** The slip-ring induction motor is preferred when
(a) low cost is important (b) high starting torque is needed (c) there is no load (d) constant speed is needed
□ (b) — Rotor resistance starting.
- 40.** A synchronous motor runs at
(a) zero speed (b) above synchronous speed (c) variable speed (d) synchronous speed only
□ (d) — Locked to supply frequency.



Long-Answer Question Bank (with section references)

- State and explain Kirchhoff's laws. Solve a two/three-mesh network by mesh analysis (1.2, 1.3, 1.12).
- Explain nodal analysis with an example; what is a supernode? (1.4)
- Derive the star-delta and delta-star transformation formulas (1.5, 1.13).
- State and explain superposition, Thevenin's and Norton's theorems with examples (1.6–1.8, 1.14).
- State and prove the maximum power transfer theorem (1.9).
- Derive the rms and average values of a sinusoidal quantity; define form and peak factors (2.2, 2.15).
- Analyse a series RLC circuit with phasor diagram; discuss power and power factor (2.4, 2.5).
- Explain series resonance; derive the resonant frequency, Q-factor and bandwidth (2.7, 2.15 E).
- Derive the relation between line and phase quantities in star and delta connections (2.10, 2.11, 2.15 F).
- Explain the two-wattmeter method and derive the expression for power factor (2.13, 2.15 G).
- Compare electric and magnetic circuits; explain the B-H curve and hysteresis (3.2, 3.3).
- Explain the construction of a DC machine with a neat sketch; derive the emf equation (3.5, 3.6).
- What is back emf? Derive the torque equation of a DC motor (3.8, 3.9).
- Explain the characteristics and applications of DC shunt and series motors (3.9, 3.14).
- Describe the methods of speed control of DC shunt motors (3.10).
- Explain the construction and working of a single-phase transformer; derive its emf equation (4.1, 4.2).
- Explain the operation of a transformer on no load and on load with phasor diagrams (4.3, 4.4, 4.13).
- Define regulation and efficiency; derive the condition for maximum efficiency (4.5, 4.6).
- Explain the production of a rotating magnetic field and the working of a 3-phase induction motor (4.8, 4.14).
- Derive the torque equation of an induction motor and the condition for maximum torque; sketch the torque-slip curve (4.10, 4.14).
- Explain the construction and working principle of a 3-phase alternator (4.11, 4.15).
- Draw and explain the structure of a power system; why is high voltage used for transmission? (5.1, 5.2)
- Describe the working of thermal, hydro and nuclear power stations; compare them (5.4).
- Write notes on solar and wind energy (5.4).



Model Question Paper with Solutions

Practice pattern · Time: 3 hours · Part A: 10 short questions (2 marks each) · Part B: long questions. Follow the exact pattern announced by your university.

Note: This is a practice paper written to match the syllabus. It is not an official university question paper.

Part A

Q1. Three $9\ \Omega$ resistors are in delta. Find the equivalent star resistance.

$$9/3 = 3\ \Omega \text{ each.}$$

Q2. A source has $V_{oc} = 12\text{ V}$ and $I_{sc} = 4\text{ A}$. Find the maximum power it can deliver.

$$R_{th} = 3\ \Omega; P_{max} = 144/12 = 12\text{ W.}$$

Q3. Find the rms value of $v = 311 \sin 314t$.

$$311/\sqrt{2} = 220\text{ V.}$$

Q4. A circuit has $R = 30\ \Omega$ and $X_L = 40\ \Omega$ in series. Find its power factor.

$$Z = 50\ \Omega; \text{pf} = 30/50 = 0.6 \text{ lagging.}$$

Q5. Find the resonant frequency for $L = 1\text{ H}$ and $C = 1\ \mu\text{F}$.

$$f_0 = 1/(2\pi \times 10^{-3}) = 159\text{ Hz.}$$

Q6. A star-connected load draws 10 A from a 400 V supply at 0.8 pf . Find the power.

$$P = \sqrt{3} \times 400 \times 10 \times 0.8 = 5.54\text{ kW.}$$

Q7. Find the emf of a 4-pole lap-wound generator with $\Phi = 0.01\text{ Wb}$, $Z = 600$, $N = 1000\text{ rpm}$.

$$E = 4 \times 0.01 \times 600 \times 1000/(60 \times 4) = 100\text{ V.}$$

Q8. A transformer has 200 primary and 50 secondary turns. The primary voltage is 240 V . Find the secondary voltage.

$$240 \times 50/200 = 60\text{ V.}$$

Q9. Find the slip of a 6-pole, 50 Hz induction motor running at 950 rpm .

$$N_s = 1000\text{ rpm}; s = 0.05.$$

Q10. Why is electrical power transmitted at high voltage?

To reduce current and hence I^2R loss and voltage drop.

Part B

Problem 1. Using Thevenin's theorem, find the current in the $4\ \Omega$ load of the network of Figure 1.3 (24 V , $6\ \Omega$, $3\ \Omega$, $2\ \Omega$). Verify by mesh analysis.

$$V_{th} = 8\text{ V}; R_{th} = 4\ \Omega; I = 8/8 = 1\text{ A (Section 1.12)}.$$

$$\text{Mesh: } 9I_1 - 3I_2 = 24, -3I_1 + 9I_2 = 0 \Rightarrow I_2 = 1\text{ A} \square.$$

Answer: 1 A.

Problem 2. A series RLC circuit with $R = 10\ \Omega$, $L = 0.1\text{ H}$, $C = 100\ \mu\text{F}$ is connected to 200 V , 50 Hz . Find the current, power factor, power and the resonant frequency.

$$X_L = 31.42\ \Omega; X_C = 31.83\ \Omega; X = -0.41\ \Omega; Z = 10.01\ \Omega. I = 19.98\text{ A}; \text{pf} \approx 0.999 \text{ leading}; P = I^2 R \approx 3.99\text{ kW}.$$

$$f_0 = 1/(2\pi\sqrt{0.1 \times 10^{-4}}) = 50.3\text{ Hz}.$$

Answer: $I \approx 20\text{ A}$; $\text{pf} \approx 1$; $P \approx 4\text{ kW}$; $f_0 \approx 50.3\text{ Hz}$ (the circuit is almost at resonance).

Problem 3. Two wattmeters connected to measure the input to a balanced 3-phase load read 10 kW and 5 kW . Find the power, power factor, and the line current if $V_L = 400\text{ V}$.

$$P = 15\text{ kW}; \tan \phi = \sqrt{3} \times 5/15 = 0.577 \Rightarrow \phi = 30^\circ, \text{pf} = 0.866.$$

$$I_L = 15000/(\sqrt{3} \times 400 \times 0.866) = 25\text{ A}.$$

Answer: 15 kW ; $\text{pf } 0.866$ lag; $I_L = 25\text{ A}$.

Problem 4. A 250 V shunt motor has $R_a = 0.5\ \Omega$ and $R_{sh} = 250\ \Omega$. It takes 21 A at 1000 rpm . Find the speed when it takes 41 A (same flux), and the torque in each case.

$$I_{a1} = 20\text{ A}, E_{b1} = 240\text{ V}; I_{a2} = 40\text{ A}, E_{b2} = 230\text{ V}. N_2 = 1000 \times 230/240 = 958\text{ rpm}.$$

$$T_1 = 9.55 \times 240 \times 20/1000 = 45.8\text{ N m}; T_2 = 9.55 \times 230 \times 40/958 = 91.7\text{ N m} (= 2 \times T_1, \text{ since } T \propto I_a).$$

Answer: $N_2 \approx 958\text{ rpm}$; $T_1 \approx 45.8\text{ N m}$, $T_2 \approx 91.7\text{ N m}$.

Problem 5. A 50 kVA , $2200/220\text{ V}$ transformer has iron loss 500 W and full-load copper loss 800 W . Find the efficiency at full load and half load at 0.8 pf , and the load for maximum efficiency.

$$\text{FL: } 40000/(40000 + 1300) = 96.85\%. \text{ HL: } 20000/(20000 + 500 + 200) = 96.62\%.$$

$$\eta_{\max} \text{ at } x = \sqrt{(500/800)} = 0.79 \Rightarrow 39.5\text{ kVA}.$$

Answer: 96.85% , 96.62% ; $\eta_{\max}$ at $\approx 39.5\text{ kVA}$.

Problem 6. A 4-pole, 50 Hz , 3-phase induction motor has $R_2 = 0.1\ \Omega$ and $X_2 = 0.5\ \Omega$ per phase. Find the speed at maximum torque and $T_{st}/T_{\max}$.

$$s_m = 0.2 \Rightarrow N = 1500 \times 0.8 = 1200\text{ rpm}. T_{st}/T_{\max} = 0.4/1.04 = 0.385.$$

Answer: 1200 rpm ; 0.385 .



Formula Sheet (Last-Minute Revision)

DC networks

$V = IR$; $P = VI = I^2R = V^2/R$; $R = \rho l/A$; $R_t = R_0(1 + \alpha_0 t)$
 Series $R = \Sigma R$; parallel $1/R = \Sigma 1/R$; divider $V_1 = VR_1/(R_1 + R_2)$; $I_1 = IR_2/(R_1 + R_2)$
 $\Delta \rightarrow Y$: $R_A = R_{AB}R_{CA}/\Sigma$; $Y \rightarrow \Delta$: $R_{AB} = R_A + R_B + R_AR_B/R_C$; balanced $R_Y = R_\Delta/3$
 Thevenin $I_L = V_{th}/(R_{th} + R_L)$; Norton $I_L = I_N R_N/(R_N + R_L)$; $R_{th} = V_{oc}/I_{sc}$
 Max power: $R_L = R_{th}$; $P_{max} = V_{th}^2/4R_{th}$; $\eta = 50\%$

AC circuits

$I_{rms} = I_m/\sqrt{2}$; $I_{av} = 2I_m/\pi$; form factor 1.11; peak factor 1.414
 $X_L = 2\pi fL$; $X_C = 1/(2\pi fC)$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$; $\cos \phi = R/Z$
 $P = VI \cos \phi$; $Q = VI \sin \phi$; $S = VI = \sqrt{P^2 + Q^2}$
 Series resonance $f_0 = 1/(2\pi\sqrt{LC})$; $Z = R$; $Q = \omega_0 L/R = (1/R)\sqrt{L/C}$; $BW = R/(2\pi L)$
 Parallel resonance $f_0 = (1/2\pi)\sqrt{1/LC - R^2/L^2}$; $Z_D = L/CR$
 Star $V_L = \sqrt{3}V_{ph}$, $I_L = I_{ph}$; Delta $V_L = V_{ph}$, $I_L = \sqrt{3}I_{ph}$; $P = \sqrt{3}V_L I_L \cos \phi$
 Two wattmeters: $P = W_1 + W_2$; $\tan \phi = \sqrt{3}(W_1 - W_2)/(W_1 + W_2)$
 pf correction $Q_C = P(\tan \phi_1 - \tan \phi_2)$; $C = Q_C/(\omega V^2)$

Magnetic circuits and DC machines

$e = -N d\Phi/dt$; $e = Blv$; $F = BIl$; $L = N\Phi/I = N^2/S$; $M = k\sqrt{L_1 L_2}$; $W = (1/2)LI^2$
 $S = l/(\mu_0 \mu_r A)$; $\Phi = NI/S$; $B = \mu_0 \mu_r H$; $H = NI/l$
 $P_h = \eta B_m^{1.6} fV$; $P_e \propto B_m^2 f^2 t^2$
 $E = P\Phi ZN/60A$ ($A = P$ lap, 2 wave); generator $E_g = V + I_a R_a$; motor $V = E_b + I_a R_a$
 $T_a = 0.159P\Phi ZI_a/A$; $T = 9.55 P/N$; $N \propto E_b/\Phi$; shunt $T \propto I_a$; series $T \propto I_a^2$

AC machines

$E = 4.44fN\Phi_m$; $K = N_2/N_1 = V_2/V_1 = I_1/I_2$
 $R_{02} = R_2 + K^2 R_1$; $X_{02} = X_2 + K^2 X_1$; %Reg $\approx I_2(R_{02} \cos \phi \pm X_{02} \sin \phi)/E_2 \times 100$
 $\eta = xS \cos \phi / (xS \cos \phi + P_i + x^2 P_{cu})$; η_{max} when $x^2 P_{cu} = P_i$
 $N_s = 120f/P$; $s = (N_s - N)/N_s$; $f_r = sf$; $E_{2r} = sE_2$
 $T = ksE_2^2 R_2 / (R_2^2 + s^2 X_2^2)$; $s_m = R_2/X_2$; $T_{max} = kE_2^2/2X_2$; $T \propto V^2$
 P_g : rotor Cu loss : $P_{mech} = 1 : s : (1 - s)$
 Alternator $f = PN/120$; $E_{ph} = 4.44f\Phi T_{ph} k_p k_d$

Power systems

Line loss $\propto 1/V^2$; $I = P/(\sqrt{3}V \cos \phi)$
 Load factor = average/maximum demand; demand factor = max demand/connected load; diversity factor = Σ individual max demands/group max demand
 Hydro $P = \rho gQH\eta$; wind $P = (1/2)\rho A v^3 C_p$





Glossary of Important Terms

Term	Meaning
Active power	Real power consumed, $P = VI \cos \phi$, in watts
Admittance	Reciprocal of impedance, $Y = 1/Z$, in siemens
Alternator	Synchronous generator producing AC
Apparent power	Product of rms voltage and current, $S = VI$, in VA
Armature	The part of a machine in which emf is induced (or which carries the load current)
Back emf	Emf induced in a motor armature, opposing the supply voltage
Bandwidth	Frequency range between the half-power points of a resonant circuit
Branch	Part of a network between two nodes
Brush	Carbon block making sliding contact with the commutator or slip rings
Coercivity	Reverse magnetising force needed to reduce flux density to zero
Commutator	Segmented copper cylinder that rectifies the armature emf in DC machines
Critical field resistance	Maximum field-circuit resistance with which a shunt generator will build up voltage
Demand factor	Maximum demand divided by connected load
Diversity factor	Sum of individual maximum demands divided by maximum demand of the group
Dynamic impedance	Impedance of a parallel resonant circuit at resonance, L/CR
Eddy currents	Circulating currents induced in a conducting core by changing flux
Efficiency	Ratio of output power to input power
Feeder	Conductor carrying power from a substation to the distribution area, without tappings
Form factor	Ratio of rms value to average value (1.11 for a sine wave)
Hysteresis	Lagging of flux density behind magnetising force in a ferromagnetic material
Impedance	Total opposition to AC, $Z = R + jX$, in ohms
Load factor	Average load divided by maximum demand
Magnetomotive force	The "driving force" of a magnetic circuit, NI (ampere-turns)
Mesh	A loop containing no other loop inside it
Node	A junction of two or more circuit elements



Term	Meaning
Peak factor	Ratio of peak value to rms value (1.414 for a sine wave)
Phase sequence	Order in which the phase voltages reach their maxima (R-Y-B)
Power factor	Cosine of the angle between voltage and current, P/S
Q-factor	Ratio of reactance to resistance at resonance; measures selectivity
Reactance	Opposition to AC by inductance ($X_L = \omega L$) or capacitance ($X_C = 1/\omega C$)
Reactive power	Power oscillating between source and reactive elements, $Q = VI \sin \phi$, in VAR
Regulation (transformer)	Change in secondary voltage from no load to full load, as % of no-load voltage
Reluctance	Opposition of a magnetic circuit to flux, $l/(\mu A)$
Residual magnetism	Flux density remaining after the magnetising force is removed
Resonance	Condition in which $X_L = X_C$, making the circuit purely resistive
RMS value	Effective value of an alternating quantity; the equivalent DC heating value
Rotating magnetic field	Constant-magnitude field rotating at synchronous speed, produced by 3-phase currents
Slip	Fractional difference between synchronous speed and rotor speed of an induction motor
Slip ring	Continuous ring used to make sliding connection to a rotating winding
Starter	Device that limits the starting current of a motor
Synchronous speed	Speed of the rotating magnetic field, $N_s = 120f/P$
Time constant	Time for a transient quantity to complete 63.2% of its change (L/R or RC)
Transformation ratio	Ratio of secondary to primary turns (or voltages), $K = N_2/N_1$